GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

June 2006

3895-8/7895-8/MS/R/06

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MARK SCHEME ON THE UNITS

Mark Scheme 4751
June 2006

Section A

1	$[r]=[\pm]\sqrt{\dfrac{3V}{\pi h}}$ o.e. 'double-decker'	3	2 for $r^2=\dfrac{3V}{\pi h}$ or $r=\sqrt{\dfrac{V}{\frac{1}{3}\pi h}}$ o.e. or M1 for correct constructive first step or for $r=\sqrt{k}$ ft their $r^2=k$	3
2	$a = ¼$	2	M1 for subst of –2 or for $-8 + 4a + 7 = 0$ o.e. obtained eg by division by $(x + 2)$	2
3	$3x + 2y = 26$ or $y = -1.5x + 13$ isw	3	M1 for $3x + 2y = c$ or $y = -1.5x + c$ M1 for subst (2, 10) to find c or for or for $y - 10$ = their gradient × $(x - 2)$	3
4	(i) P $\Leftarrow$ Q (ii) P $\Leftrightarrow$ Q	1 1	condone omission of P and Q	2
5	$x + 3(3x + 1) = 6$ o.e. $10x = 3$ or $10y = 19$ o.e. (0.3, 1.9) or $x = 0.3$ and $y = 1.9$ o.e.	M1 A1 A1	for subst or for rearrangement and multn to make one pair of coefficients the same or for both eqns in form '$y =$' (condone one error) graphical soln: (must be on graph paper) M1 for each line, A1 for (0.3, 1.9) o.e cao; allow B3 for (0.3, 1.9) o.e.	3
6	$-3 < x < 1$ [condone $x < 1$, $x > -3$]	4	B3 for –3 and 1 or M1 for $x^2 + 2x - 3$ [< 0]or $(x + 1)^2 <$ / $= 4$ and M1 for $(x + 3)(x - 1)$ or $x= (-2 \pm 4)/2$ or for $(x + 1)$ and ±2 on opp. sides of eqn or inequality; if 0, then SC1 for one of $x < 1$, $x > -3$	4
7	(i) $28\sqrt{6}$ (ii) $49 - 12\sqrt{5}$ isw	2 3	1 for $30\sqrt{6}$ or $2\sqrt{6}$ or $2\sqrt{2}\sqrt{3}$ or $28\sqrt{2}\sqrt{3}$ 2 for 49 and 1 for $- 12\sqrt{5}$ or M1 for 3 correct terms from $4 - 6\sqrt{5} - 6\sqrt{5} + 45$	5
8	20 –160 or ft for –8 × their 20	2 2	0 for just 20 seen in second part; M1 for $6!/(3!3!)$ or better condone $-160x^3$; M1 for $[-]2^3$ × [their] 20 seen or for [their] 20 × $(-2x)^3$; allow B1 for 160	4
9	(i) 4/27 (ii) $3a^{10}b^8c^{-2}$ or $\dfrac{3a^{10}b^8}{c^2}$	2 3	1 for 4 or 27 2 for 3 'elements' correct, 1 for 2 elements correct, −1 for any adding of elements; mark final answer; condone correct but unnecessary brackets	5
10	$x^2 + 9x^2 = 25$ $10x^2 = 25$ $x= \pm(\sqrt{10})/2$ or $\pm\sqrt{(5/2)}$ or $\pm5/\sqrt{10}$ oe $y = [\pm]\ 3\sqrt{(5/2)}$ o.e. eg $y = [\pm]\ \sqrt{22.5}$	M1 M1 A2 B1	for subst for x or y attempted or $x^2 = 2.5$ o.e.; condone one error from start [allow $10x^2 - 25 = 0$ + correct substn in correct formula] allow $\pm\sqrt{2.5}$; A1 for one value ft 3 × their x value(s) if irrational; condone not written as coords.	5

Section B

11	**i**	grad AB = 8/4 or 2 or $y = 2x - 10$	1	or M1 for $AB^2 = 4^2 + 8^2$ or 80 and $BC^2 = 2^2 + 1^2$ or 5 and $AC^2 = 6^2 + 7^2$ or 85; M1 for $AC^2 = AB^2 + BC^2$ and 1 for [Pythag.] true so AB perp to BC; if 0, allow G1 for graph of A, B, C	
		grad BC = 1/–2 or – ½ or $y = -\frac{1}{2}x + 2.5$	1		
		product of grads = –1 [so perp] (allow seen or used)	1		3
	ii	midpt E of AC = (6, 4.5)	1		
		$AC^2 = (9 - 3)^2 + (8 - 1)^2$ or 85	M1	allow seen in (i) only if used in (ii); or $AE^2 = (9 - \text{their } 6)^2 + (8 - \text{their } 4.5)^2$ or	
		rad = ½ √85 o.e.	A1	rad.^{2} = 85/4 o.e. e.g. in circle eqn	
		$(x - 6)^2 + (y - 4.5)^2 = 85/4$ o.e.	B2	M1 for $(x - a)^2 + (y - b)^2 = r^2$ soi or for lhs correct	
		$(5-6)^2 + (0-4.5)^2 = 1 + 81/4$ [= 85/4]	1	some working shown; or 'angle in semicircle [=90°]'	6
	iii	$\overrightarrow{BE} = \overrightarrow{ED} = \begin{pmatrix} 1 \\ 4.5 \end{pmatrix}$	M1	o.e. ft their centre; or for $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$	
		D has coords (6 + 1, 4.5 + 4.5) ft or (5 + 2, 0 + 9)	M1	or (9 − 2, 8 + 1); condone mixtures of vectors and coords. throughout part iii	
		= (7, 9)	A1	allow B3 for (7,9)	3
12	**i**	f(–2) used	M1	or M1 for division by $(x + 2)$ attempted as far as $x^3 + 2x^2$ then A1 for $x^2 + 7x + 6$ with no remainder	
		–8 + 36 – 40 + 12 = 0	A1		2
	ii	divn attempted as far as $x^2 + 3x$	M1	or inspection with $b = 3$ or $c = 2$ found;	
		$x^2 + 3x + 2$ or $(x + 2)(x + 1)$	A1	B2 for correct answer	2
	iii	$(x + 2)(x + 6)(x + 1)$	2	allow seen earlier; M1 for $(x + 2)(x + 1)$	2
	iv	sketch of cubic the right way up	G1	with 2 turning pts; no 3rd tp	
		through 12 marked on y axis	G1	curve must extend to $x > 0$	
		intercepts –6, –2, –1 on x axis	G1	condone no graph for $x < -6$	3
	v	$[x](x^2 + 9x + 20)$	M1	or other partial factorisation	
		$[x](x + 4)(x + 5)$	M1		
		$x = 0, -4, -5$	A1	or B1 for each root found e.g. using factor theorem	3
13	**i**	$y = 2x + 3$ drawn on graph	M1		
		$x = 0.2$ to 0.4 and -1.7 to -1.9	A2	1 each; condone coords; must have line drawn	3
	ii	$1 = 2x^2 + 3x$	M1	for multiplying by x correctly	
		$2x^2 + 3x - 1$ [= 0]	M1	for correctly rearranging to zero (may be earned first) or suitable step re completing square if they go on	
		attempt at formula or completing square	M1	ft, but no ft for factorising	
		$x = \dfrac{-3 \pm \sqrt{17}}{4}$	A2	A1 for one soln	5
	iii	branch through (1,3),	1	and approaching $y = 2$ from above	
		branch through (−1,1),approaching $y = 2$ from below	1	and extending below x axis	2
	iv	–1 and ½ or ft intersection of their curve and line [tolerance 1 mm]	2	1 each; may be found algebraically; ignore y coords.	2

Mark Scheme 4752
June 2006

Section A

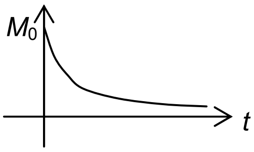
1	1, 3	1,1		2	17
2	$r = 0.2$	3	M1 for 10 = 8/(1 – r), then M1 dep't for any correct step	3	
3	1/√15 i.s.w. not +/–	3	M2 for √15 seen M1 for rt angled triangle with side 1 and hyp 4, or $\cos^2\theta = 1 - 1/4^2$.	3	
4	$x^5/5 - 3x^{-1}/-1 + x$ [value at 2 – value at 1] attempted 5.7 c.a.o.	B3 M1 A1	1 each term dep't on B2	5	
5	[y =] $3x - x^3/3$ + c subst of (6, 1) in their eqn with c $y = 3x - x^3/3 + 55$ c.a.o	B1 B1 M1 A1	Dep't on integration attempt Dep't on B0B1 Allow $c = 55$ isw	4	
6	(i) 3, 8, 13, 18 (ii) use of $n/2[2a + (n-1)d]$ (S_{100} =) 25 050 or (S_{50} =) 6275 (S_{49} =) 6027 or (S_{51} =) 6528 their($S_{100} - S_{50}$) dep't on M1 18 775 cao	B1 M1 A1 M1 A1	Ignore extras Use of $a + (n-1)d$ $u_{51} = 253$ $u_{100} = 498$ $u_{50} = 248$ $u_{52} = 258$ 50/2(their($u_{51} + u_{100}$)) dep't on M1 or 50/2[2 × their(u_{51}) + 49 × 5]	5	19
7	(i) sketch of correct shape correct period and amplitude period halved for $y = \cos 2x$; amplitude unchanged (ii) 30, 150, 210, 330	G1 G1 G1 B2	Not ruled lines need 1 and –1 indicated; nos. on horiz axis not needed if one period shown B1 for 2 of these, ignore extras outside range.	5	
8	$\sqrt{x} = x^{\frac{1}{2}}$ soi $18x^2$, $\frac{1}{2}x^{-\frac{1}{2}}$ $36x$ $Ax^{-3/2}$ (from $Bx^{-\frac{1}{2}}$)	B1 B1B1 B1 B1	–1 if d/dx(3) not = 0 any A,B	5	
9	$3x \log 5 = \log 100$ $3x = \log 100/\log 5$ $x = 0.954$	M1 M1 A2	allow any or no base or $3x = \log_5 100$ dep't A1 for other rot versions of 0.9537... SC B2/4 for 0.954 with <u>no</u> log wkg SC B1 r.o.t. 0.9537...	4	

Section B

10	**i** (*A*)	$5.2^2 + 6.3^2 - 2\times 5.2 \times 6.3 \times \cos$ "57"	M2	M1 for recognisable attempt at cos rule. or greater accuracy		
		ST = 5.6 or 5.57 cao	A1		3	
	i (*B*)	sin T/5.2 = sin(their 57)/their ST	M1	Or sin S/6.3 = … or cosine rule		
		T=51 to 52 or S = 71 to 72	A1			
		bearing 285 + their T	B1	If outside 0 to 360, must be adjusted		
		or 408 – their S			3	
	ii	5.2θ, $24 \times 26/60$	B1B1			
		$\theta = 1.98$ to 2.02	B1	Lost for all working in degrees		
		$\theta =$ their $2 \times 180/\pi$ or 114.6°...	M1	Implied by 57.3		
		Bearing = 293 to 294 cao	A1		5	11
11	**i**	$y' = 3x^2 - 6x$	B1	condone one error		
		use of $y' = 0$	M1			
		(0, 1) or (2, –3)	A2	A1 for one correct or $x = 0$, 2 SC B1 for (0,1) from their y'		
		sign of y'' used to test or y' either side	T1	Dep't on M1 or y either side or clear cubic sketch	5	
	ii	$y'(-1) = 3 + 6 = 9$	B1			
		$3x^2 - 6x = 9$	M1	ft for their y'		
		$x = 3$	A1	implies the M1		
		At P $y = 1$	B1			
		grad normal = –1/9 cao	B1			
		$y - 1 = -1/9\ (x - 3)$	M1	ft their (3, 1) and their grad, not 9		
		intercepts 12 and 4/3or use of $\int_0^{12} \frac{4}{3} - \frac{1}{9}x$ dx (their normal)	B1	ft their normal (linear)		
		½ × 12 × 4/3 cao	A1		8	13
12	**i**	$\log_{10} P = \log_{10} a + \log_{10} 10^{bt}$	B1	condone omission of base		
		$\log_{10} 10^{bt} = bt$	B1			
		intercept indicated as $\log_{10} a$	B1		3	
	ii	3.9(0), 3.94, 4(.00), 4.05, 4.11	T1	to 3 sf or more; condone one error		
		plots ft	P1	1 mm		
		line of best fit ft	L1	ruled and reasonable	3	
	iii	(gradient =) 0.04 to 0.06 seen	M1			
		(intercept =) 3.83 to 3.86 seen	M1			
		(a =) 6760 to 7245 seen	A1			
		$P = 7000 \text{ x } 10^{0.05t}$ oe	A1	7000×1.12^t SC $P = 10^{0.05t + 3.85}$ left A2	4	
	iv	17 000 to 18 500	B2	14 000 to 22 000 B1	2	12

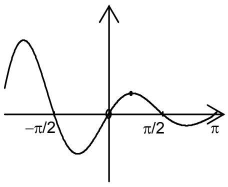
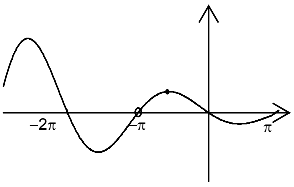
Mark Scheme 4753
June 2006

1 $\lvert 3x-2\rvert = x$ ⇒ $3x - 2 = x \Rightarrow 2x = 2 \Rightarrow x = 1$ or $2 - 3x = x \Rightarrow 2 = 4x \Rightarrow x = ½$ *or* $(3x - 2)^2 = x^2$ $\Rightarrow 8x^2 - 12x + 4 = 0 \Rightarrow 2x^2 - 3x + 1 = 0$ $\Rightarrow (x - 1)(2x - 1) = 0,$ ⇒ $x = 1, ½$	B1 M1 A1 M1 A1 A1 [3]	$x = 1$ solving correct quadratic
2 let $u = x$, $dv/dx = \sin 2x \Rightarrow v = -½\cos 2x$ $\Rightarrow \int_0^{\pi/6} x\sin 2x dx = \left[x.-\frac{1}{2}\cos 2x\right]_0^{\pi/6} + \int_0^{\pi/6}\frac{1}{2}\cos 2x.1.dx$ $= \frac{\pi}{6}.-\frac{1}{2}\cos\frac{\pi}{3} - 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ $= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$ $= \frac{3\sqrt{3}-\pi}{24}$ *	M1 A1 B1ft M1 B1 E1 [6]	parts with $u = x$, $dv/dx = \sin 2x$ $... + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ substituting limits cos π/3 = ½ , sin π/3 = √3/2 soi www
3 (i) $x - 1 = \sin y$ ⇒ $x = 1 + \sin y$ ⇒ $dx/dy = \cos y$ **(ii)** When $x = 1.5$, $y = \arcsin(0.5) = \pi/6$ $\frac{dy}{dx} = \frac{1}{\cos y}$ $= \frac{1}{\cos \pi/6}$ $= 2/\sqrt{3}$	M1 A1 E1 M1 A1 M1 A1 [7]	 www condone 30° or 0.52 or better or $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-1)^2}}$ or equivalent, but must be exact
4(i) $V = \pi h^2 - \frac{1}{3}\pi h^3$ $\Rightarrow \frac{dV}{dh} = 2\pi h - \pi h^2$ **(ii)** $\frac{dV}{dt} = 0.02$ $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = \frac{0.02}{dV/dh} = \frac{0.02}{2\pi h - \pi h^2}$ When $h = 0.4$, $\Rightarrow \frac{dh}{dt} = \frac{0.02}{0.8\pi - 0.16\pi} = 0.0099\,\text{m/min}$	M1 A1 B1 M1 M1dep A1cao [6]	expanding brackets (correctly) or product rule oe soi $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ oe substituting $h = 0.4$ into their $\frac{dV}{dh}$ and $\frac{dV}{dt} = 0.02$ 0.01 or better or 1/32π

5(i)	$a^2 + b^2 = (2t)^2 + (t^2 - 1)^2$	M1	substituting for *a*, *b* and *c* in terms of *t*
	$= 4t^2 + t^4 - 2t^2 + 1$		Expanding brackets correctly
	$= t^4 + 2t^2 + 1$	M1	www
	$= (t^2 + 1)^2 = c^2$	E1	
(ii)	$c = \sqrt{(20^2 + 21^2)} = 29$	B1	
	For example:		Attempt to find *t*
	$2t = 20 \Rightarrow t = 10$	M1	Any valid argument
$\Rightarrow$	$t^2 - 1 = 99$ which is not consistent with 21	E1	or E2 'none of 20, 21, 29 differ by two'.
		[6]	
6 (i)		B1	Correct shape
		B1	Passes through (0, M_0)
(ii)	$\frac{M}{M_0} = e^{-0.000121\times5730} = e^{-0.6933...} \approx \frac{1}{2}$	M1	substituting $k = -0.00121$ and $t = 5730$ into equation (or ln eqn)
		E1	showing that $M \approx ½ M_0$
(iii)	$\frac{M}{M_0} = e^{-kT} = \frac{1}{2}$	M1	substituting $M/M_0 = ½$ into equation (oe)
	$\Rightarrow \ln\frac{1}{2} = -kT$	M1	taking lns correctly
	$\Rightarrow \ln 2 = kT$		
	$\Rightarrow T = \frac{\ln 2}{k} *$	E1	
(iv)	$T = \frac{\ln 2}{2.88\times10^{-5}} \approx 24\,000 \text{ years}$	B1	24 000 or better
		[8]	

Section B

7(i) $x = 1$	B1 [1]	
(ii) $\frac{dy}{dx} = \frac{(x-1)2x-(x^2+3).1}{(x-1)^2}$	M1	Quotient rule
$= \frac{2x^2-2x-x^2-3}{(x-1)^2}$	A1	correct expression
$= \frac{x^2-2x-3}{(x-1)^2}$		
$\mathrm{d}y/\mathrm{d}x = 0$ when $x^2 - 2x - 3 = 0$	M1	their numerator = 0
$\Rightarrow (x-3)(x+1) = 0$	M1	solving quadratic by any valid method
$\Rightarrow x = 3$ or -1	A1	$x = 3$ from correct working
When $x = 3$, $y = (9 + 3)/2 = 6$	B1ft	$y = 6$
So P is (3, 6)	[6]	
(iii) Area $= \int_2^3 \frac{x^2+3}{x-1} dx$	M1	Correct integral and limits
$u = x - 1 \Rightarrow \mathrm{d}u/\mathrm{d}x = 1$, $\mathrm{d}u = \mathrm{d}x$ When $x = 2$, $u = 1$; when $x = 3$, $u = 2$	B1	Limits changed, and substituting $\mathrm{d}x = \mathrm{d}u$
$= \int_1^2 \frac{(u+1)^2+3}{u} du$	B1	substituting $\frac{(u+1)^2+3}{u}$
$= \int_1^2 \frac{u^2+2u+4}{u} du$		
$= \int_1^2 (u+2+\frac{4}{u})du$ *	E1	www
$= \left[\frac{1}{2}u^2+2u+4\ln u\right]_1^2$	B1	[½ $u^2 + 2u + 4\ln u$]
= (2 + 4 + 4ln2) − (½ + 2 + 4ln1)	M1	substituting correct limits
= 3½ + 4ln2	A1cao	
	[7]	
(iv) $e^y = \frac{x^2+3}{x-1}$		
$\Rightarrow \quad e^y \frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$	M1	$e^y \mathrm{d}y/\mathrm{d}x$ = their $f'(x)$ or $xe^y - e^y = x^2 + 3$
$\Rightarrow \quad \frac{dy}{dx} = e^{-y}\frac{x^2-2x-3}{(x-1)^2}$	A1ft	$\Rightarrow e^y + xe^y \frac{dy}{dx} - e^y \frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x-e^y}{e^y(x-1)}$
When $x = 2$, $e^y = 7 \Rightarrow$	B1	y = ln 7 or 1.95… or $e^y = 7$
$\Rightarrow \quad \mathrm{d}y/\mathrm{d}x = \frac{1}{7}.\frac{4-4-3}{1} = -\frac{3}{7}$	A1cao	or $\frac{dy}{dx} = \frac{4-7}{7(2-1)} = -\frac{3}{7}$ or −0.43 or better
	[4]	

8 (i) (*A*)  (*B*) 	B1 B1 M1 A1 [4]	Zeros shown every π/2. Correct shape, from –π to π Translated in *x*-direction π to the left
(ii) $f'(x) = -\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x$ $f'(x) = 0$ when $-\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x = 0$ $\Rightarrow \frac{1}{5}e^{-\frac{1}{5}x}(-\sin x + 5\cos x) = 0$ $\Rightarrow \sin x = 5 \cos x$ $\Rightarrow \frac{\sin x}{\cos x} = 5$ $\Rightarrow \tan x = 5$* $\Rightarrow x = 1.37(34\ldots)$ $\Rightarrow y = 0.75$ or $0.74(5\ldots)$	B1 B1 M1 E1 B1 B1 [6]	$e^{-\frac{1}{5}x}\cos x$ $\ldots -\frac{1}{5}e^{-\frac{1}{5}x}\sin x$ dividing by $e^{-\frac{1}{5}x}$ www 1.4 or better, must be in radians 0.75 or better
(iii) $f(x+\pi) = e^{-\frac{1}{5}(x+\pi)}\sin(x+\pi)$ $= e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin(x+\pi)$ $= -e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin x$ $= -e^{-\frac{1}{5}\pi}f(x)$* $\int_{\pi}^{2\pi} f(x)dx$ let $u = x - \pi$, $du = dx$ $= \int_0^{\pi} f(u+\pi)du$ $= \int_0^{\pi} -e^{-\frac{1}{5}\pi}f(u)du$ $= -e^{-\frac{1}{5}\pi}\int_0^{\pi} f(u)du$* Area enclosed between π and 2π $= (-)e^{-\frac{1}{5}\pi} \times$ area between 0 and π.	M1 A1 A1 E1 B1 B1dep E1 B1 [8]	$e^{-\frac{1}{5}(x+\pi)} = e^{-\frac{1}{5}x}.e^{-\frac{1}{5}\pi}$ $\sin(x+\pi) = -\sin x$ www $\int f(u+\pi)du$ limits changed using above result or repeating work or multiplied by 0.53 or better

Mark Scheme 4754
June 2006

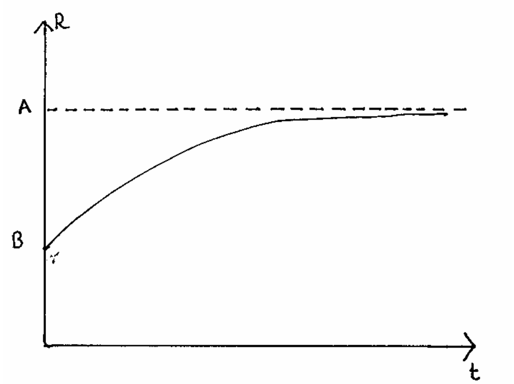
1 $\sin x - \sqrt{3}\cos x = R\sin(x-\alpha)$		
$= R(\sin x\cos\alpha - \cos x\sin\alpha)$		
$\Rightarrow$ $R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$		
$\Rightarrow$ $R^2 = 1^2 + (\sqrt{3})^2 = 4$, $R = 2$	B1	$R = 2$
$\tan\alpha = \sqrt{3}/1 = \sqrt{3} \Rightarrow \alpha = \pi/3$	M1	$\tan\alpha = \sqrt{3}$ or $\sin\alpha=\sqrt{3}$/their R or $\cos\alpha=1$/their R
	A1	$\alpha = \pi/3$, 60° or 1.05 (or better) radians www
$\Rightarrow \sin x - \sqrt{3}\cos x = 2\sin(x-\pi/3)$	M1	Using x-their $\alpha=\pi/2$or 90° $\alpha\neq 0$
x coordinate of P is when $x - \pi/3 = \pi/2$	A1ft	exact radians only (not $\pi/2$)
$\Rightarrow x = 5\pi/6$	B1ft	their R (exact only)
$y = 2$		
So coordinates are $(5\pi/6, 2)$	[6]	
2(i) $\frac{3+2x^2}{(1+x)^2(1-4x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{1-4x}$		
$\Rightarrow$ $3+2x^2 = A(1+x)(1-4x) + B(1-4x) + C(1+x)^2$	M1	Clearing fractions (or any 2 correct equations)
$x = -1 \Rightarrow 5 = 5B \Rightarrow B = 1$	B1	$B = 1$ www
$x = ¼ \Rightarrow 3\frac{1}{8} = \frac{25}{16}C \Rightarrow C = 2$	B1	$C = 2$ www
coeff^t of x^2: $2 = -4A + C \Rightarrow A = 0$	E1	$A = 0$ needs justification
.	[4]	
(ii) $(1+x)^{-2} = 1 + (-2)x + (-2)(-3)x^2/2! + \ldots$	M1	Binomial series (coefficients unsimplified - for either)
$= 1 - 2x + 3x^2 + \ldots$	A1	
$(1-4x)^{-1} = 1 + (-1)(-4x) + (-1)(-2)(-4x)^2/2! + \ldots$		
$= 1 + 4x + 16x^2 + \ldots$	A1	
$\frac{3+2x^2}{(1+x)^2(1-4x)} = (1+x)^{-2} + 2(1-4x)^{-1}$		or $(3+2x^2)(1+x)^{-2}(1-4x)^{-1}$ expanded
$\approx 1 - 2x + 3x^2 + 2(1 + 4x + 16x^2)$		
$= 3 + 6x + 35x^2$	A1ft	theirA,B,C and their expansions
	[4]	
3 $\sin(\theta + \alpha) = 2\sin\theta$		Using correct Compound angle formula
$\Rightarrow$ $\sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\sin\theta$	M1	in a valid equation
$\Rightarrow$ $\tan\theta\cos\alpha + \sin\alpha = 2\tan\theta$	M1	dividing by $\cos\theta$
$\Rightarrow$ $\sin\alpha = 2\tan\theta - \tan\theta\cos\alpha$		
$= \tan\theta\ (2 - \cos\alpha)$	M1	collecting terms in $\tan\theta$ or $\sin\theta$ or dividing by $\tan\theta$ oe
$\Rightarrow$ $\tan\theta = \frac{\sin\alpha}{2-\cos\alpha}$ *	E1	www (can be all achieved for the method in reverse)
$\sin(\theta + 40°) = 2\sin\theta$		
$\Rightarrow$ $\tan\theta = \frac{\sin 40}{2-\cos 40} = 0.5209$	M1	
$\Rightarrow$ $\theta = 27.5°, 207.5°$	A1 A1	$\tan\theta = \frac{\sin 40}{2-\cos 40}$ -1 if given in radians
	[7]	-1 extra solutions in the range

4 (a) $\frac{dx}{dt}=k\sqrt{x}$	M1	$\frac{dx}{dt}=\ldots$
	A1	$k\sqrt{x}$
	[2]	
(b) $\frac{dy}{dt}=\frac{10000}{\sqrt{y}}$		
$\Rightarrow \int\sqrt{y}dy=\int 10000dt$	M1	separating variables
$\Rightarrow \frac{2}{3}y^{\frac{3}{2}}=10000t+c$	A1	condone omission of c
When $t = 0$, $y = 900 \Rightarrow 18000 = c$	B1	evaluating constant for their integral
$\Rightarrow y=[\frac{3}{2}(10000t+18000)]^{\frac{2}{3}}$ $=(1500(10t+18))^{\frac{2}{3}}$	A1	any correct expression for y =
When $t = 10$, $y = 3152$	M1	for method allow substituting t=10 in their expression
	A1	cao
	[6]	
5 (i) $\int xe^{-2x}dx$ let $u = x$, $dv/dx = e^{-2x}$	M1	Integration by parts with $u = x$, $dv/dx = e^{-2x}$
$\Rightarrow v=-\frac{1}{2}e^{-2x}$ $=-\frac{1}{2}xe^{-2x}+\int\frac{1}{2}e^{-2x}dx$ $=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+c$	A1	$=-\frac{1}{2}xe^{-2x}+\int\frac{1}{2}e^{-2x}dx$
$=-\frac{1}{4}e^{-2x}(1+2x)+c$ *	E1	condone omission of c
or $\frac{d}{dx}[-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+c]=-\frac{1}{2}e^{-2x}+xe^{-2x}+\frac{1}{2}e^{-2x}$	M1	product rule
	A1	
$=xe^{-2x}$	E1	
	[3]	
(ii) $V=\int_0^2\pi y^2dx$ $=\int_0^2\pi(x^{1/2}e^{-x})^2dx$	M1	Using formula condone omission of limits
$=\pi\int_0^2 xe^{-2x}dx$	A1	$y^2=xe^{-2x}$ condone omission of limits and π
$=\pi\left[-\frac{1}{4}e^{-2x}(1+2x)\right]_0^2$ $=\pi(-\frac{1}{4}e^{-4}.5+\frac{1}{4})$	DM1	condone omission of π (need limits)
$=\frac{1}{4}\pi(1-\frac{5}{e^4})$ *	E1	
	[4]	

Section B

6 (i) At E, $\theta = 2\pi$ ⇒ $x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ ⇒ $y = a(1 - \cos \pi) = 2a$	M1 A1 M1 A1 [4]	$\theta=\pi$, 180°,cosθ=-1
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{\sin\theta}{(1-\cos\theta)}$	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin\theta) = \cos\theta, \frac{d}{d\theta}(\cos\theta) = -\sin\theta$ both or equivalent www condone uncancelled a
(iii) tan 30° = 1/√3 ⇒ $\frac{\sin\theta}{(1-\cos\theta)} = \frac{1}{\sqrt{3}}$ ⇒ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$ * When $\theta = 2\pi/3$, sin $\theta = \sqrt{3}/2$ $(1 - \cos\theta)/\sqrt{3} = (1 + ½)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + ½) = 3a/2$* OF = $a(2\pi/3 - \sqrt{3}/2)$	M1 E1 M1 E1 E1 B1 [6]	Or gradient=1/√3 sin $\theta = \sqrt{3}/2$, cos $\theta = -½$ or equiv.
(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ = 20 ⇒ $a = 2.22$ m	B1ft M1 A1 M1 A1 [5]	their OE -2their OF

7 (i) AE = √($15^2 + 20^2 + 0^2$) = 25	M1 A1 [2]	
(ii) $\overrightarrow{\mathrm{AE}} = \begin{pmatrix}15\\-20\\0\end{pmatrix} = 5\begin{pmatrix}3\\-4\\0\end{pmatrix}$ Equation of BD is $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\\0\end{pmatrix}$ BD = 15 ⇒ $\lambda = 3$ ⇒ D is (8, −19, 11)	M1 A1 M1 A1cao [4]	Any correct form or $\mathbf{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}15\\-20\\0\end{pmatrix}$ $\lambda = 3$ or 3/5 as appropriate
(iii) At A: −3 × 0 + 4 × 0 + 5 × 6 = 30 At B: −3 × (−1) + 4 × (−7) + 5 × 11 = 30 At C: −3 × (−8) + 4 × (−6) + 5 × 6 = 30 Normal is $\begin{pmatrix}-3\\4\\5\end{pmatrix}$	M1 A2,1,0 B1 [4]	One verification (OR B1 Normal, M1 scalar product with 1 vector in the plane, A1two correct, A1 verification with a point OR M1 vector form of equation of plane eg r=0i+0j+6k+μ(i+7j-5k)+ν(8i+6j+0k) M1 elimination of both parameters A1 equation of plane B1 Normal *)
(iv) $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AE} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}15\\-20\\0\end{pmatrix} = 60-60=0$ $\begin{pmatrix}4\\3\\5\end{pmatrix}.\overrightarrow{AB} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}-1\\-7\\5\end{pmatrix} = -4-21+25=0$ ⇒ $\begin{pmatrix}4\\3\\5\end{pmatrix}$ is normal to plane Equation is $4x + 3y + 5z = 30$.	M1 E1 M1 A1 [4]	scalar product with one vector in plane = 0 scalar product with another vector in plane = 0 $4x + 3y + 5z = \ldots$ 30 OR as * above OR M1 for subst 1 point in 4x+3y+5z= ,A1 for subst 2 further points =30 A1 correct equation , B1 Normal
(v) Angle between planes is angle between normals $\begin{pmatrix}4\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\4\\5\end{pmatrix}$ $\cos\theta = \dfrac{4\times(-3)+3\times4+5\times5}{\sqrt{50}\times\sqrt{50}} = \dfrac{1}{2}$ ⇒ $\theta = 60°$	M1 M1 A1 A1 [4]	Correct method for any 2 vectors their normals only (rearranged) or 120° cao

	Comprehension Paper 2		
Qu	Answer	Mark	Comment
1.	$\left(26+\frac{385}{1760}\right)\times 4$ minutes 1 hour 44 minutes 52.5 seconds	M1 A1	Accept all equivalent forms, with units. Allow ….52 and 53 seconds.
2.	$R=259.6-0.391(T-1900)$ $\therefore\ 259.6-0.391(T-1900)=0$ $\Rightarrow T=2563.9$ *R* will become negative in 2563	M1 A1 A1	R=0 and attempting to solve. T=2563,2564,2563.9…any correct cao
3.	The value of *L* is 120.5 and this is over 2 hours or (120 minutes)	E1	or R>120.5minutes or showing there is no solution for $120=120.5+54.5\mathrm{e}^{-\dots}$
4.(i)	Substituting $t=0$ in $R=L+(U-L)\mathrm{e}^{-kt}$ gives $R=L+(U-L)\times 1$ $=U$	M1 A1 E1	$\mathrm{e}^0=1$
4.(ii)	As $t\to\infty$, $\mathrm{e}^{-kt}\to 0$ and so $R\to L$	M1 E1	
5.(i)		M1 A1 A1	Increasing curve Asymptote *A* and *B* marked correctly
5.(ii)	Any field event: long jump, high jump, triple jump, pole vault, javelin, shot, discus, hammer, etc.	B1	
6.(i)	$t=104$	B1	
6.(ii)	$R=115+(175-115)\mathrm{e}^{-0.0467t^{0.797}}$ $R=115+60\times\mathrm{e}^{-0.0467\times 104^{0.797}}$ $R=115+60\times\mathrm{e}^{-1.892}$ $R=124.047...$ 2 hours 4 minutes 3 seconds	M1 A1	Substituting their t 124, 124.05, etc.

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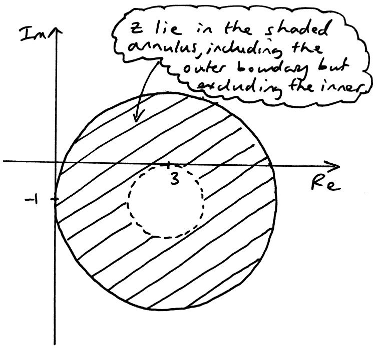
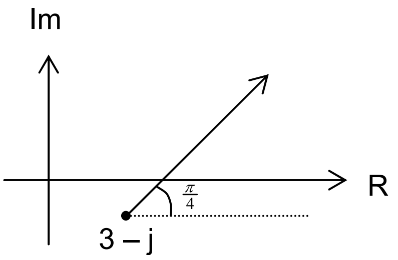
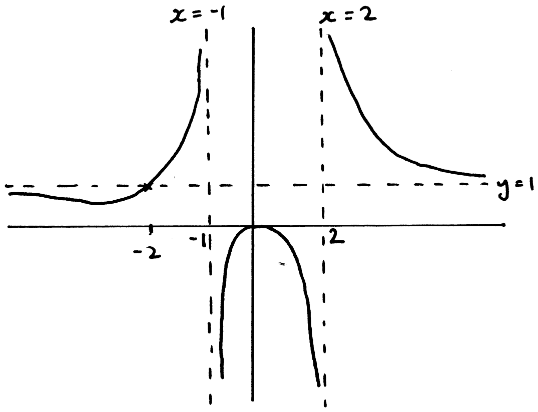
Qu	Answer	Mark	Comment
Section A			
1 (i)	Reflection in the x-axis.	B1 **[1]**	
1(ii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **[1]**	
1(iii)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	M1 A1 c.a.o. **[2]**	Multiplication of their matrices in the correct order or B2 for correct matrix without working
2	$(x+2)(Ax^2+Bx+C)+D$ $= Ax^3+Bx^2+Cx+2Ax^2+2Bx+2C+D$ $= Ax^3+(2A+B)x^2+(2B+C)x+2C+D$ $\Rightarrow A=2,\ B=-7,\ C=15,\ D=-32$	M1 B1 B1 F1 F1 **OR** B5 **[5]**	Valid method to find all coefficients For A = 2 For D = -32 F1 for each of B and C For all correct
3(i)	$\alpha+\beta+\gamma=-4$ $\alpha\beta+\beta\gamma+\alpha\gamma=-3$ $\alpha\beta\gamma=-1$	B1 B1 B1 **[3]**	
3(ii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$ $=16+6=22$	M1 A1 E1 **[3]**	Attempt to use $(\alpha+\beta+\gamma)^2$ Correct Result shown
4 (i)	Argand diagram with solid circle, centre 3 – j, radius 3, with values of z on and within the circle clearly indicated as satisfying the inequality.	B1 B1 B1 **[3]**	Circle, radius 3, shown on diagram Circle centred on 3 - j Solution set indicated (solid circle with region inside)
4(ii)		B1 B1 **[2]**	Hole, radius 1, shown on diagram Boundaries dealt with correctly

Qu	Answer	Mark	Comment
Section A (continued)			
4(iii)		B1 B1 B1 **[3]**	Line through their 3 – j Half line $\frac{\pi}{4}$ to real axis
5(i)	$\begin{pmatrix}-1 & 2\\ -3 & 4\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}=\begin{pmatrix}1\\ 1\end{pmatrix}$	B1	
	$\mathbf{S}^{-1}=\frac{1}{2}\begin{pmatrix}4 & -2\\ 3 & -1\end{pmatrix}$	M1, A1	Attempt to divide by determinant and manipulate contents Correct
	$\frac{1}{2}\begin{pmatrix}4 & -2\\ 3 & -1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}=\begin{pmatrix}1\\ 1\end{pmatrix}$	E1 **[4]**	
5(ii)	$\mathbf{T}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$		
	$\Rightarrow \mathbf{T}^{-1}\mathbf{T}\begin{pmatrix}x\\ y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\ y\end{pmatrix}$	M1	Pre-multiply by $\mathbf{T}^{-1}$
	$\Rightarrow \begin{pmatrix}x\\ y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\ y\end{pmatrix}$	A1 **[2]**	Invariance shown
6	$3+6+12+........+3\times 2^{n-1}=3\left(2^{n}-1\right)$		
	n = 1, LHS = 3, RHS = 3	B1	
	Assume true for *n* = *k*	E1	Assuming true for *k*
	Next term is $3\times 2^{k+1-1}=3\times 2^{k}$	B1	(*k* + 1)th term.
	Add to both sides $\text{RHS}=3\left(2^{k}-1\right)+3\times 2^{k}$	M1	Add to both sides
	$=3\left(2^{k}-1+2^{k}\right)$ $=3\left(2\times 2^{k}-1\right)$ $=3\left(2^{k+1}-1\right)$	A1	Working must be valid
	But this is the given result with *k* + 1 replacing *k*. Therefore if it is true for *k* it is true for *k* + 1.	E1	Dependent on previous A1and E1
	Since it is true for *k* = 1, it is true for all positive integers *n*.	E1 **[7]**	Dependent on B1 and previous E1
			Section A Total: 36

Section B			
7(i)	$x=2$, $x=-1$ and $y=1$	B1 B1B1 **[3]**	One mark for each
7(ii) **(*A*)**	Large positive x, $y \to 1^+$ (from above) (e.g. consider $x=100$)	M1 B1	Evidence of method needed for M1
(*B*)	Large negative x, $y \to 1^-$ (from below) (e.g. consider $x=-100$)	B1 **[3]**	
7(iii)	Curve 3 branches	B1	With correct approaches to vertical asymptotes
	Correct approaches to horizontal asymptote	B1	Consistent with their (i) and (ii)
	Asymptotes marked	B1	Equations or values at axes clear
	Through origin	B1 **[4]**	
			
7(iv)	$x<-1$, $x>2$	B1B1, B1, **[3]**	s.c. 1 for inclusive inequalities Final B1 for all correct with no other solutions

8(i)	$(2+\mathrm{j})^2=3+4\mathrm{j}$ $(2+\mathrm{j})^3=2+11\mathrm{j}$ Substituting into $2x^3-11x^2+22x-15$: $2(2+11\mathrm{j})-11(3+4\mathrm{j})+22(2+\mathrm{j})-15$ $=4+22\mathrm{j}-33-44\mathrm{j}+44+22\mathrm{j}-15$ $=0$ So 2 + j is a root.	B1 B1 M1 A1 A1 **[5]**	 Attempt at substitution Correctly substituted Correctly cancelled (Or other valid methods)
8(ii)	2 - j	B1 **[1]**	
8(iii)	$(x-(2+\mathrm{j}))(x-(2-\mathrm{j}))$ $=(x-2-\mathrm{j})(x-2+\mathrm{j})$ $=x^2-2x+\mathrm{j}x-2x+4-2\mathrm{j}-\mathrm{j}x+2\mathrm{j}+1$ $=x^2-4x+5$	M1 A1	Use of factor theorem
	$(x^2-4x+5)(ax+b)=2x^3-11x^2+22x-15$ $(x^2-4x+5)(2x-3)=2x^3-11x^2+22x-15$	M1	Comparing coefficients or long division
	$(2x-3)=0\Rightarrow x=\frac{3}{2}$	A1 **[4]**	Correct third root
	OR		
	Sum of roots = $\frac{11}{2}$ **or** product of roots = $\frac{15}{2}$ leading to	M1 A1	
	$\alpha+2+\mathrm{j}+2-\mathrm{j}\ =\frac{11}{2}$	M1	
	$\Rightarrow\alpha=\frac{3}{2}$	A1 **[4]**	
	or		
	$\alpha(2+\mathrm{j})(2-\mathrm{j})=\frac{15}{2}$	M1 A1	
	$\Rightarrow 5\alpha=\frac{15}{2}\Rightarrow\alpha=\frac{3}{2}$	M1 A1 **[4]**	(Or other valid methods)

9(i)	$r(r+1)(r+2)-(r-1)r(r+1)$ $\equiv(r^2+r)(r+2)-r^3-r$ $\equiv r^3+2r^2+r^2+2r-r^3+r$ $\equiv 3r^2+3r \equiv 3r(r+1)$	M1 E1 **[2]**	Accept '=' in place of '≡' throughout working Clearly shown
9(ii)	$\sum_{r=1}^{n} r(r+1)$ $=\frac{1}{3}\sum_{r=1}^{n}[r(r+1)(r+2)-(r-1)r(r+1)]$ $=\frac{1}{3}[(1\times2\times3-0\times1\times2)+(2\times3\times4-1\times2\times3)+$ $(3\times4\times5-2\times3\times4)+......$ $+(n(n+1)(n+2)-(n-1)n(n+1))]$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	M1 M1 A2 M1 A1 **[6]**	Using identity from (i) Writing out terms in full At least 3 terms correct (minus 1 each error to minimum of 0) Attempt at eliminating terms (telescoping) Correct result
9(iii)	$\sum_{r=1}^{n} r(r+1)=\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r$ $=\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$ $=\frac{1}{6}n(n+1)[(2n+1)+3]$ $=\frac{1}{6}n(n+1)(2n+4)$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	B1 B1 M1 A1 E1 **[5]**	Use of standard sums (1 mark each) Attempt to combine Correctly simplified to match result from (ii)
			Section B Total: 36
			Total: 72

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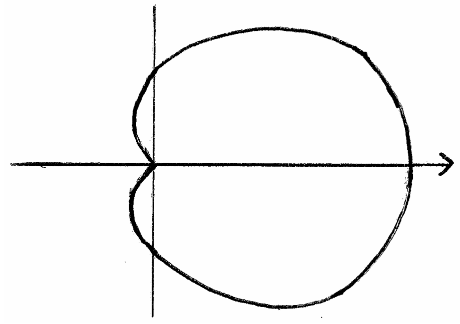
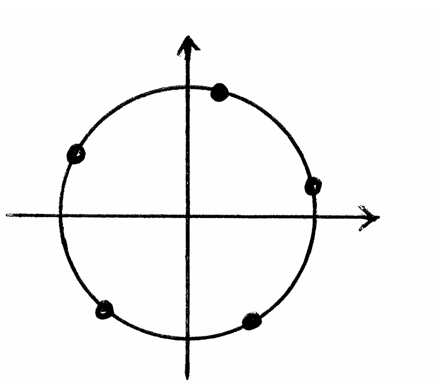
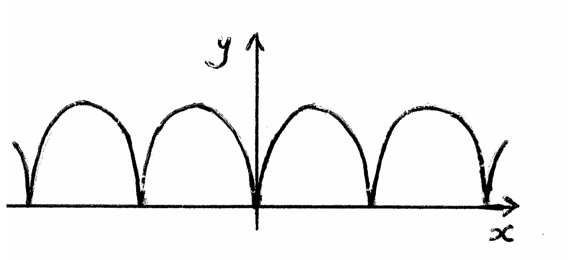
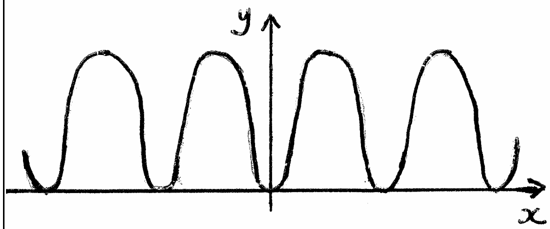
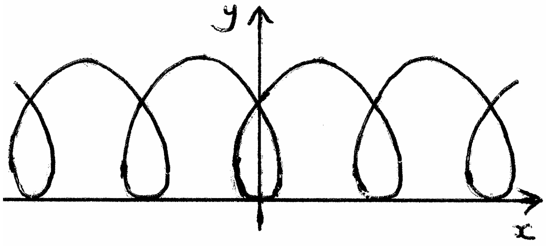
1(a)(i)		B1 B1 **2**	Correct shape for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ including maximum in 1st quadrant Correct form at O and no extra sections
(ii)	Area is $\int \frac{1}{2}r^2\,\mathrm{d}\theta = \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} \frac{1}{2}a^2(\sqrt{2}+2\cos\theta)^2\,\mathrm{d}\theta$ $= \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} a^2(1+2\sqrt{2}\cos\theta+1+\cos 2\theta)\,\mathrm{d}\theta$ $= \left[\, a^2(2\theta+2\sqrt{2}\sin\theta+\frac{1}{2}\sin 2\theta) \,\right]_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi}$ $= 3(\pi+1)a^2$	M1 A1 B1 B1B1 ft M1 A1 **7**	For integral of $(\sqrt{2}+2\cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)* Using $2\cos^2\theta = 1+\cos 2\theta$ Integration of $\cos\theta$ and $\cos 2\theta$ Evaluation using $\sin\frac{3}{4}\pi = (\pm)\frac{1}{\sqrt{2}}$
(b)(i)	$\mathrm{f}'(x) = \sec^2(\frac{1}{4}\pi + x)$ $\mathrm{f}''(x) = 2\sec^2(\frac{1}{4}\pi + x)\tan(\frac{1}{4}\pi + x)$ $\mathrm{f}(0) = 1,\ \mathrm{f}'(0) = 2,\ \mathrm{f}''(0) = 4$ $\mathrm{f}(x) = 1 + 2x + 2x^2 + \ldots$	B1 B1 M1 B1A1A1 **6**	Any correct form Evaluating $\mathrm{f}'(0)$ or $\mathrm{f}''(0)$
	OR $\mathrm{g}'(u) = \sec^2 u$ (where $\mathrm{g}(u) = \tan u$) B1 $\mathrm{g}''(u) = 2\sec^2 u \tan u$ B1 $\mathrm{g}(\frac{1}{4}\pi) = 1,\ \mathrm{g}'(\frac{1}{4}\pi) = 2,\ \mathrm{g}''(\frac{1}{4}\pi) = 4$ M1 $\mathrm{f}(x) = \mathrm{g}(\frac{1}{4}\pi + x) = 1 + 2x + 2x^2 + \ldots$ B1A1A1		Condone $\sec^2 x$ etc Evaluating $\mathrm{g}'(\frac{1}{4}\pi)$ or $\mathrm{g}''(\frac{1}{4}\pi)$
(ii)	$\int_{-h}^{h} x^2(1+2x+2x^2+\ \ldots)\,\mathrm{d}x$ $= \left[\, \frac{1}{3}x^3 + \frac{1}{2}x^4 + \frac{2}{5}x^5 + \ldots \,\right]_{-h}^{h}$ $\approx (\frac{1}{3}h^3 + \frac{1}{2}h^4 + \frac{2}{5}h^5) - (-\frac{1}{3}h^3 + \frac{1}{2}h^4 - \frac{2}{5}h^5)$ $= \frac{2}{3}h^3 + \frac{4}{5}h^5$	M1 A1 ft A1 (ag) **3**	Using series and integrating (ft requires three non-zero terms) Correctly shown Allow ft from $1+kx+2x^2$ with $k \neq 0$

2 **(a)(i)**	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2\mathrm{j}\sin n\theta$	B1B1 **2**	
(ii)	$\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = 64\sin^4\theta\cos^2\theta$ $= z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$ $= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$ $\sin^4\theta\cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$ $(A = \frac{1}{32},\ B = -\frac{1}{16},\ C = -\frac{1}{32},\ D = \frac{1}{16})$	B1 M1 A1 M1 A1 ft A1 **6**	Expansion $z^6 + \ldots + z^{-6}$ Using $z^n + \frac{1}{z^n} = 2\cos n\theta$ with $n = 2,\ 4$ or 6. Allow M1 if used in partial expansion, or if 2 omitted, etc
(b)(i)	$\lvert 4 + 4\mathrm{j} \rvert = \sqrt{32}$, $\arg(4 + 4\mathrm{j}) = \frac{1}{4}\pi$	B1B1 **2**	Accept $5.7;\ 0.79,\ 45°$
(ii)	$r = \sqrt{2}$ $\theta = -\frac{3}{4}\pi,\ -\frac{7}{20}\pi,\ \frac{1}{20}\pi,\ \frac{9}{20}\pi,\ \frac{17}{20}\pi$ 	B1 B3 B2 **6**	Accept $32^{\frac{1}{10}},\ 1.4,\ \sqrt[5]{4\sqrt{2}}$ etc Accept $-2.4, -1.1, 0.16, 1.4, 2.7$ Give B2 for three correct Give B1 for one correct Deduct 1 mark (maximum) if degrees used $(-135°,\ -63°,\ 9°,\ 81°,\ 153°)$ $\frac{1}{20}\pi + \frac{2}{5}k\pi$ earns B2; with $k = -2,\ -1,\ 0,\ 1,\ 2$ earns B3 Give B1 for four points correct, or B1 ft for five points
(iii)	$\sqrt{2}\mathrm{e}^{-\frac{3}{4}\pi\mathrm{j}} = \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{j}\right)$ $= -1 - \mathrm{j}$ $p = -1,\ q = -1$	M1 A1 **2**	Exact evaluation of a fifth root Give B2 for correct answer stated or obtained by any other method

3 (i)	$\mathbf{M}^{-1}=\frac{1}{5-k}\begin{pmatrix}1 & 5k-13 & 5-2k\\ 1 & 52-8k & 3k-20\\ -1 & -12 & 5\end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluating determinant For $(5-k)$ *must be simplified* Finding at least four cofactors At least 6 signed cofactors correct Transposing matrix of cofactors and dividing by determinant Fully correct
	OR Elementary row operations applied to **M** (LHS) and **I** (RHS), and obtaining at least two zeros in LHS M1 Obtaining one row in LHS consisting of two zeros and a multiple of $(5-k)$ A1 Obtaining one row in RHS which is a multiple of a row of the inverse matrix A1 Obtaining two zeros in every row in LHS M1 Completing process to find inverse M1A1		or elementary column operations
(ii)	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=-\frac{1}{2}\begin{pmatrix}1 & 22 & -9\\ 1 & -4 & 1\\ -1 & -12 & 5\end{pmatrix}\begin{pmatrix}12\\ m\\ 0\end{pmatrix}$ $x=-11m-6,\quad y=2m-6,\quad z=6m+6$	M1 M1 M1 A2 ft **5**	Substituting $k=7$ into inverse Correct use of inverse Evaluating matrix product Give A1 ft for one correct *Accept unsimplified forms or solution left in matrix form*
	OR e.g. eliminating *x* , $3y-z=-24$ $5y-z=4m-36$ M2 $y=2m-6$ M1 $x=-11m-6,\quad y=2m-6,\quad z=6m+6$ A2		Eliminating one variable in two different ways Obtaining one of *x* , *y* , *z* Give M3 for any other valid method leading to one of *x* , *y* , *z* in terms of *m* Give A1 for one correct
(iii)	Eliminating *x* , $3y+3z=-24$ $5y+5z=4p-36$ For solutions, $4p-36=-24\times\frac{5}{3}$	M2 A1 M1	Eliminating one variable in two different ways Two correct equations *Dependent on previous M2*
	OR Replacing one column of matrix with column from RHS, and evaluating determinant M2 determinant $12+12p$ or $-12-12p$ A1 For solutions, det $=0$ M1		*Dependent on previous M2*

	OR Any other method leading to an equation from which *p* could be found M3 Correct equation A1		
	$p = -1$ Let $z = \lambda$, $x = 5 - \lambda,\quad y = -8 - \lambda,\quad z = \lambda$	A1 M1 (or M3) A1 **7**	Obtaining a line of solutions Give M3 when M0 for finding *p* or $x = 13 + \lambda,\quad y = \lambda,\quad z = -8 - \lambda$ or $x = \lambda,\quad y = -13 + \lambda,\quad z = 5 - \lambda$ Accept $x = 5 - z,\ y = -8 - z$ or $x = y + 13 = 5 - z$ etc

4 (i)	$1+2\sinh^2 x = 1+2[\,\frac{1}{2}(\mathrm{e}^{x}-\mathrm{e}^{-x})\,]^2$		
	$= 1+\frac{1}{2}(\mathrm{e}^{2x}-2+\mathrm{e}^{-2x})$	B1	For $(\mathrm{e}^{x}-\mathrm{e}^{-x})^2 = \mathrm{e}^{2x}-2+\mathrm{e}^{-2x}$
	$= \frac{1}{2}(\mathrm{e}^{2x}+\mathrm{e}^{-2x})$	B1	For $\cosh 2x = \frac{1}{2}(\mathrm{e}^{2x}+\mathrm{e}^{-2x})$
	$= \cosh 2x$	B1 (ag) **3**	For completion
(ii)	$2(1+2\sinh^2 x)+\sinh x = 5$	M1	Using (i)
	$4\sinh^2 x+\sinh x-3 = 0$		
	$(4\sinh x-3)(\sinh x+1) = 0$	M1	Solving to obtain a value of $\sinh x$
	$\sinh x = \frac{3}{4},\ -1$	A1A1	
	$x = \operatorname{arsinh}(\,\frac{3}{4}\,) = \ln(\,\frac{3}{4}+\sqrt{\frac{9}{16}+1}\,) = \ln 2$	A1 ft	
	$x = \operatorname{arsinh}(-1) = \ln(\,-1+\sqrt{1+1}\,) = \ln(\sqrt{2}-1)$	A1 ft **6**	or $-\ln(\sqrt{2}+1)$ *SR* Give A1 for $\pm\ln 2,\ \pm\ln(\sqrt{2}-1)$
	OR $2\mathrm{e}^{4x}+\mathrm{e}^{3x}-10\mathrm{e}^{2x}-\mathrm{e}^{x}+2 = 0$		
	M2		Obtaining a linear or quadratic factor
	$(\mathrm{e}^{x}-2)(2\mathrm{e}^{x}+1)(\mathrm{e}^{2x}+2\mathrm{e}^{x}-1) = 0$ A1A1		For $(\mathrm{e}^{x}-2)$ and $(\mathrm{e}^{2x}+2\mathrm{e}^{x}-1)$
	$x = \ln 2,\ \ln(\sqrt{2}-1)$ A1A1 ft		
(iii)	$\int_0^{\ln 3} \frac{1}{2}(\cosh 2x-1)\,\mathrm{d}x$	M1	Expressing in integrable form or $\int \frac{1}{4}(\mathrm{e}^{2x}-2+\mathrm{e}^{-2x})\,\mathrm{d}x$
	$= \left[\ \frac{1}{4}\sinh 2x-\frac{1}{2}x\ \right]_0^{\ln 3}$	A1A1	or $(\,\frac{1}{8}\mathrm{e}^{2x}-\frac{1}{8}\mathrm{e}^{-2x}\,)-\frac{1}{2}x$
	$= \frac{1}{8}\left(9-\frac{1}{9}\right)-\frac{1}{2}\ln 3$	M1	For $\mathrm{e}^{2\ln 3} = 9$ and $\mathrm{e}^{-2\ln 3} = \frac{1}{9}$ M0 for just stating $\sinh(2\ln 3) = \frac{40}{9}$ etc
	$= \frac{10}{9}-\frac{1}{2}\ln 3$	A1 (ag) **5**	Correctly obtained
(iv)	Put $x = 3\cosh u$	M1	Any cosh substitution
	when $x = 3,\ u = 0$		
	when $x = 5,\ u = \operatorname{ar}\cosh\frac{5}{3} = \ln 3$	B1	For $\ln 3$ *Not awarded for* $\operatorname{arcosh}\frac{5}{3}$
	$\int_3^5 \sqrt{x^2-9}\ \mathrm{d}x\ = \int_0^{\ln 3} (3\sinh u)(3\sinh u\ \mathrm{d}u)$	A1	*Limits not required*
	$= 9\int_0^{\ln 3} \sinh^2 u\ \mathrm{d}u$		
	$= 10-\frac{9}{2}\ln 3$	A1 **4**	

5 (i)		B2	At least two cusps clearly shown Give B1 for at least two arches
	Has cusps Periodic / Symmetrical in *y*-axis / Has maxima / Is never below the *x*-axis	B1 B1 **4**	 Any other feature
(ii)		B2	At least two minima (zero gradient) clearly shown Give B1 for general shape correct (at least two cycles)
	The curve has no cusps	B1 **3**	For description of any *difference*
(iii)	(*A*) 	B2 **2**	At least two loops Give B1 for general shape correct (at least one cycle)
	(*B*) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sin\theta}{1-2\cos\theta}$	M1 A1 **2**	Correct method of differentiation *Allow M1 if inverted* Allow $\frac{\sin\theta}{1-k\cos\theta}$
	(*C*) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is infinite when $1-2\cos\theta=0$	M1	
	$\theta=\frac{1}{3}\pi$	A1	Any correct value of θ
	$x=\frac{1}{3}\pi-2\sin\frac{1}{3}\pi$ $=-(\sqrt{3}-\frac{1}{3}\pi)$	M1	
	Hence width of loop is $2(\sqrt{3}-\frac{1}{3}\pi)$	M1	Finding width of loop
	$=2\sqrt{3}-\frac{2\pi}{3}$	A1 (ag) **5**	Correctly obtained *Condone negative answer*
(iv)	$k=4.6$	B2 **2**	Give B1 for a value between 4 and 5 (inclusive)

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1 (i)	$\begin{pmatrix}-1\\4\\3\end{pmatrix}\times\begin{pmatrix}-k\\4\\k+2\end{pmatrix}=\begin{pmatrix}4k-4\\2-2k\\4k-4\end{pmatrix}$ [$=2(k-1)\begin{pmatrix}2\\-1\\2\end{pmatrix}$]	B1 M1 A2 **4**	$\overrightarrow{AB}$ and $\overrightarrow{CD}$ *(Condone $\overrightarrow{BA}$ and $\overrightarrow{DC}$)* Evaluating vector product Give A1 ft for one element correct
(ii)(*A*)	$k=1$	B1 **1**	
(*B*)	$\overrightarrow{CA}\times\overrightarrow{AB}=\begin{pmatrix}-3\\-8\\4\end{pmatrix}\times\begin{pmatrix}-1\\4\\3\end{pmatrix}=\begin{pmatrix}-40\\5\\-20\end{pmatrix}$ Distance is $\dfrac{\lvert\overrightarrow{CA}\times\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}=\dfrac{45}{\sqrt{26}}$ (≈ 8.825)	M1 M1 A1 M1 M1 A1 **6**	For appropriate vector product Evaluation *Dependent on previous M1* Method for finding shortest distance *Dependent on first M1* Calculating magnitudes *Dependent on previous M1* Accept 8.82 to 8.83
	OR $\overrightarrow{CP}.\overrightarrow{AB}=\begin{pmatrix}-2-\lambda-1\\-3+4\lambda-5\\2+3\lambda+2\end{pmatrix}\cdot\begin{pmatrix}-1\\4\\3\end{pmatrix}=0$ M2A1 $\overrightarrow{CP}=\dfrac{1}{26}\begin{pmatrix}-95\\-140\\155\end{pmatrix}$ Distance is $\dfrac{\sqrt{52650}}{26}$ M1 M1A1		Finding $\overrightarrow{CP}$ *Dependent on previous M1* *Dependent on previous M1*
(*C*)	Normal vector is $\overrightarrow{CA}\times\overrightarrow{AB}=\begin{pmatrix}-40\\5\\-20\end{pmatrix}=-5\begin{pmatrix}8\\-1\\4\end{pmatrix}$ Equation of plane is $8x-y+4z=-16+3+8$ $8x-y+4z+5=0$	M1 M1 A1 **3**	*Dependent on previous M1* Allow $-40x+5y-20z=25$ etc
(iii)	$\dfrac{\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})}{\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert}=\dfrac{\begin{pmatrix}k+2\\8\\-4\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\2\end{pmatrix}(2k-2)}{3(2k-2)}$ Shortest distance is $\left\lvert\dfrac{2k-12}{3}\right\rvert$	M1 M1 A1 ft A1 **4**	For $\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})$ Fully correct method *(evaluation not required) Dependent on previous M1* Correct evaluated expression for distance ft from (i) Simplified answer *Modulus not required*

(iv)	Intersect when $k = 6$ $-2-\lambda = 6-6\mu$ $-3+4\lambda = 5+4\mu$ $2+3\lambda = -2+8\mu$ Solving, $\lambda = 4,\ \mu = 2$ Point of intersection is $(-6,\ 13,\ 14)$	B1 ft M1 A1 ft M1 A1 A1 **6**	 Forming at least two equations Two correct equations Solving to obtain λ or μ *Dependent on previous M1* One value correct
	$-2-\lambda = k-k\mu$ OR $-3+4\lambda = 5+4\mu$ M1 $2+3\lambda = -2+(k+2)\mu$ A1 Solving, $k = 6$ M1A1 $\lambda = 4,\ \mu = 2$ A1 Point of intersection is $(-6,\ 13,\ 14)$ A1		Forming three equations All equations correct *Dependent on previous M1* One value correct

2 (i)	Normal vector is $\begin{pmatrix}2x-4y\\-4x+6y\\-4z\end{pmatrix}$	M1 A1 A1 A1 **4**	Partial differentiation Condone $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}+\lambda\begin{pmatrix}2x-4y\\-4x+6y\\-4z\end{pmatrix}$ *For 4 marks the normal must appear as a vector (isw)*
(ii)	At Q normal vector is $\begin{pmatrix}18\\-44\\-4\end{pmatrix}$ Tangent plane is $18x-44y-4z=306-176-4=126$ $9x-22y-2z=63$	M1 M1 M1 A1 **4**	For $18x-44y-4z$ *Dependent on previous M1* Using Q to find constant Accept any correct form
(iii)	$18\delta x-44\delta y-4\delta z\approx 0$ $18h-44p-4(-h)\approx 0$ $p\approx\frac{1}{2}h$	M1 A1 ft M1 A1 **4**	For $18\delta x-44\delta y-4\delta z$ *If left in terms of x, y, z: M1A0M1A0*
	OR $9(17+h)-22(4+p)-2(1-h)\approx 63$ M2A1 ft $p\approx\frac{1}{2}h$ A1		
	OR $(17+h)^2-4(17+h)(4+p)+\ldots=0$ $-44p+22h\approx 0$ M2A1 $p\approx\frac{1}{2}h$ A1		Neglecting second order terms
	OR $p=\dfrac{4h+44\pm\sqrt{28h^2+88h+1936}}{6}$ M2A1 $p\approx\frac{1}{2}h$ A1		
(iv)	Normal parallel to *z*-axis requires $2x-4y=0$ and $-4x+6y=0$ $x=y=0$; then $-2z^2-63=0$ No solutions; hence no such points	M1A1 ft M1 A1 (ag) **4**	Correctly shown
	OR $2x-4y=-4x+6y$, so $y=\frac{3}{5}x$ $-\frac{8}{25}x^2-2z^2-63=0$, hence no points M2A2		Similarly if only $2x-4y=0$ used
(v)	$2x-4y=5\lambda$ $-4x+6y=-6\lambda$ $-4z=2\lambda$ $x=-\frac{3}{2}\lambda$, $y=-2\lambda$, $z=-\frac{1}{2}\lambda$ Substituting into equation of surface $\frac{9}{4}\lambda^2-12\lambda^2+12\lambda^2-\frac{1}{2}\lambda^2-63=0$ $\lambda=\pm 6$	M1A1 ft M1 M1 M1 M1	Obtaining *x, y, z* in terms of λ or $x=3z$, $y=4z$ Obtaining a value of λ (or equivalent)

	Point $(-9,\ -12,\ -3)$ gives $k=-45+72-6=21$ Point $(9,\ 12,\ 3)$ gives $k=45-72+6=-21$	A1 A1 **8**	Using a point to find *k* *If $\lambda=1$ is assumed: M0M1M0M0M1*

3 (i)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=(6t^2-6)^2+(12t)^2$ $=36t^4+72t^2+36$ $=36(t^2+1)^2$ Arc length is $\int_0^1 6(t^2+1)\,\mathrm{d}t$ $=\left[\ 2t^3+6t\ \right]_0^1$ $=8$	M1A1 A1 M1 A1 A1 **6**	Using $\int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ For $2t^3+6t$
(ii)	Curved surface area is $\int 2\pi\,y\,\mathrm{d}s=\int_0^1 2\pi(6t^2)6(t^2+1)\,\mathrm{d}t$ $=\pi\left[\ \frac{72}{5}t^5+24t^3\ \right]_0^1$ $=\frac{192\pi}{5}\quad(\approx 120.6)$	M1 A1 M1 A1 A1 **5**	Using $\int ...y\,\mathrm{d}s$ (in terms of *t*) with 'd*s*' the same as in (i) Any correct integral form in terms of *t* *(limits required)* Integration For $\pi(\frac{72}{5}t^5+24t^3)$
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{12t}{6t^2-6}\quad\left(=\frac{2t}{t^2-1}\right)$ Equation of normal is $y-6t^2=\frac{1-t^2}{2t}(x-2t^3+6t)$ $y-6t^2=\frac{1}{2}\left(\frac{1}{t}-t\right)x-t^2(1-t^2)+3(1-t^2)$ $y=\frac{1}{2}\left(\frac{1}{t}-t\right)x+2t^2+t^4+3$	M1 A1 M1 A1 (ag) **4**	Method of differentiation *At least one intermediate step required* Correctly obtained
(iv)	Differentiating partially with respect to *t* $0=\frac{1}{2}\left(-\frac{1}{t^2}-1\right)x+4t+4t^3$ $\frac{1}{2t^2}(1+t^2)x=4t(1+t^2)$ $x=8t^3$ $t=\frac{1}{2}x^{\frac{1}{3}},$ so $y=\frac{1}{2}(2x^{-\frac{1}{3}}-\frac{1}{2}x^{\frac{1}{3}})x+\frac{1}{2}x^{\frac{2}{3}}+\frac{1}{16}x^{\frac{4}{3}}+3$ $y=\frac{3}{2}x^{\frac{2}{3}}-\frac{3}{16}x^{\frac{4}{3}}+3$	M1 A2 M1 M1 A1 **6**	Give A1 if just one error or omission For obtaining $a\,x=b\,t^3$ Eliminating *t*

(v)	P lies on the envelope of the normals Hence $a = \frac{3}{2} \times 64^{\frac{2}{3}} - \frac{3}{16} \times 64^{\frac{4}{3}} + 3$ $= -21$	M1 M1 A1 **3**	Or a fully correct method for finding the centre of curvature at a general pt [$(8t^3,\ 6t^2 - 3t^4 + 3)$] Or $t = 2$ and $a = 6 \times 2^2 - 3 \times 2^4 + 3$

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<tr><td>I</td><td>I</td><td>J</td><td>K</td><td>L</td><td>–I</td><td>–J</td><td>–K</td><td>–L</td></tr>
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6</td>
<td>Give B5 for 30 (bold) entries correct
Give B4 for 24 (bold) entries correct
Give B3 for 18 (bold) entries correct
Give B2 for 12 (bold) entries correct
Give B1 for 6 (bold) entries correct</td>
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<tr><td>Inverse</td><td>**I**</td><td>**–J**</td><td>**–K**</td><td>**–L**</td><td>**–I**</td><td>**J**</td><td>**K**</td><td>**L**</td></tr>
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<td>Give B2 for six correct
Give B1 for three correct</td>
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<tr><td>Element</td><td>**I**</td><td>**J**</td><td>**K**</td><td>**L**</td><td>**–I**</td><td>**–J**</td><td>**–K**</td><td>**–L**</td></tr>
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<td>B3
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<td>Give B2 for six correct
Give B1 for three correct</td>
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<td>(iv)</td>
<td>Only two elements of *G* do not have order 4; so any subgroup of order 4 must contain an element of order 4
A subgroup of order 4 is cyclic if it contains an element of order 4
Hence any subgroup of order 4 is cyclic</td>
<td>M1A1
B1
A1
4</td>
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(may be implied)
For completion</td>
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<td>OR If a group of order 4 is not cyclic, it contains three elements of order 2 B1
G has only one element of order 2; so this cannot occur M1A1
So any subgroup of order 4 is cyclic A1</td>
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<td>(v)</td>
<td>$\{\mathbf{I},\ -\mathbf{I}\}$
$\{\mathbf{I},\ \mathbf{J},\ -\mathbf{I},\ -\mathbf{J}\}$
$\{\mathbf{I},\ \mathbf{K},\ -\mathbf{I},\ -\mathbf{K}\}$
$\{\mathbf{I},\ \mathbf{L},\ -\mathbf{I},\ -\mathbf{L}\}$</td>
<td>B1
B1
B1
B1
B1
5</td>
<td>For $\{\mathbf{I},\ -\mathbf{I}\}$, at least one correct subgroup of order 4, and no wrong subgroups. This mark is lost if *G* or $\{\mathbf{I}\}$ is included</td>
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(vi)	The symmetry group has 5 elements of order 2 (4 reflections and rotation through $180°$) *G* has only one element of order 2, hence *G* is not isomorphic to the symmetry group	M1 A1 A1	 **3**	Considering elements of order 2 (or self-inverse elements) Identification of at least two elements of order 2 in the symmetry group For completion

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.8&0.1&0\\0.2&0.6&0.15\\0&0.3&0.85\end{pmatrix}$	B1B1B1 **3**	For the three columns
(ii)	$\mathbf{P}^7\begin{pmatrix}0.6\\0.4\\0\end{pmatrix}=\begin{pmatrix}0.3204&0.1545&0.0927\\0.3089&0.2895&0.2780\\0.3706&0.5560&0.6293\end{pmatrix}\begin{pmatrix}0.6\\0.4\\0\end{pmatrix}=\begin{pmatrix}0.254\\0.301\\0.445\end{pmatrix}$ Division 3 is the most likely	M1 M1 A1 M1 A1 A1 **6**	Considering $\mathbf{P}^7$ (or $\mathbf{P}^8$ or $\mathbf{P}^6$) Evaluating a power of **P** For $\mathbf{P}^7$ (Allow ± 0.001 throughout) Evaluation of probabilities One probability correct Correctly determined
(iii)	$\mathbf{P}^n\to\begin{pmatrix}0.1429&0.1429&0.1429\\0.2857&0.2857&0.2857\\0.5714&0.5714&0.5714\end{pmatrix}$ Equilibrium probabilities are 0.143, 0.286, 0.571	M1 M1 A1 **3**	Considering powers of **P** Obtaining limit *Must be accurate to 3 dp if given as decimals*
	OR $\mathbf{P}\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}p\\q\\r\end{pmatrix}\Rightarrow\begin{array}{l}0.8p+0.1q=p\\0.2p+0.6q+0.15r=q\\0.3q+0.85r=r\end{array}$ M1 $q=2p,\ r=2q=4p$ and $p+q+r=1$ M1 $p=\frac{1}{7},\ q=\frac{2}{7},\ r=\frac{4}{7}$ A1		Obtaining at least two equations Solving (must use $p+q+r=1$)
(iv)	$\mathbf{Q}=\begin{pmatrix}0.8&0.1&0&0\\0.2&0.6&0.15&0\\0&0.3&0.75&0\\0&0&0.1&1\end{pmatrix}$	B1 B1 B1 **3**	Third column Fourth column Fully correct
(v)	$\mathbf{Q}^5\begin{pmatrix}0\\1\\0\\0\end{pmatrix}=\begin{pmatrix}0.4122&0.1566&0.0592&0\\0.3131&0.2767&0.2052&0\\0.2369&0.4105&0.4030&0\\0.0378&0.1563&0.3326&1\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $=\begin{pmatrix}0.1566\\0.2767\\0.4105\\0.1563\end{pmatrix}$ P(still in league) $=1-0.1563$ $=0.844$	M1 M1 A1 M1 A1 ft **5**	Considering $\mathbf{Q}^5$ (or $\mathbf{Q}^6$ or $\mathbf{Q}^4$) Evaluating a power of **Q** For 0.1563 (Allow 0.156 ± 0.001) For $1-a_{4,2}$ ft dependent on M1M1M1
(vi)	P(out of league) is element $a_{4,2}$ in $\mathbf{Q}^n$ When $n=15$, $a_{4,2}=0.4849$ When $n=16$, $a_{4,2}=0.5094$ First year is 2031	M1 M1 A1 A1 **4**	Considering $\mathbf{Q}^n$ for at least two more values of *n* Considering $a_{4,2}$ *Dep on previous M1* For $n=16$ *SR* With no working, $n=16$ stated B3 2031 stated B4

Post-multiplication by transition matrix

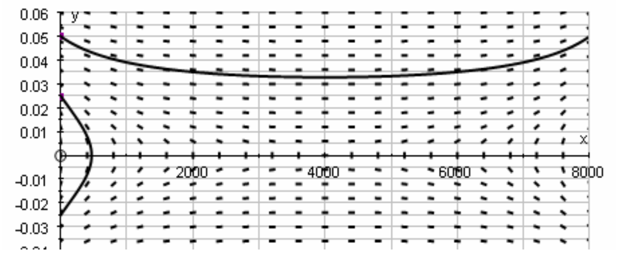
5 (i)	$\mathbf{P}=\begin{pmatrix}0.8&0.2&0\\0.1&0.6&0.3\\0&0.15&0.85\end{pmatrix}$	B1B1B1 **3**	For the three rows
(ii)	$(0.6\quad 0.4\quad 0)\mathbf{P}^7$ $=(0.6\quad 0.4\quad 0)\begin{pmatrix}0.3204&0.3089&0.3706\\0.1545&0.2895&0.5560\\0.0927&0.2780&0.6293\end{pmatrix}$ $=(0.254\quad 0.301\quad 0.445)$ Division 3 is the most likely	M1 M1 A1 M1 A1 A1 **6**	Considering $\mathbf{P}^7$ (or $\mathbf{P}^8$ or $\mathbf{P}^6$) Evaluating a power of **P** For $\mathbf{P}^7$ (Allow ± 0.001 throughout) Evaluation of probabilities One probability correct Correctly determined
(iii)	$\mathbf{P}^n \to \begin{pmatrix}0.1429&0.2857&0.5714\\0.1429&0.2857&0.5714\\0.1429&0.2857&0.5714\end{pmatrix}$ Equilibrium probabilities are 0.143, 0.286, 0.571	M1 M1 A1 **3**	Considering powers of **P** Obtaining limit *Must be accurate to 3 dp if given as decimals*
	OR $(p\quad q\quad r)\mathbf{P}=(p\quad q\quad r)$ $0.8p+0.1q=p$ $0.2p+0.6q+0.15r=q$ M1 $0.3q+0.85r=r$ $q=2p,\ r=2q=4p$ and $p+q+r=1$ M1 $p=\frac{1}{7},\ q=\frac{2}{7},\ r=\frac{4}{7}$ A1		Obtaining at least two equations Solving (must use $p+q+r=1$)
(iv)	$\mathbf{Q}=\begin{pmatrix}0.8&0.2&0&0\\0.1&0.6&0.3&0\\0&0.15&0.75&0.1\\0&0&0&1\end{pmatrix}$	B1 B1 B1 **3**	Third row Fourth row Fully correct
(v)	$(0\quad 1\quad 0\quad 0)\mathbf{Q}^5$ $=(0\quad 1\quad 0\quad 0)\begin{pmatrix}0.4122&0.3131&0.2369&0.0378\\0.1566&0.2767&0.4105&0.1563\\0.0592&0.2052&0.4030&0.3326\\0&0&0&1\end{pmatrix}$ $=(0.1566\quad 0.2767\quad 0.4105\quad 0.1563)$ P(still in league) $=1-0.1563$ $=0.844$	M1 M1 A1 M1 A1 ft **5**	Considering $\mathbf{Q}^5$ (or $\mathbf{Q}^6$ or $\mathbf{Q}^4$) Evaluating a power of **Q** For 0.1563 (Allow 0.156 ± 0.001) For $1-a_{2,4}$ ft dependent on M1M1M1

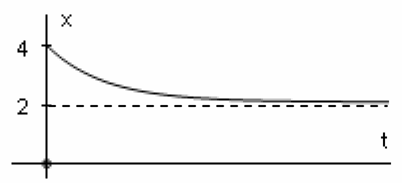
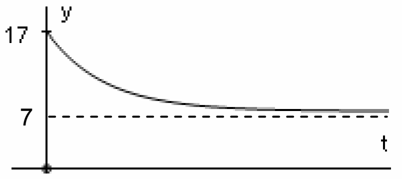
(vi)	P(out of league) is element $a_{2,4}$ in $\mathbf{Q}^n$ When $n=15,\ a_{2,4}=0.4849$ When $n=16,\ a_{2,4}=0.5094$ First year is 2031	M1 M1 A1 A1 **4**	Considering $\mathbf{Q}^n$ for at least two more values of *n* Considering $a_{2,4}$ *Dep on previous M1* For $n=16$ *SR* With no working, $n=16$ stated B3 2031 stated B4

Mark Scheme 4758
June 2006

1(i)	$\lambda = 0$	B1		
	$x = A\cos\sqrt{5}t + B\sin\sqrt{5}t$	M1	$\cos\sqrt{5}t$ or $\sin\sqrt{5}t$ or $A\cos\omega t + B\sin\omega t$ seen or GS for their λ	
		A1		
				3
(ii)	$(2\lambda)^2 - 4\cdot 5 < 0$	M1	Use of discriminant	
		A1	Correct inequality	
	$0 < \lambda < \sqrt{5}$	A1	Accept lower limit omitted or $-\sqrt{5}$	
				3
(iii)	$\alpha^2 + 2\alpha + 5 = 0$	M1	Auxiliary equation	
	$\alpha = -1 \pm 2j$	A1		
	$x = \mathrm{e}^{-t}\left(C\cos 2t + D\sin 2t\right)$	F1	CF for their roots	
				3
(iv)	$x_0 = C$	M1	Condition on *x*	
	$\dot{x} = -\mathrm{e}^{-t}\left(C\cos 2t + D\sin 2t\right) + \mathrm{e}^{-t}\left(-2C\sin 2t + 2D\cos 2t\right)$	M1	Differentiate (product rule)	
	$0 = -C + 2D$	M1	Condition on $\dot{x}$	
	$D = \frac{1}{2}x_0$			
	$x = x_0\,\mathrm{e}^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$	A1	cao	
				4
(v)	$\cos 2t + \frac{1}{2}\sin 2t = 0$	M1		
	$\tan 2t = -2$	M1		
	$t = 1.017$	A1	cao	
				3
(vi)	$\alpha^2 + 6\alpha + 5$	M1	Auxiliary equation	
	$\alpha = -1, -5$	A1		
	$x = E\,\mathrm{e}^{-t} + F\,\mathrm{e}^{-5t}$	F1	CF for their roots	
	$x_0 = E + F$	M1	Condition on *x*	
	$\dot{x} = -E\,\mathrm{e}^{-t} - 5F\,\mathrm{e}^{-5t}$			
	$0 = -E - 5F$	M1	Condition on $\dot{x}$	
	$E = \frac{5}{4}x_0, F = -\frac{1}{4}x_0$			
	$x = \frac{1}{4}x_0\left(5\,\mathrm{e}^{-t} - \mathrm{e}^{-5t}\right)$	A1	cao	
	$x = \frac{1}{4}x_0\,\mathrm{e}^{-t}\left(5 - \mathrm{e}^{-4t}\right)$	M1	Attempt complete method	
	$t > 0 \Rightarrow 5 > \mathrm{e}^{-4t}, x_0 > 0, \mathrm{e}^{-t} > 0 \Rightarrow x > 0$ i.e. never zero	E1	Fully justified (only $\neq 0$ required)	
				8

2(i)	$\lambda+2=0\Rightarrow\lambda=-2$	M1		
	CF $x=A\mathrm{e}^{-2t}$	A1		
	PI $x=at+b$	B1		
	$a+2(at+b)=t+1$	M1	Differentiate and substitute	
	$2a=1,\ a+2b=1$	M1	Compare	
	$a=\frac{1}{2}, b=\frac{1}{4}$	A1		
	$x=\frac{1}{2}t+\frac{1}{4}+A\mathrm{e}^{-2t}$	F1	CF + PI	
	$t=0, x=1\Rightarrow 1=\frac{1}{4}+A$	M1	Condition on *x*	
	$x=\frac{1}{2}t+\frac{1}{4}+\frac{3}{4}\mathrm{e}^{-2t}$	F1	Follow a non-trivial GS	
	Alternatively:			
	$I=\exp\left(\int 2\,\mathrm{d}t\right)=\mathrm{e}^{2t}$	M1 A1	Integrating factor	
	$\mathrm{e}^{2t}\frac{\mathrm{d}x}{\mathrm{d}t}+2\mathrm{e}^{2t}x=\mathrm{e}^{2t}\left(t+1\right)$	B1	Multiply DE by their *I*	
	$\mathrm{e}^{2t}x=\int \mathrm{e}^{2t}\left(t+1\right)\mathrm{d}t$	M1	Attempt integral	
	$=\frac{1}{2}\mathrm{e}^{2t}\left(t+1\right)-\int\frac{1}{2}\mathrm{e}^{2t}\,\mathrm{d}t$	M1	Integration by parts	
	$\mathrm{e}^{2t}x=\frac{1}{2}\mathrm{e}^{2t}\left(t+1\right)-\frac{1}{4}\mathrm{e}^{2t}+A$	A1		
	$x=\frac{1}{2}t+\frac{1}{4}+A\mathrm{e}^{-2t}$	F1	Divide by their *I* (must also divide constant)	
	$t=0, x=1\Rightarrow 1=\frac{1}{4}+A$	M1	Condition on *x*	
	$x=\frac{1}{2}t+\frac{1}{4}+\frac{3}{4}\mathrm{e}^{-2t}$	F1	Follow a non-trivial GS	
				9
(ii)	$\frac{2}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}$	M1	Separate	
	$\int\frac{2}{y}\,\mathrm{d}y=\int\frac{1}{x}\,\mathrm{d}x$	M1	Integrate	
	$2\ln y=\ln x+c$			
	$y=B\sqrt{x}$	M1	Make *y* subject, dealing properly with constant	
	$\left(t=0\right), x=1, y=4\Rightarrow y=4\sqrt{x}$	M1	Condition	
	$y=4\sqrt{\frac{1}{2}t+\frac{1}{4}+\frac{3}{4}\mathrm{e}^{-2t}}$	F1	*y* = 4√(their *x* in terms of *t*)	
				5
(iii)	$\frac{\mathrm{d}z}{\mathrm{d}x}+\frac{2}{x}z=6$	M1	Divide DE by *x*	
	$I=\exp\left(\int\frac{2}{x}\,\mathrm{d}x\right)$	M1	Attempt integrating factor	
	$=x^2$	A1	Simplified	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2z\right)=6x^2$	F1	Follow their integrating factor	
	$x^2z=2x^3+C$	A1		
	$z=2x+Cx^{-2}$	F1	Divide by their *I* (must also divide constant)	
	$\left(t=0\right), x=1, z=3\Rightarrow C=1$	M1	Condition on *z*	
	$z=2x+x^{-2}$	A1	cao (in terms of *x*)	
	$t=1\Rightarrow x=0.852$			
	$y=3.69$	B1	Any 2 values (at least 3sf)	
	$z=3.08$	B1	All 3 correct (and 3sf)	
				10

3(i)	$\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1}{v}\mathrm{f}(x)$ so $\left(\text{unless } \mathrm{f}(x)=0\right), v\to 0\Rightarrow\frac{\mathrm{d}v}{\mathrm{d}x}\to\pm\infty$	M1	Consider $\frac{\mathrm{d}v}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}v}$ when *v* = 0, but not if $\frac{\mathrm{d}v}{\mathrm{d}x}=0$	
	i.e. gradient parallel to *v*-axis (vertical)	E1	Must conclude about direction	
	$x=4000\Rightarrow v\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1}{5000^2}-\frac{1}{5000^2}=0$	M1	Consider $\frac{\mathrm{d}v}{\mathrm{d}x}$ when *x* = 4000	
	so if $v\neq 0$ then gradient parallel to *x*-axis (horizontal)	E1	Must conclude about direction	
		M1	Add to tangent field	
		A1	Several vertical direction indicators on *x*-axis	6
(ii)		M1	Attempt one curve	
		A1		
		M1	Attempt second curve	
		A1		
	$V_0=0.05\Rightarrow$ probe reaches B	B1	Must be consistent with their curve	
	$V_0=0.025\Rightarrow$ probe returns to A	B1	Must be consistent with their curve N.B. Cannot score these if curve not drawn	6
(iii)	$\int v\,\mathrm{d}v=\int\left((9000-x)^{-2}-(1000+x)^{-2}\right)\mathrm{d}x$	M1	Separate	
	$\frac{1}{2}v^2=\frac{1}{9000-x}+\frac{1}{1000+x}+c$	M1 B1 A1	Integrate LHS RHS	
	$\frac{1}{2}{V_0}^2=\frac{1}{9000}+\frac{1}{1000}+c$	M1	Condition	
	$v^2=\frac{2}{9000-x}+\frac{2}{1000+x}+{V_0}^2-\frac{1}{450}$	A1		6
(iv)	minimum when *x* = 4000	B1	Clearly stated	
	${v_{\min}}^2=\frac{2}{5000}+\frac{2}{5000}+{V_0}^2-\frac{1}{450}$	M1	Substitute their *x* into *v* or v^2	
		F1	Their v^2 or *v* when *x* = 4000	
	need ${v_{\min}}^2>0$	M1	For ${v_{\min}}^2>0$	
	${v_{\min}}^2>0$ if ${V_0}^2>\frac{1}{450}-\frac{4}{5000}$	M1	Attempt inequality for ${V_0}^2$	
	$V_0>0.0377$	A1	cao	6

4(i)	$\ddot{x} = 2\dot{x} - \dot{y}$	M1	Differentiate first equation	
	$= 2\dot{x} - (5x - 4y + 18)$	M1	Substitute for $\dot{y}$	
	$y = 2x + 3 - \dot{x}$	M1	y in terms of $x, \dot{x}$	
	$\ddot{x} = 2\dot{x} - 5x + 4(2x + 3 - \dot{x}) - 18$	M1	Substitute for y	
	$\ddot{x} + 2\dot{x} - 3x = -6$	E1	LHS	
		E1	RHS	
				6
(ii)	$\lambda^2 + 2\lambda - 3 = 0$	M1	Auxiliary equation	
	$\lambda = 1$ or -3	A1		
	CF $x = A\mathrm{e}^{-3t} + B\mathrm{e}^{t}$	F1	CF for their roots	
	PI $x = a$	B1	Constant PI	
	$-3a = -6 \Rightarrow a = 2$	B1	PI correct	
	$x = 2 + A\mathrm{e}^{-3t} + B\mathrm{e}^{t}$	F1	Their CF + PI	
	$y = 2x + 3 - \dot{x}$	M1	y in terms of $x, \dot{x}$	
	$= 4 + 2A\mathrm{e}^{-3t} + 2B\mathrm{e}^{t} + 3 - (-3A\mathrm{e}^{-3t} + B\mathrm{e}^{t})$	M1	Differentiate x and substitute	
	$y = 7 + 5A\mathrm{e}^{-3t} + B\mathrm{e}^{t}$	A1	Constants must correspond with those in x	
				9
(iii)	$4 = 2 + A + B$	M1	Condition on x	
	$17 = 7 + 5A + B$	M1	Condition on y	
	$A = 2, B = 0$	M1	Solve	
	$x = 2 + 2\mathrm{e}^{-3t}$	F1	Follow their GS	
	$y = 7 + 10\mathrm{e}^{-3t}$	F1	Follow their GS	
		B1 B1	Sketch of x starts at 4 and decreases Asymptote x = 2	
		B1 B1	Sketch of y starts at 17 and decreases Asymptote y =7	
				9

Mark Scheme 4761
June 2006

Q 1		mark		Sub
	$0 = u - 9.8 \times 3$	M1	*uvast* leading to *u* with $t = 3$ or $t = 6$	
	$u = 29.4$ so 29.4 m s^{-1}	A1	Signs consistent	
		M1	*uvast* leading to *s* with $t = 3$ or $t = 6$ or **their** *u*	
	$s = 0.5 \times 9.8 \times 9 = 44.1$ so 44.1 m	F1	FT **their** *u* if used with $t = 3$. Signs consistent. Award for 44.1, 132.3 or 176.4 seen. [Award maximum of 3 if one answer wrong]	
				4
				4

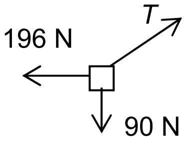
Q 2		mark		Sub
(i)	$\sqrt{(-6)^2 + 13^2} = 14.31782...$	M1	Accept $\sqrt{-6^2 + 13^2}$	
	so 14.3 N (3 s. f.)	A1		
				2
(ii)	Resultant is $\begin{pmatrix} -6 \\ 13 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$	B1	May not be explicit. If diagram used it must have correct orientation. Give if final angle correct.	
	Require 270 + $\arctan \dfrac{8}{3}$	M1	Use of $\arctan\left(\pm\dfrac{8}{3}\right)$ or $\arctan\left(\pm\dfrac{3}{8}\right)$ (±20.6° or ±69.4°) or equivalent on **their** resultant	
	so 339.4439…° so 339°	A1	cao. Do not accept -21°.	
				3
(iii)	$\begin{pmatrix} -3 \\ 5 \end{pmatrix} = 5\mathbf{a}$	M1	Use of N2L with accn *used* in vector form	
	so ($-0.6\,\mathbf{i} + \mathbf{j}$) m s^{-2}	A1	Any form. Units not required. isw.	
	change in velocity is ($-6\,\mathbf{i} + 10\,\mathbf{j}$) m s^{-1}	F1	10**a** seen. Units not required. Must be a vector. [SC1 for $a = \sqrt{3^2 + 5^2}\,/\,5 = 1.17$]	
				3
				8

Q 3		mark		Sub
(i)	$F = 14000 \times 0.25$	M1	Use of N2L . Allow $F = mga$ and wrong mass. No extra forces.	
	so 3500 N	A1		
				2
(ii)	$4000 - R = 3500$ so 500 N	B1	FT F from (i). Condone negative answer.	
				1
(iii)	$1150 - R_{\text{T}} = 4000 \times 0.25$	M1	N2L applied to truck (or engine) using all forces required. No extras. Correct mass. Do not allow use of $F = mga$. Allow sign errors.	
	so 150 N	A1	cao	
				2
(iv)	**either**			
	Component of weight down slope is	M1	Attempt to find cpt of *weight* (allow wrong mass). Accept $\sin \leftrightarrow \cos$. Accept use of $m\sin\theta$.	
	Extra driving force is cpt of *mg* down slope	M1	May be implied. Correct mass. No extra forces. Must have resolved weight component. Allow $\sin \leftrightarrow \cos$	
	$14000g\sin 3°$ $= 14000 \times 9.8 \times 0.0523359... = 7180.49...$			
	so 7180 N (3 s. f.)	A1		
	or			
		M1	Attempt to find cpt of *weight* (allow wrong mass). Accept $\sin \leftrightarrow \cos$. Accept use of $m\sin\theta$.	
	$D - 500 - 14000g\sin 3 = 14000 \times 0.25$	M1	N2L with all terms present with correct signs and mass. No extras. FT 500 N. Accept **their** 500 + 150 for resistance. Must have resolved weight component. Allow $\sin \leftrightarrow \cos$.	
	D = 11180.49… so extra is 7180 N (3 s. f.)	A1	Must be the extra force.	
				3 8

Q 4		**mark**		**Sub**
(i)	**either**			
	Need **j** cpt 0 so $18t^2 - 1 = 0$	M1	Need not solve	
	$\Rightarrow t^2 = \frac{1}{18}$. Only one root as $t > 0$	E1	Must establish only one of the two roots is valid	
	or			
	Establish sign change in **j** cpt	B1		
	Establish only one root	B1		
				2
(ii)	$\mathbf{v} = 3\,\mathbf{i} + 36t\,\mathbf{j}$	M1 A1	Differentiate. Allow **i** or **j** omitted	
	Need **i** cpt 0 and this never happens	E1	Clear explanation. Accept '**i** cpt always there' or equiv	3
(iii)	$x = 3t$ and $y = 18t^2 - 1$	B1	Award for these two expressions seen.	
	Eliminate *t* to give			
	$y = 18\left(\frac{x}{3}\right)^2 - 1$	M1	*t* properly eliminated. Accept any form and brackets missing	
	so $y = 2x^2 - 1$	A1	cao	
				3 8

Q 5		**mark**		**Sub**
(i)	$0^2 = V^2 - 2 \times 9.8 \times 22.5$	M1	Use of appropriate *uvast*. Give for correct expression	
	$V = 21$ so 21 m s^{-1}	E1	Clearly shown. Do not allow $v^2 = 0 + 2gs$ without explanation. Accept using $V = 21$ to show $s = 22.5$.	
				2
(ii)	$28\sin\theta = 21$	M1	Attempt to find angle of projection. Allow $\sin \leftrightarrow \cos$.	
	so $\theta = 48.59037...$	A1		
				2
(iii)	Time to highest point is $\frac{21}{9.8} = \frac{15}{7}$	B1	Or equivalent (time of whole flight)	
	Distance is $2 \times \frac{15}{7} \times 28 \times \cos(\textbf{their}\,\theta)$..	M1	Valid method for horizontal distance. Accept ½ time.	
			Do not accept 28 used for horizontal speed or vertical speed when calculating time.	
		B1	Horizontal speed correct	
	79.3725… so 79.4 m (3 s. f.)	A1	cao. Accept answers rounding to 79 or 80. [If angle with vertical found in (ii) allow up to full marks in (iii). If $\sin \leftrightarrow \cos$ allow up to B1 B1 M0 A1] [If $u^2 \sin 2\theta / g$ used then M1* Correct formula used. FT their angle. M1 Dep on *. Correct subst. FT their angle. A2 cao]	
				4 8

Q 6		**mark**		**Sub**
(i)	$0.5\times2\times12+0.5\times4\times12$	M1	Attempt at sum of areas or equivalent. No extra areas.	
	so 36 m	A1		
				2
(ii)	$8-\frac{36}{12}=5$ seconds	B1	cao	
				1
(iii)	$-6\ \text{m s}^{-2}$	M1	Attempt at accn for $0\le t\le2$	
		B1	must be - ve or equivalent	
				2
(iv)	$58.5=12\times6+0.5\times a\times36$	M1	Use of *uvast* with 12 and 58.5	
	so $a=-0.75$	A1		
				2
(v)	$a=-10+\frac{9}{2}t-\frac{3}{8}t^2$	M1	Differentiation	
		A1		
	$a(1)=-10+\frac{9}{2}-\frac{3}{8}=-5.875$	A1	cao	
				3
(vi)	$s=\int\left(12-10t+\frac{9}{4}t^2-\frac{1}{8}t^3\right)\text{d}t$	M1	Attempt to integrate	
		A1	At least one term correct	
	$=12t-5t^2+\frac{3}{4}t^3-\frac{1}{32}t^4+C$	A1	All correct. Accept + C omitted	
	$s=0$ when $t=0$ so $C=0$	A1*	Clearly shown	
	$s(8)=32$	A1	cao (award even if A1* is not given)	
				5
	either			
(vii)	$s(2)=9.5$ and $s(4)=8$	B1	Both calculated correctly from **their** s. No further marks if **their** $s(2)\le s(4)$	
	Displacement is negative	E1		
	Car going backwards	E1	Do *not* need car going backwards *throughout* the interval.	
	or			
	Evaluate $v(t)$ where $2<t<4$ or appeal to shape of the graph	B1	e.g. $v(3)=-1.125$	
			No further marks if **their** $v\ge0$	
	Velocity is negative	E1		
	Car going backwards	E1	Do *not* need car going backwards *throughout* the interval [Award WW2 for 'car going backwards'; WW1 for velocity or displacement negative]	
				3
				18

Q 7		**mark**		**Sub**
(i)	$T_{AB}\sin\alpha = 147$	M1	Attempt at resolving. Accept sin ↔ cos . Must have *T* resolved and equated to 147.	
	so $T_{AB} = \frac{147}{0.6}$	B1	Use of 0.6. Accept correct subst for angle in wrong expression.	
	= 245 so 245 N	A1	Only accept answers agreeing to 3 s. f. [Lami: M1 pair of ratios attempted; B1 correct sub;A1]	3
(ii)	$T_{BC} = 245\cos\alpha$	M1	Attempt to resolve 245 and equate to *T*, or equiv Accept sin ↔ cos	
	$= 245\times 0.8 = 196$	E1	Substitution of 0.8 clearly shown [SC1 $245\times 0.8 = 196$] [Lami: M1 pair of ratios attempted; E1]	2
(iii)	Geometry of A, B and C and weight of B the same and these determine the tension	E1	Mention of two of: same weight: same direction AB: same direction BC	
		E1	Specific mention of same geometry & weight or recognition of same force diagram	2
(iv)			No extra forces.	
		B1	Correct orientation and arrows	
		B1	'*T*' 196 and 90 labelled. Accept 'tension' written out.	
	either Realise that 196 N and 90 N are horiz and vert forces where resultant has magnitude and line of action of the tension	M1	Allow for only β or *T* attempted	
	$\tan\beta = 90/196$	B1	Use of arctan (196/90) or arctan (90/196) or equiv	
	$\beta = 24.6638...$ so 24.7 (3 s. f.)	A1		
	$T = \sqrt{196^2 + 90^2}$	M1	Use of Pythagoras	
	T = 215.675… so 216 N (3 s. f.)	E1		
	or			
	↑ $T\sin\beta - 90 = 0$	B1	Allow if *T* = 216 assumed	
	→ $T\cos\beta - 196 = 0$	B1	Allow if *T* = 216 assumed	
	Solving $\tan\beta = \frac{90}{196} = 0.45918...$	M1	Eliminating *T*, or…	
	$\beta = 24.6638...$ so 24.7 (3 s. f.)	A1	[If *T* = 216 assumed, B1 for β ; B1 for check in 2nd	
	T = 215.675… so 216 N (3 s. f.)	E1	equation; E0]	7
(v)	Tension on block is 215.675.. N (pulley is smooth and string is light)	B1	May be implied. Reasons not required.	
	$M\times 9.8\times\sin 40 = 215.675... + 20$	M1	*Equating* their tension on the block unresolved ± 20 to weight component. If equation in any other direction, normal reaction must be present.	
		A1	Correct	
	M = 37.4128… so 37.4 (3 s. f.)	A1	Accept answers rounding to 37 and 38	
				4 18

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Q 1		mark		Sub
(a) (i) (A)	PCLM → +ve $2\times4-6\times2=8v$ v = -0.5 so 0.5 m s^{-1} in opposite direction to initial motion of P	M1 A1 A1	Use of PCLM and correct mass on RHS Any form Direction must be negative and consistent or clear. Accept use of a diagram.	3
(B)	$0.5\times2\times4^2+0.5\times6\times2^2-0.5\times8\times(-0.5)^2$ = 27 J	M1 A1	Use of KE. Must sum initial terms. Must have correct masses FT **their** (A) only	2
(ii) (A)	PCLM → +ve $2\times4-6\times2=2v_P+6v_Q$ $v_P+3v_Q=-2$ NEL → +ve $\frac{v_Q-v_P}{-2-4}=-\frac{2}{3}$ $v_Q-v_P=4$ $v_Q=0.5$ so 0.5 m s^{-1} in orig direction of P $v_P=-3.5$ so 3.5 m s^{-1} in opp to orig dir of P	M1 A1 M1 A1 A1 A1	Use of PCLM Any form NEL Any form cao. Direction need not be made clear. cao. Direction must be negative and consistent or clear (e.g diag)	6
(B)	→ +ve $2\times-3.5-2\times4=-15$ N s so 15 N s in opp to orig direction	M1 A1	Use of change in momentum with correct mass. FT (A). Dir must be clear (e.g. diag)	2
(b)	Let $\alpha=\arcsin(12/13)$ and $\beta=\arcsin(3/5)$ Parallel: $26\cos\alpha=u\cos\beta$ so $26\times\frac{5}{13}=u\times\frac{4}{5}$ and u = 12.5 Perp: $e=\frac{u\sin\beta}{26\sin\alpha}$ $=\frac{12.5\times\frac{3}{5}}{26\times\frac{12}{13}}=\frac{5}{16}$	M1 A1 A1 M1 F1 F1	PCLM parallel to plane attempted. At least one resolution correct NEL on normal components attempted. FT **their** u FT **their** u	6
				19

Q 2		**mark**		**Sub**
(i)	Diagrams	B1	Internal force at B must be shown	
	cw moments about A $2 \times 90 - 3R_B = 0$	M1	1st moments equation attempted for either force.	
	$R_B = 60$ so 60 N upwards	A1	Accept direction not specified	
	cw moments about R: $T \downarrow$ $75 \times 1 + 3T - 60 \times 0.5 = 0$	M1	2nd moments equation for other force. All forces present. No extra forces.	
		A1	Allow only sign errors	
	$T = -15$ so 15 N upwards	A1	Direction must be clear (accept diag)	6
(ii)	cw moments about A $90 \times 2\cos 30 - V \times 3\cos 30 - U \times 3\cos 60 = 0$	M1	Moments equation with resolution. Accept terms missing	
		A1	All correct. Allow only sign errors.	
	giving $60\sqrt{3} = U + V\sqrt{3}$	E1	Clearly shown	3
(iii)	Diagram	B1	*U* and *V* correct with labels and arrows	1
(iv)	ac moments about C $75 \times 2\cos 30 + 3.5V\cos 30 - 3.5U\cos 60 = 0$	M1	Moments equation with resolution. Accept term missing	
		B1	At least two terms correct (condone wrong signs)	
	$\frac{300}{7}\sqrt{3} = U - V\sqrt{3}$	A1	Accept any form	
	Solving for *U* and *V*	M1	Any method to eliminate one variable	
	$U = \frac{360\sqrt{3}}{7}$ (= 89.0768…)	A1	Accept any form and any reasonable accuracy	
	$V = \frac{60}{7}$ (= 8.571428…)	F1	Accept any form and any reasonable accuracy [Either of *U* and *V* is cao. FT the other]	
	Resolve → on BC *F* = *U*	M1		
	so frictional force is $\frac{360\sqrt{3}}{7}$ N (= 89.1 N (3 s. f.))	F1		8
				18

Q 3		mark		Sub
(a)	$20000 = \left(R + 900g \times 0.1\right) \times 16$	M1	Use of *P* = *Fv*, may be implied.	
		B1	Correct weight term	
		A1	All correct	
	R = 368 so 368 N	A1		4
(b) (i)	$F_{\max} = \mu mg \cos\alpha$	B1	Correct expression for $F_{\max}$ **or** wt cpt down slope (may be implied and in any form)	
	Force down slope is weight cpt $mg\sin\alpha$	B1	Identifying $\sin\alpha$ as $5/13$ or equivalent	
	Require $\mu mg\cos\alpha \ge mg\sin\alpha$			
	so $\mu \ge \tan\alpha = \frac{5}{12}$	E1	Proper use of $F \le \mu R$ or equivalent.	
			[$\mu = \tan\alpha$ used WW; SC1]	3
(ii)	**either**			
	$0.5 \times 11 \times v^2$	M1	Use of work energy with at least three required terms attempted	
	$= 11g \times 1.5 \times \frac{5}{13} + 0.2 \times 11g \times 1.5 \times \frac{12}{13} + 9$	B1	Any term RHS. Condone sign error.	
		B1	Another term RHS. Condone sign error.	
		A1	All correct . Allow if trig consistent but wrong	
	$v^2 = 18.3717...$			
	v = 4.2862… so 4.29 m s^{-1} (3 s. f.)	A1	cao	
	or			5
	+ ve up the slope			
	$-11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$	M1	Use of N2L	
		B1	Any correct term on LHS	
	a = - 6.1239 m s^{-2}	**A1**		
	$v^2 = -3a$	**M1**	use of appropriate *uvast*	
	v = 4.286 m s^{-1}	**A1**	c.a.o.	
(iii)	**continued overleaf**			

3	**continued**			
(iii)	**either**			
	Extra GPE balances WD against resistances	M1	Or equivalent	
	$mgx\sin\alpha$	B1		
	$= 6(x+3) + 0.2\times 11g\times\cos\alpha(x+3)$	B1	One of 1st three terms on RHS correct	
		B1	Another of 1st 3 terms on RHS correct	
		A1	All correct. FT **their** *v* if used.	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	6
	or	M1	Allow 1 term missing	
	$0.5\times 11\times 18.3717...$	B1	KE. FT **their** *v*	
	$= (1.5+x)\times 11g\times\frac{5}{13} - 6(1.5+x)$	B1	Use of 1.5 + *x* (may be below)	
	$-(1.5+x)\times 0.2\times 11g\times\frac{12}{13}$	B1	WD against friction	
		A1	All correct	
	x = 4.99386… so 4.99 m (3 s. f.)	A1	cao.	
	or			
	+ ve down the slope			
	$11g\times {}^{5}\!/\!_{13} - 0.2\times 11g\times {}^{12}\!/\!_{13} - 6 = 11a$	M1	N2L with all terms present	
		A1	all correct except condone sign errors	
	$a = 1.4145...$ m s^{-2}	A1		
	$4.286^2 = 2a(1.5+x)$	M1	use of appropriate *uvast*	
		B1	for (1.5 + *x*) (may be implied)	
	x = 4.99	A1	c.a.o.	
				18

Q 4		mark		Sub
(i)	$100\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=10\begin{pmatrix}5\\0\end{pmatrix}+30\begin{pmatrix}10\\15\end{pmatrix}+30\begin{pmatrix}20\\15\end{pmatrix}+30\begin{pmatrix}25\\30\end{pmatrix}$	M1 B1 B1	Correct method for c.m. Total mass correct One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$100\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\begin{pmatrix}1700\\1800\end{pmatrix}$ $\bar{x}=17$ $\bar{y}=18$	A1 A1	cao cao. [Allow SC 4/5 for $\bar{x}=18$ and $\bar{y}=17$]	5
(ii)	$(17,18,20)$	B1 B1	*x*- and *y*- coordinates. FT from (i). *z* coordinate	2
(iii)	cw moments about horizontal edge thro' D *x* component $P\times20-60\times(20-17)=0$ *P* = 9	M1 B1 B1 A1	Or equivalent with all forces present One moment correct (accept use of mass or length) correct use of **their** $\bar{x}$ in a distance FT only **their** $\bar{x}$	4
(iv)	Diagram	B1	Normal reaction must be indicated acting vertically upwards at edge on O*z* and weight be in approximately the correct place.	1
(v)	On point of toppling ac moments about edge along O*z* $30\times Q-60\times17=0$ *Q* = 34 Resolving horizontally *F* = *Q* As 34 > 30, slips first	M1 B1 F1 B1 B1	Or equivalent with all forces present Any moment correct (accept use of mass or length) FT only **their** $\bar{x}$ FT **their** Q correctly argued.	5
				17

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1(a)(i)	[Force] $=\mathrm{M\,L\,T^{-2}}$ [Power] = [Force] × [Distance] ÷ [Time] = [Force] $\times\mathrm{L\,T^{-1}}$ $=\mathrm{M\,L^{2}\,T^{-3}}$	B1 M1 A1 **3**	or [Energy] $=\mathrm{M\,L^{2}\,T^{-2}}$ or [Energy] $\times\mathrm{T^{-1}}$
(ii)	$[\mathrm{RHS}]=\dfrac{(\mathrm{L})^{3}(\mathrm{L\,T^{-1}})^{2}(\mathrm{M\,L^{-3}})}{\mathrm{M\,L^{2}\,T^{-3}}}$ $=\mathrm{T}$ [LHS] = L so equation is not consistent	B1B1 M1 A1 E1 **5**	For $(\mathrm{L\,T^{-1}})^{2}$ and $(\mathrm{M\,L^{-3}})$ Simplifying dimensions of RHS With all working correct (cao) *SR* '$\ldots\ \mathrm{L}=\frac{28}{9}\pi\,\mathrm{T}$, so inconsistent' can earn B1B1M1A1E0
(iii)	[RHS] needs to be multiplied by $\mathrm{L\,T^{-1}}$ which are the dimensions of *u* Correct formula is $x=\dfrac{28\pi r^{3}u^{3}\rho}{9P}$	M1 A1 A1 cao **3**	RHS must appear correctly
	OR $x=k\,r^{\alpha}\,u^{\beta}\,\rho^{\gamma}\,P^{\delta}$ M1 $\beta=3$ A1 $x=\dfrac{28\pi r^{3}u^{3}\rho}{9P}$ A1		Equating powers of one dimension
(b)(i)	Elastic energy is $\frac{1}{2}\times150\times0.8^{2}$ $=48\ \mathrm{J}$	M1 A1 **2**	*Treat use of modulus* $\lambda=150\ \mathrm{N}$ *as MR*
(ii)	In extreme position, length of string is $2\sqrt{1.2^{2}+0.9^{2}}\quad(=3)$ elastic energy is $\frac{1}{2}\times150\times1.4^{2}\quad(=147)$ By conservation of energy, $147-48=\frac{1}{2}\times m\times10^{2}$ Mass is 1.98 kg	B1 M1 M1 A1 A1 **5**	for $\sqrt{1.2^{2}+0.9^{2}}$ or 1.5 or 3 allow M1 for $(2\times)\frac{1}{2}\times150\times0.7^{2}$ Equation involving EE and KE

2 (a)(i)	Vertically, $T\cos 55° = 0.6\times 9.8$ Tension is 10.25 N	M1 A1 **2**	
(ii)	Radius of circle is $r = 2.8\sin 55° \quad (=2.294)$	B1	
	Towards centre, $T\sin 55° = 0.6\times\dfrac{v^2}{2.8\sin 55°}$	M2	Give M1 for one error
	OR $T\sin 55° = 0.6\times(2.8\sin 55°)\times\omega^2$ M1 $\omega = 2.47$ $v = (2.8\sin 55°)\,\omega$ M1		or $T = 0.6\times 2.8\times\omega^2$ *Dependent on previous M1*
	Speed is 5.67 m s^{-1}	A1 **4**	
(b)(i)	Tangential acceleration is $r\alpha = 1.4\times 1.12$ $F_1 = 0.5\times 1.4\times 1.12$ $= 0.784\text{ N}$ Radial acceleration is $r\omega^2 = 1.4\omega^2$ $F_2 = 0.5\times 1.4\omega^2$ $= 0.7\omega^2\text{ N}$	M1 A1 M1 A1 **4**	*SR* $F_1 = -0.784,\ F_2 = -0.7\omega^2$ *penalise once only*
(ii)	Friction $F = \sqrt{F_1^{\,2} + F_2^{\,2}}$ Normal reaction $R = 0.5\times 9.8$ About to slip when $F = \mu\times 0.5\times 9.8$ $\sqrt{0.784^2 + 0.49\omega^4} = 0.65\times 0.5\times 9.8$ $\omega = 2.1$	M1 M1 A1 A1 A1 cao **5**	For LHS and RHS *Both dependent on M1M1*
(iii)	$\tan\theta = \dfrac{F_1}{F_2}$ $= \dfrac{0.784}{0.7\times 2.1^2}$ Angle is $14.25°$	M1 A1 A1 **3**	Allow M1 for $\tan\theta = \dfrac{F_2}{F_1}$ etc Accept 0.249 rad

3 (i)	$T_{\text{AP}} = \frac{1323}{3} \times 2 \quad (= 882)$ $T_{\text{BP}} = \frac{1323}{4.5} \times 2.5 \quad (= 735)$ $T_{\text{AP}} - mg - T_{\text{BP}} = 882 - 15 \times 9.8 - 735 = 0$ so P is in equilibrium	B1 B1 E1 **3**	
	OR $\frac{1323}{3}(\text{AP} - 3) = \frac{1323}{4.5}(\text{BP} - 4.5) + 15 \times 9.8$ B2 $\text{AP} + \text{BP} = 12$ and solving, $\text{AP} = 5$ E1		Give B1 for one tension correct
(ii)	Extension of AP is $5 - x - 3 = 2 - x$ $T_{\text{AP}} = \frac{1323}{3}(2 - x) = 441(2 - x)$ Extension of BP is $7 + x - 4.5 = 2.5 + x$ $T_{\text{BP}} = \frac{1323}{4.5}(2.5 + x) = 294(2.5 + x)$	E1 B1 B1 **3**	
(iii)	$441(2 - x) - 15 \times 9.8 - 294(2.5 + x) = 15\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -49x$ Motion is SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.898 \text{ s}$	M1 A1 M1 A1 **4**	Equation of motion involving 3 forces Obtaining $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x \ (+c)$ Accept $\frac{2}{7}\pi$
(iv)	Centre of motion is AP = 5 If minimum value of AP is 3.5, amplitude is 1.5 Maximum value of AP is 6.5 m	B1 **1**	
(v)	When $\text{AP} = 4.1, \quad x = 0.9$ Using $v^2 = \omega^2(A^2 - x^2)$ $v^2 = 49(1.5^2 - 0.9^2)$ Speed is 8.4 m s^{-1}	M1 A1 A1 **3**	Accept ± 8.4 or -8.4
	OR $x = 1.5 \sin 7t$ When $x = 0.9, \ 7t = 0.6435 \quad (t = 0.0919)$ $v = 7 \times 1.5 \cos 7t$ M1 $= 10.5 \cos(0.6435)$ A1 $= 8.4$ A1		or $x = 1.5 \cos 7t$ or $7t = 0.9273 \quad (t = 0.1325)$ or $v = -7 \times 1.5 \sin 7t$ $= (-)\, 10.5 \sin(0.9273)$

(vi)	$x = 1.5\cos 7t$	M1 A1	For $\cos(\sqrt{49}\,t)$ or $\sin(\sqrt{49}\,t)$ or $x = 1.5\sin 7t$ *M1A1 above can be awarded in (v) if not earned in (vi)*
	When $1.5\cos 7t = 0.5$	M1	or other fully correct method to find the required time e.g. $0.400 - 0.224$ or $0.224 - 0.049$
	Time taken is 0.176 s	A1 **4**	Accept 0.17 or 0.18

4 (i)	$\int \pi y^2 \, dx = \int_1^4 \pi x \, dx$ $= \left[\tfrac{1}{2}\pi x^2\right]_1^4 = 7.5\pi$ $\int \pi x y^2 \, dx$ $= \int_1^4 \pi x^2 \, dx \quad = \left[\tfrac{1}{3}\pi x^3\right]_1^4 \quad (= 21\pi)$ $\bar{x} = \frac{21\pi}{7.5\pi}$ $= 2.8$	M1 A1 M1 A1 M1 A1 **6**	π *may be omitted throughout*
(ii)	Cylinder has mass $3\pi\rho$ Cylinder has CM at $x = 2.5$ $(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$ $\bar{x} = 3$	B1 B1 M1 A1 E1 **5**	Or volume 3π Relating three CMs *(* ρ *and / or* π *may be omitted)* or equivalent, e.g. $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$ Correctly obtained
(iii)(*A*)	Moments about A, $S \times 3 - 96 \times 2 = 0$ $S = 64$ N Vertically, $R + S = 96$ $R = 32$ N	M1 A1 M1 A1 **4**	Moments equation or another moments equation *Dependent on previous M1*
(*B*)	Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$ Vertically, $R + S = 96 + 6$ $R = 35$ N, $S = 67$ N	M1 A1 A1 **3**	Moments equation Both correct
	OR Add 3 N to each of *R* and *S* M1 $R = 35$ N, $S = 67$ N A2		*Provided* $R \neq S$ Both correct

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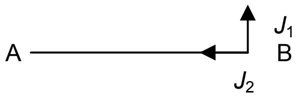
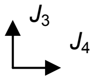
1(i)	$m = \frac{4}{3}\pi r^3 \rho$	M1	Expression for *m*	
	$\frac{\mathrm{d}m}{\mathrm{d}t} = 4\pi r^2 \rho \frac{\mathrm{d}r}{\mathrm{d}t}$	M1	Relate $\frac{\mathrm{d}m}{\mathrm{d}t}$ to $\frac{\mathrm{d}r}{\mathrm{d}t}$	
	$\lambda \cdot 4\pi r^2 = 4\pi r^2 \rho \frac{\mathrm{d}r}{\mathrm{d}t}$	M1	Use of $\frac{\mathrm{d}m}{\mathrm{d}t}$ proportional to surface area	
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\lambda}{\rho} = k$	E1	Accept alternative symbol for constant if used correctly (here and subsequently)	
	$r = r_0 + kt$	M1	Integrate and use condition	
	$m = \frac{4}{3}\pi\rho(r_0 + kt)^3$	A1		
				6
(ii)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(mv\right) = mg$	M1	N2L	
	$mv = \int mg \ \mathrm{d}t = \int \frac{4}{3}\pi\rho(r_0 + kt)^3 g \ \mathrm{d}t$	M1	Express *mv* as an integral	
	$= \frac{4}{3}\pi\rho g\left[\frac{1}{4k}(r_0 + kt)^4 + c\right]$	M1	Integrate	
	$t = 0, v = 0 \Rightarrow c = -\frac{1}{4k}{r_0}^4$	M1	Use condition	
	$\frac{4}{3}\pi\rho(r_0 + kt)^3 v = \frac{4}{3}\pi\rho g \cdot \frac{1}{4k}\left[(r_0 + kt)^4 - {r_0}^4\right]$	M1	Substitute for *m*	
	$v = \frac{g}{4k}\left[r_0 + kt - \frac{{r_0}^4}{(r_0 + kt)^3}\right]$	A1		
				6
2(i)	$AP = 2a\cos\theta$	M1	Attempt AP in terms of θ	
	$PB = \frac{5}{2}a - 2a\cos\theta$	E1		
	$V = -mg \cdot PB - mg \cdot PA\cos\theta$	M1	Attempt *V* in terms of θ	
	$= -mg\left(\frac{5}{2}a - 2a\cos\theta\right) - mg\left(2a\cos\theta\right)\cos\theta$			
	$= -mga\left(2\cos^2\theta - 2\cos\theta + \frac{5}{2}\right)$	E1		
				4
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\sin\theta\left(4\cos\theta - 2\right)$	M1	Differentiate	
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 \Rightarrow \sin\theta = 0 \text{ or } \cos\theta = \frac{1}{2}$	M1	Solve	
	$\Rightarrow \theta = 0 \text{ or } \pm\frac{1}{3}\pi$	A1	For 0 and either of $\frac{1}{3}\pi$ or $-\frac{1}{3}\pi$	
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga\sin\theta\left(-4\sin\theta\right) + mga\cos\theta\left(4\cos\theta - 2\right)$	M1	Differentiate again	
		A1		
	$\theta = 0 \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 2mga > 0 \Rightarrow$ stable	M1	Consider sign of V'' in one case	
		F1	Correct deduction for one value of θ	
	$\theta = \pm\frac{1}{3}\pi \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -3mga < 0 \Rightarrow$ unstable	F1	Correct deduction for another value of θ	
			N.B. Each F mark is dependent on both M marks. To get both F marks, the two values of θ must be physically possible (i.e. in the first or fourth quadrant) and not be equivalent or symmetrical positions.	
				8

3(i)	$P = Fv = mv\dfrac{\mathrm{d}v}{\mathrm{d}x}v$	M1	Use of $P = Fv$	
	$v^2\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.0004\left(10\,000v + v^3\right)$	A1	Or equivalent	
	$\int \dfrac{v}{10\,000 + v^2}\,\mathrm{d}v = \int 0.0004\,\mathrm{d}x$	M1	Separate variables	
	$\frac{1}{2}\ln\left\lvert 10\,000 + v^2\right\rvert = 0.0004x + c$	M1	Integrate	
	$v^2 = A\mathrm{e}^{0.0008x} - 10\,000$	M1	Rearrange	
	$x = 0, v = 0 \Rightarrow A = 10\,000$	M1	Use condition	
	$v = 100\sqrt{\mathrm{e}^{0.0008x} - 1}$	A1		
	$x = 900 \Rightarrow v = 102.7 > 80$ so successful			
	or $v = 80 \Rightarrow x = 618.37 < 900$ so successful	E1	Show that their *v* implies successful take off	
				8
(ii)	$v\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0.0004\left(10\,000v + v^3\right)$	F1	Follow previous DE	
	$\int \dfrac{1}{10\,000 + v^2}\,\mathrm{d}v = \int 0.0004\,\mathrm{d}t$	M1	Separate variables	
	$\frac{1}{100}\tan^{-1}\left(\frac{1}{100}v\right) = 0.0004t + k$	M1	Integrate	
		A1		
	$t = 0, v = 0 \Rightarrow k = 0$	M1	Use condition	
	$\Rightarrow v = 100\tan(0.04t)$	E1	Clearly shown	
	$v \to \infty$ at finite time suggests model invalid	B1		
				7
(iii)	$t = 11 \Rightarrow v = 47.0781$	B1	At least 3sf	
	Hence maximum $P = 230.049m$	M1	Attempt to calculate maximum *P*	
	$v = 47.0781 \Rightarrow x = 250.237$	M1	Use solution in (i) to calculate *x*	
	$v^2\dfrac{\mathrm{d}v}{\mathrm{d}x} = 230.049$	M1	Set up DE for $t \ldots 11$. Constant acceleration formulae $\Rightarrow$ M0.	
	$\frac{1}{3}v^3 = 230.049x + B$	M1	Separate variables and integrate	
		F1	Follow their maximum *P* (condone no constant)	
	$v = 47.0781, x = 250.237 \Rightarrow B = -22786.3$	M1	Use condition on *x*, *v* (not $v = 0$, not $x = 0$ unless clearly compensated for when making conclusion). Constant acceleration formulae $\Rightarrow$ M0.	
	$v = 80 \Rightarrow x = 840.922$ or $x = 900 \Rightarrow v = 82.0696$	M1	Relevant calculation. Must follow solving a DE.	
	so successful	A1	All correct (accept 2sf or more)	
				9

4(i)	Considering elements of length $\delta x \Rightarrow I = \int_0^{2a} \rho x^2 \,\mathrm{d}x$	M1	Set up integral	
	$= \frac{M}{8a^2}\int_0^{2a}\left(5ax^2 - x^3\right)\mathrm{d}x$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2}\left[\frac{5}{3}ax^3 - \frac{1}{4}x^4\right]_0^{2a}$	M1	Integrate	
	$= \frac{7}{6}Ma^2$	E1		
	Considering elements of length $\delta x \Rightarrow M\bar{x} = \int_0^{2a} \rho x \,\mathrm{d}x$	M1	Set up integral	
	$= \frac{M}{8a^2}\int_0^{2a}\left(5ax - x^2\right)\mathrm{d}x$	M1	Substitute for ρ in predominantly correct integral	
	$= \frac{M}{8a^2}\left[\frac{5}{2}ax^2 - \frac{1}{3}x^3\right]_0^{2a}$	M1	Integrate	
	$\bar{x} = \frac{11}{12}a$	E1		
				8
(ii)	$\frac{1}{2}I\dot{\theta}^2 = Mg\cdot\frac{11}{12}a(1-\cos\theta)$	M1	KE term in terms of angular velocity	
		B1	$\pm Mg\cdot\frac{11}{12}a\cos\theta$ seen	
		M1	energy equation	
	$\dot{\theta} = \sqrt{\frac{11g}{7a}(1-\cos\theta)}$	A1		
				4
(iii)	 $\theta = \frac{1}{2}\pi \Rightarrow \dot{\theta} = \sqrt{\frac{11g}{7a}}$	F1	Their $\dot{\theta}$ at $\theta = \frac{1}{2}\pi$	
	$2a\cdot\left(-J_1\right) = I\left(0 - \sqrt{\frac{11g}{7a}}\right)$	M1	Use of angular momentum	
		A1	Correct equation (their $\dot{\theta}$)	
	$J_1 = \frac{1}{12}M\sqrt{77ag}$	E1		
	$J_2 = \frac{1}{12}M\sqrt{77ag}$	B1	Correct answer or follow their J_1	
				5
(iv)	 $J_4 = J_2$	M1	Consider horizontal impulses	
	$= \frac{1}{12}M\sqrt{77ag}$	F1	Follow their J_2	
	$J_3 + J_1 = M\cdot\frac{11}{12}a\sqrt{\frac{11g}{7a}}$	M1	Vertical impulse-momentum equation	
		M1	Use of $r\dot{\theta}$	
	$J_3 = \frac{1}{21}M\sqrt{77ag}$	A1	cao	
	angle $= \tan^{-1}\left(\frac{J_3}{J_4}\right) = \tan^{-1}\left(\frac{\frac{1}{21}M\sqrt{77ag}}{\frac{1}{12}M\sqrt{77ag}}\right)$	M1	Must substitute	
	$= \tan^{-1}\left(\frac{4}{7}\right) \approx 0.519\,\text{rad} \approx 29.7°$	A1	cao (any correct form)	
				7

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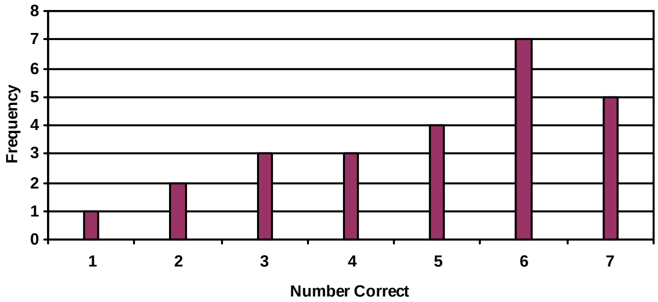
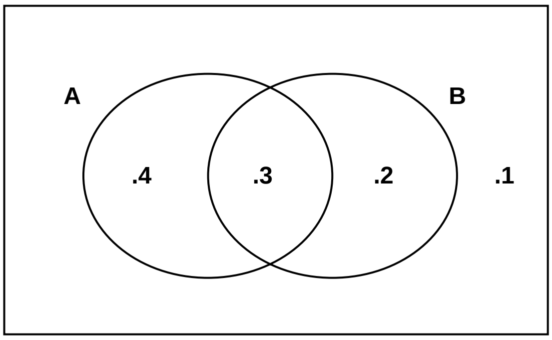
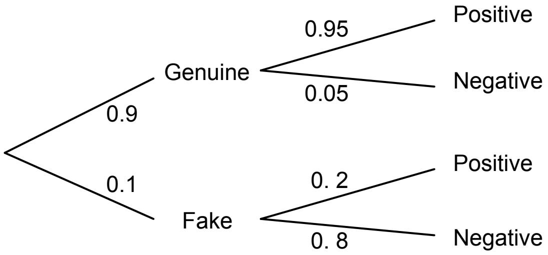
Q1 **(i)**		G1 Labelled linear scales G1 Height of lines	**2**
(ii)	Negative (skewness)	B1	**1**
(iii)	Σfx = 123 so mean = 123/25 = 4.92 o.e. $S_{xx} = 681 - \frac{123^2}{25} = 75.84$ M.s.d = $\frac{75.84}{25} = 3.034$	B1 M1 for S_{xx} attempted A1 FT their 4.92	**3**
(iv)	Total for 25 days is 123 and totals for 31 days is 155. Hence total for next 6 days is 32 and so mean = 5.33	M1 31 x 5 – 25xtheir 4.92 A1 FT their 123	**2**
		TOTAL	**8**
Q2 **(i)**	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \frac{7}{10} \times \frac{3}{7}$ $\rightarrow \mathrm{P}(A \cap B) = 0.3$ o.e.	M1 Product of these fractions A1	**2**
(ii)		B1FT either 0.4 or 0.2 in correct place B1FT all correct and labelled	**2**
(iii)	P(*B*|*A*) ≠ P(*B*), 3/7 ≠ 0.5 Unequal so not independent	E1 Correct comparison E1*dep* for ‘not independent’	**2**
(iv)	3/7 < 0.5 so Isobel is less likely to score when her parents attend	E1 for comparison E1*dep*	**2**
		TOTAL	**8**

Q3 (i)	P(X = 1) = 7k, P(X = 2) = 12k, P(X = 3) = 15k, P(X = 4) = 16k 50k = 1 so k = 1/50	M1 for addition of four multiples of k A1 **ANSWER GIVEN**	**2**
(ii)	E(X) = 1 x 7k + 2 x 12k + 3 x 15k + 4 x 16k = 140k = 2.8 OR E(X) = 1 x $^{7}/_{50}$ + 2 x $^{12}/_{50}$ + 3 x $^{15}/_{50}$ + 4 x $^{16}/_{50}$ = $^{140}/_{50}$ = 2.8 oe Var(X) = 1 x 7k + 4 x 12k + 9 x 15k + 16 x 16k -7.84 = 1.08 OR Var(X) = 1 x $^{7}/_{50}$ + 4 x $^{12}/_{50}$ + 9 x $^{15}/_{50}$ + 16 x $^{16}/_{50}$ -7.84 = 8.92 – 7.84 = 1.08	M1 for Σxp (at least 3 terms correct) A1 CAO M1 Σx^2p (at least 3 terms correct) M1*dep* for – their E(X)² NB provided Var(X) > 0 A1 FT their E(X)	**5**
		TOTAL	**7**
Q4 (i)	4 x 5 x 3 = 60	M1 for 4 x 5 x 3 A1 CAO	**2**
(ii)	**(*A*)** $\binom{4}{2}$ = 6 **(*B*)** $\binom{4}{2}\binom{5}{2}\binom{3}{2}$ = 180	B1 **ANSWER GIVEN** B1 CAO	**2**
(iii)	**(*A*)** 1/5 **(*B*)** $\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}=\frac{2}{5}$	B1 CAO M1 for $\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}$ A1	**3**
		TOTAL	**7**
Q5 (i)	$\mathrm{P}(X=2)=\binom{3}{2}\times 0.87^2\times 0.13=0.2952$	M1 0.87^2 x 0.13 M1 $\binom{3}{2}$ x p^2q **with p+q=1** A1 CAO	**3**
(ii)	In 50 throws expect 50 (0.2952) = 14.76 times	B1 FT	**1**
(iii)	P (two 20's twice) = $\binom{4}{2}\times 0.2952^2\times 0.7048^2=0.2597$	M1 $0.2952^2\times 0.7048^2$ A1 FT their 0.2952	**2**
		TOTAL	**6**

Q6 **(i)**		G1 for left hand set of branches fully correct including labels and probabilities G1 for right hand set of branches fully correct	**2**
(ii)	P (test is positive) = (0.9)(0.95) + (0.1)(0.2) = 0.875	M1 Two correct pairs added A1 CAO	**2**
(iii)	P (test is correct) = (0.9)(0.95) + (0.1)(0.8) = 0.935	M1 Two correct pairs added A1 CAO	**2**
(iv)	P (Genuine|Positive) = 0.855/0.875 = 0.977	M1 Numerator M1 Denominator A1 CAO	**3**
(v)	P (Fake|Negative) = 0.08/0.125 = 0.64	M1 Numerator M1 Denominator A1 CAO	**3**
(vi)	EITHER: A positive test means that the painting is almost certain to be genuine so no need for a further test. However, more than a third of those paintings with a negative result are genuine so a further test is needed. NOTE: Allow sensible alternative answers	E1FT E1FT	**2**
(vii)	P (all 3 genuine) = $(0.9 \times 0.05 \times 0.96)^3$ $= (0.045 \times 0.96)^3$ $= (0.0432)^3$ = 0.0000806	M1 for 0.9 x 0.05 (=0.045) M1 for complete correct triple product M1*indep* for cubing A1 CAO	**4**
		TOTAL	**18**

Q7 (i)	$X \sim B(20, 0.1)$ (***A***) $P(X = 1) = \binom{20}{1} \times 0.1 \times 0.9^{19} = 0.2702$ OR from tables $0.3917 - 0.1216 = 0.2701$ (***B***) $P(X \geq 1) = 1 - 0.1216 = 0.8784$	M1 0.1×0.9^{19} M1 $\binom{20}{1} \times pq^{19}$ A1 CAO OR: M2 for 0.3917 – 0.1216 A1 CAO M1 P(X=0) *provided that P(X≥1)=1–P(X≤1) not seen* M1 1-P(X=0) A1 CAO	**3** **3**
(ii)	EITHER: $1 - 0.9^n \geq 0.8$ $0.9^n \leq 0.2$ Minimum $n = 16$ OR (using trial and improvement): Trial with 0.9^{15} or 0.9^{16} or 0.9^{17} $1 - 0.9^{15} = 0.7941 < 0.8$ and $1 - 0.9^{16} = 0.8147 > 0.8$ Minimum $n = 16$ NOTE: $n = 16$ unsupported scores SC1 only	M1 for 0.9^n M1 for inequality A1 CAO M1 M1 A1 CAO	**3**
(iii)	(***A***) Let p = probability of a randomly selected rock containing a fossil (for population) H_0: $p = 0.1$ H_1: $p < 0.1$	B1 for definition of p B1 for H_0 B1 for H_1	**3**
	(***B***) Let $X \sim B(30, 0.1)$ $P(X \leq 0) = 0.0424 < 5\%$ $P(X \leq 1) = 0.0424 + 0.1413 = 0.1837 > 5\%$ So critical region consists only of 0.	M1 for attempt to find P(X ≤ 0) or P(X ≤ 1) using binomial M1 for both attempted M1 for comparison of either of the above with 5% A1 for critical region dep on both comparisons **(NB Answer given)**	**4**
	(***C***) 2 does not lie in the critical region. So there is insufficient evidence to reject the null hypothesis and we conclude that it seems that 10% of rocks in this area contain fossils.	M1 for comparison A1 for conclusion **in context**	**2**
		TOTAL	**18**

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(i)	$P(X = 1) = 8 \times 0.1^1 \times 0.9^7$ $= 0.383$	M1 for binomial probability P(X=1) A1 (at least 2sf) CAO	**2**
(ii)	$\lambda = 30 \times 0.1 = 3$	B1 for mean SOI	**1**
	(A) $P(X = 6) = e^{-3}\dfrac{3^6}{6!} = 0.0504$(3 s.f.) or from tables $= 0.9665 - 0.9161 = 0.0504$	M1 for calculation or use of tables to obtain P(X=6) A1 (at least 2sf) CAO	**2**
	(B) Using tables: $P(X \geq 8) = 1 - P(X \leq 7)$ $= 1 - 0.9881 = 0.0119$	M1 for correct probability calc' A1 (at least 2sf) CAO	**2**
(iii)	*n* is large and *p* is small	B1, B1 Allow appropriate numerical ranges	**2**
(iv)	$\mu = np = 120 \times 0.1 = 12$ $\sigma^2 = npq = 120 \times 0.1 \times 0.9 = 10.8$	B1 B1	**2**
(v)	$P(X > 15.5) = P\left(Z > \dfrac{15.5 - 12}{\sqrt{10.8}}\right)$ $= P(Z > 1.065) = 1 - \Phi(1.065) = 1 - 0.8566$ $= 0.1434$ NB Allow full marks for use of N(12,12) as an approximation to Poisson(12) leading to $1 - \Phi(1.010) = 1 - 0.8438 = 0.1562$	B1 for correct continuity correction. M1 for probability using correct tail A1 **cao, (**but FT wrong or omitted CC)	**3**
(vi)	From tables $\Phi^{-1}(0.99) = 2.326$ $\dfrac{x + 0.5 - 12}{\sqrt{10.8}} \geq 2.326$ $x = 11.5 + 2.326 \times \sqrt{10.8} \geq 19.14$ So 20 breakfasts should be carried NB Allow full marks for use of N(12,12) leading to $x \geq 11.5 + 2.326 \times \sqrt{12} = 19.56$	B1 for 2.326 seen M1 for equation in *x* and positive *z*-value A1 CAO (condone 19.64) A1FT for rounding appropriately (i.e. round up if c.c. used o/w rounding should be to nearest integer)	**4**
			18

Question 2

(i)	$X \sim N(49.7, 1.6^2)$ (*A*) $P(X > 51.5) = P\left(Z > \frac{51.5 - 49.7}{1.6}\right)$ $= P(Z > 1.125)$ $= 1 - \Phi(1.125) = 1 - 0.8696 = 0.1304$ (*B*) $P(X < 48.0) = P\left(Z < \frac{48.0 - 49.7}{1.6}\right)$ $= P(Z < -1.0625) = 1 - \Phi(1.0625)$ $= 1 - 0.8560 = 0.1440$ $P(48.0 < X < 51.5) = 1 - 0.1304 - 0.1440 = 0.7256$	M1 for standardizing M1 for prob. calc. A1 (at least 2 s.f.) M1 for appropriate prob' calc. A1 (0.725 – 0.726)	**5**
(ii)	P(one over 51.5, three between 48.0 and 51.5) $= \binom{4}{1} \times 0.7256 \times 0.2744^3 = 0.0600$	M1 for coefficient M1 for 0.7256×0.2744^3 A1 FT (at least 2 sf)	**3**
(iii)	From tables, $\Phi^{-1}(0.60) = 0.2533$, $\Phi^{-1}(0.30) = -0.5244$ $49.0 = \mu + 0.2533\sigma$ $47.5 = \mu - 0.5244\sigma$ $1.5 = 0.7777\sigma$ $\sigma = 1.929$, $\mu = 48.51$	B1 for 0.2533 or 0.5244 seen M1 for at least one correct equation μ & σ M1 for attempt to solve two correct equations A1 CAO for both	**4**
(iv)	Where μ denotes the mean circumference of the entire population of organically fed 3-year-old boys. $n = 10$, Test statistic $Z = \frac{50.45 - 49.7}{1.6/\sqrt{10}} = \frac{0.75}{0.5060} = 1.482$ 10% level 1 tailed critical value of z is 1.282 1.482 > 1.282 so significant. There is sufficient evidence to reject H_0 and conclude that organically fed 3-year-old boys have a higher mean head circumference.	E1 M1 A1(at least 3sf) B1 for 1.282 M1 for comparison leading to a conclusion A1 for conclusion in context	**6**
			18

Question 3

(i)	**EITHER:** $S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x \Sigma y = 6235575 - \frac{1}{10} \times 4715 \times 13175$ $= 23562.5$ $S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 2237725 - \frac{1}{10} \times 4715^2 =$ 14602.5 $S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 17455825 - \frac{1}{10} \times 13175^2 =$ 97762.5 $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{23562.5}{\sqrt{14602.5 \times 97762.5}} = 0.624$ **OR:** $\text{cov}(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = 6235575/10 - 471.5 \times 1317.5$ $= 2356.25$ $\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(14602.5/10)} = \sqrt{1460.25} = 38.21$ $\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(97762.5/10)} = \sqrt{9776.25} = 98.87$ $r = \frac{\text{cov(x,y)}}{rmsd(x)rmsd(y)} = \frac{2356.25}{38.21 \times 98.87} = 0.624$	M1 for method for S_{xy} M1 for method for at least one of S_{xx} or S_{yy} A1 for at least one of S_{xy}, S_{xx} or S_{yy} correct M1 for structure of *r* A1 (0.62 to 0.63) M1 for method for cov (*x,y*) M1 for method for at least one msd A1 for at least one of S_{xy}, S_{xx} or S_{yy} correct M1 for structure of *r* A1 (0.62 to 0.63)	**5**
(ii)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ (two-tailed test) where ρ is the population correlation coefficient For *n* = 10, 5% critical value = 0.6319 Since 0.624 < 0.6319 we cannot reject H_0: There is not sufficient evidence at the 5% level to suggest that there is any correlation between length and circumference.	B1 for H_0, H_1 in symbols B1 for defining ρ B1FT for critical value M1 for sensible comparison leading to a conclusion A1 FT for result B1 FT for conclusion in context	**6**
(iii)	*(A)* This is the probability of rejecting H_0 when it is in fact true. *(B)* Advantage of 1% level – less likely to reject H_0 when it is true. Disadvantage of 1% level – less likely to accept H_1 when H_0 is false.	B1 for 'P(reject H_0)' B1 for 'when true' B1, B1 Accept answers in context	**2** **2**

(iv)	The student's approach is not valid. If a statistical procedure is repeated with a new sample, we should not simply ignore one of the two outcomes. The student could combine the two sets of data into a single set of twenty measurements.	E1 E1 – allow suitable alternatives. E1 for combining samples.	**3**
			18

Question 4

<table>
<tr><td>(i)</td><td>H_0: no association between musical preference and age;
H_1: some association between musical preference and age;</td><td>B1</td><td>1</td></tr>
<tr><td></td><td>
<table>
<tr><th colspan="2" rowspan="2">Observed</th><th colspan="3">Musical preference</th><th rowspan="2">Row totals</th></tr>
<tr><th>Pop</th><th>Classical</th><th>Jazz</th></tr>
<tr><td rowspan="3">Age group</td><td>Under 25</td><td>57</td><td>15</td><td>12</td><td>84</td></tr>
<tr><td>25 – 50</td><td>43</td><td>21</td><td>21</td><td>85</td></tr>
<tr><td>Over 50</td><td>22</td><td>32</td><td>27</td><td>81</td></tr>
<tr><td colspan="2">Column totals</td><td>122</td><td>68</td><td>60</td><td>250</td></tr>
</table>
<table>
<tr><th colspan="2" rowspan="2">Expected</th><th colspan="3">Musical preference</th><th rowspan="2">Row totals</th></tr>
<tr><th>Pop</th><th>Classical</th><th>Jazz</th></tr>
<tr><td rowspan="3">Age group</td><td>Under 25</td><td>40.992</td><td>22.848</td><td>20.160</td><td>84</td></tr>
<tr><td>25 – 50</td><td>41.480</td><td>23.120</td><td>20.400</td><td>85</td></tr>
<tr><td>Over 50</td><td>39.528</td><td>22.032</td><td>19.440</td><td>81</td></tr>
<tr><td colspan="2">Column totals</td><td>122</td><td>68</td><td>60</td><td>250</td></tr>
</table>
</td><td>M1 A2 for expected values (at least 1 dp) (allow A1 for at least one row or column correct)</td><td></td></tr>
<tr><td></td><td>
<table>
<tr><th colspan="2" rowspan="2">Contributions</th><th colspan="3">Musical preference</th><th rowspan="2"></th></tr>
<tr><th>Pop</th><th>Classical</th><th>Jazz</th></tr>
<tr><td rowspan="3">Age group</td><td>Under 25</td><td>6.25</td><td>2.70</td><td>3.30</td><td></td></tr>
<tr><td>25 – 50</td><td>0.06</td><td>0.19</td><td>0.02</td><td></td></tr>
<tr><td>Over 50</td><td>7.77</td><td>4.51</td><td>2.94</td><td></td></tr>
<tr><td colspan="2"></td><td></td><td colspan="2"></td><td></td></tr>
</table>

$X^2 = 27.74$</td><td>M1 for valid attempt at $(O-E)^2/E$
A1 for all correct
M1dep for summation
A1 for X^2 (27.7 – 27.8)</td><td>7</td></tr>
<tr><td></td><td>Refer to χ_4^2
Critical value at 5% level = 9.488
Result is significant
There is some association between age group and musical preference.
NB if H_0 H_1 reversed, or ‘correlation’ mentioned, do not award first B1or final E1</td><td>B1 for 4 deg of f
B1 CAO for cv
B1FT
E1 (conclusion in context)</td><td>4</td></tr>
</table>

(ii)	The values of 6.25 and 7.77 show that under 25's have a strong positive association with pop whereas over 50's have a strong negative association with pop. The values of 4.51 and 2.94 show that over 50's have a reasonably strong positive association with both classical and jazz. The values of 2.70 and 3.30 show that under 25's have a reasonably strong negative associations with both classical and jazz. The 25-50 group's preferences differ very little from the overall preferences.	B1, B1 for specific reference to a value from the table of contributions followed by an appropriate comment B1, B1 (as above for second value) B1, B1 (as above for third value)	**6**
			18

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<table>
<tr><td>Q1</td><td>$f(x) = 12x^3 - 24x^2 + 12x, \qquad 0 \le x \le 1$</td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>$E(X) = \int_0^1 x f(x)\,dx$</td><td>M1</td><td>Integral for E(X) including limits (which may appear later).</td><td></td></tr>
<tr><td></td><td>$= 12\left[\frac{x^5}{5} - 2\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$</td><td>A1</td><td>Successfully integrated.</td><td></td></tr>
<tr><td></td><td>$= 12\left[\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right] = 12 \times \frac{1}{30} = \frac{2}{5}$</td><td>A1</td><td>Correct use of limits leading to final answer. C.a.o.</td><td></td></tr>
<tr><td></td><td>For mode, $f'(x) = 0$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>$f'(x) = 12(3x^2 - 4x + 1) = 12(3x - 1)(x - 1)$
$\therefore f'(x) = 0$ for $x = 1$ and $x = \frac{1}{3}$</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td>Any convincing argument (e.g. $f''(x)$) that $\frac{1}{3}$ (and not 1) is the mode.</td><td>A1</td><td></td><td>6</td></tr>
<tr><td>(ii)</td><td>Cdf $F(x) = \int_0^x f(t)\,dt$
$= 12\left(\frac{x^4}{4} - 2\frac{x^3}{3} + \frac{x^2}{2}\right)$
$= 3x^4 - 8x^3 + 6x^2$</td><td>M1
A1</td><td>Definition of cdf, including limits (or use of “+c” and attempt to evaluate it), possibly implied later. Some valid method must be seen.
Or equivalent expression; condone absence of domain [0,1].</td><td></td></tr>
<tr><td></td><td>$F\left(\frac{1}{4}\right) = \frac{3}{256} - \frac{8}{64} + \frac{6}{16} = \frac{3-32+96}{256} = \frac{67}{256}$
$F\left(\frac{1}{2}\right) = \frac{3}{16} - \frac{8}{8} + \frac{6}{4} = \frac{3-16+24}{16} = \frac{11}{16}$
$F\left(\frac{3}{4}\right) = \frac{3\times81}{256} - \frac{8\times27}{64} + \frac{6\times9}{16} = \frac{243}{256}$</td><td>B1</td><td>For all three; answers given; must show convincing working (such as common denominator)! Use of decimals is not acceptable.</td><td>3</td></tr>
<tr><td>(iii)</td><td><table><tr><td>o_i</td><td>126</td><td>209</td><td>131</td><td>46</td></tr><tr><td>e_i</td><td>134</td><td>352 – 134 = 218</td><td>486 – 352 = 134</td><td>26</td></tr></table></td><td>B2</td><td>For e_i.
B1 if any 2 correct, provided Σ = 512.</td><td></td></tr>
<tr><td></td><td>$X^2 = 0{\cdot}4776 + 0{\cdot}3716 + 0{\cdot}0672 + 15{\cdot}3846 = 16{\cdot}30(1)$</td><td>M1
A1</td><td></td><td></td></tr>
<tr><td></td><td>Refer to χ_3^2.</td><td>M1</td><td>Must be some clear evidence of reference to χ_3^2, probably implicit by reference to a critical point (5% : 7·815; 1% : 11·34). No ft (to the A marks) if incorrect χ^2 used, but E marks are still available.</td><td></td></tr>
<tr><td></td><td>Very highly significant.
Very strong evidence that the model does not fit.</td><td>A1
A1</td><td>There must be at least one reference to “very …”, i.e. the extremeness of the test statistic.</td><td></td></tr>
<tr><td></td><td>The main feature is that we observe many</td><td></td><td>Or e.g. “big/small” contributions</td><td></td></tr>
</table>

	more loads at the "top end" than expected. The other observations are below expectation, but discrepancies are comparatively small.	E1 E1	to X^2 gets E1, … … and directions of discrepancies gets E1.	9
				18

Q2	A to B : $X \sim$ N(26, $\sigma = 3$) B to C : $Y \sim$ N(15, $\sigma = 2$)		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	P(X < 24) = $\mathrm{P}\left(Z < \frac{24-26}{3} = -0 \cdot 6667\right)$ = 1 – 0·7476 = 0·2524	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	$X + Y \sim \mathrm{N}(41,$ $\sigma^2 = 9 + 4 = 13 \ [\sigma = 3 \cdot 6056])$ P(this < 42) = $\mathrm{P}\left(Z < \frac{42-41}{3 \cdot 6056} = 0 \cdot 2774\right) = 0 \cdot 6093$	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iii)	$0 \cdot 85X \sim \mathrm{N}(22 \cdot 1,$ $\sigma^2 = (0 \cdot 85)^2 \times 9 = 6 \cdot 5025 \ [\sigma = 2 \cdot 55])$ P(this < 24) = $\mathrm{P}\left(Z < \frac{24-22 \cdot 1}{2 \cdot 55} = 0 \cdot 7451\right)$ = 0·7719	B1 B1 A1	Mean. Variance. Accept sd. c.a.o.	3
(iv)	$0 \cdot 9X + 0 \cdot 8Y \sim N(23 \cdot 4 + 12 = 35 \cdot 4,$ $\sigma^2 = (0 \cdot 9)^2 \times 9 + (0 \cdot 8)^2 \times 4 = 9 \cdot 85 \ [\sigma = 3 \cdot 1385])$ Require t such that 0·75 = P(this < t) $= \mathrm{P}\left(Z < \frac{t - 35 \cdot 4}{3 \cdot 1385}\right) = \mathrm{P}(Z < 0 \cdot 6745)$ $\therefore t - 35 \cdot 4 = 3 \cdot 1385 \times 0 \cdot 6745 = 2 \cdot 1169$ $\Rightarrow t = 37 \cdot 52$ Must therefore take scheduled time as 38	B1 B1 M1 B1 A1 M1	Mean. Variance. Accept sd. Formulation of requirement (using c's parameters). Any use of a continuity correction scores M0 (and hence A0). 0·6745 c.a.o. Round to next integer above c's value for t.	6
(v)	CI is given by $13 \cdot 4 \pm 1 \cdot 96 \frac{2}{\sqrt{15}}$ = 13·4 ± 1·0121 = (12·38(79), 14·41(21))	M1 B1 A1	If <u>both</u> 13·4 and $2/\sqrt{15}$ are correct. (N.B. 13·4 is given as $\bar{x}$ in the question.) (If $3/\sqrt{15}$ used, treat as mis-read and award this M1, but not the final A1.) For 1·96 c.a.o. Must be expressed as an interval.	3
				18

Q3				
(i)	Simple random sample might not be representative - e.g. it might contain only managers.	E1 E1	Or other sensible comment.	2
(ii)	Presumably there is a list of staff, so systematic sampling would be possible. List is likely to be alphabetical, in which case systematic sampling might not be representative. But if the list is in categories, systematic sampling could work well.	E1 E1 E1	Or other sensible comments.	3
(iii)	Would cover the entire population. Can get information for each category.	E1 E1		2
(iv)	5, 11, 24	B1	(4·8, 11·2, 24)	1
(v)	$\overline{x}$ = 345818, s_{n-1} = 69241 Underlying Normality H_0: μ = 300000, H_1: μ > 300000		All given in the question.	
	Test statistic is $\dfrac{345818 - 300000}{\frac{69241}{\sqrt{11}}}$	M1	Allow alternatives: 300000 + (c's 1·812) × $\frac{69241}{\sqrt{11}}$ (= 337829) for subsequent comparison with 345818. or 345818 – (c's 1·812) × $\frac{69241}{\sqrt{11}}$ (= 307988) for comparison with 300000.	
	=2·19(47).	A1	c.a.o. but ft from here in any case if wrong. Use of $\mu - \overline{d}$ scores M1A0, but ft.	
	Refer to t_{10}. Upper 5% point is 1·812. Significant. Evidence that mean wealth is greater than 300000.	M1 A1 A1 A1	No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Special case: (t_{11} and 1·796) can score 1 of these last 2 marks if either form of conclusion is given.	
	CI is given by 345818 ± 2·228 × $\frac{69241}{\sqrt{11}}$	 M1 B1 M1		
	= 345818 ± 46513·84 = (299304(·2),	A1	c.a.o. Must be expressed as an	10

	392331(·8))		interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{10} is OK.	
				18

<table>
<tr><td>Q4</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="7">(i)</td><td><table><tr><th>Differences</th><th>Rank of |diff|</th></tr><tr><td>– 2</td><td>2</td></tr><tr><td>– 1</td><td>1</td></tr><tr><td>– 6</td><td>5</td></tr><tr><td>– 3</td><td>3</td></tr><tr><td>4</td><td>4</td></tr><tr><td>– 12</td><td>9</td></tr><tr><td>7</td><td>6</td></tr><tr><td>– 8</td><td>7</td></tr><tr><td>– 10</td><td>8</td></tr></table></td><td>M1
M1
A1</td><td>For differences.
ZERO in this section if differences not used.
For ranks.
FT from here if ranks wrong</td><td></td></tr>
<tr><td>T = 4 +6 = 10 (or 1+2+3+5+7+8+9 = 35)</td><td>B1</td><td></td><td></td></tr>
<tr><td>Refer to tables of Wilcoxon paired (/single sample) statistic.</td><td>M1</td><td>No ft from here if wrong.</td><td></td></tr>
<tr><td>Lower (or upper if 35 used) 5% tail is needed.</td><td>M1</td><td>i.e. a 1-tail test. No ft from here if wrong.</td><td></td></tr>
<tr><td>Value for n = 9 is 8 (or 37 if 35 used).</td><td>A1</td><td>No ft from here if wrong.</td><td></td></tr>
<tr><td>Result is not significant.</td><td>A1</td><td>ft only c’s test statistic.</td><td></td></tr>
<tr><td>No evidence to suggest a real change.</td><td>A1</td><td>ft only c’s test statistic.</td><td>9</td></tr>
<tr><td rowspan="10">(ii)</td><td>Normality of <u>differences</u> is required.</td><td>B1</td><td></td><td></td></tr>
<tr><td>CI MUST be based on DIFFERENCES.</td><td></td><td>ZERO/6 for the CI if differences not used.</td><td></td></tr>
<tr><td>Differences are 53, 15, 32, 13, 61, 82, 70</td><td></td><td>Accept negatives throughout.</td><td></td></tr>
<tr><td>$\bar{d} = 46\cdot5714$ $s_{n-1} = 27\cdot0485$</td><td>B1</td><td>Accept $s_{n-1}^2 = 731\cdot62$ …
[s_n = 25·0420, but do <u>NOT</u> allow this here or in construction of CI.]</td><td></td></tr>
<tr><td>CI is given by
46·5714 ±</td><td>M1</td><td>Allow c’s $\bar{d}$ ± …</td><td></td></tr>
<tr><td>3·707</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td>B1</td><td>If t_6 used.
99% 2-tail point for c’s t distribution. (Independent of previous mark.)</td><td></td></tr>
<tr><td>$\times \frac{27\cdot0485}{\sqrt{7}}$</td><td>M1</td><td>Allow c’s s_{n-1}.</td><td></td></tr>
<tr><td>= 46·5714 ± 37·8980 = (8·67(34), 84·47)</td><td>A1</td><td>c.a.o. Must be expressed as an interval. [Upper boundary is 84·4694]</td><td></td></tr>
<tr><td>Cannot base CI on Normal distribution because
sample is small
population s.d. is not known</td><td>E1
E1</td><td>Insist on “population”, but allow “σ”.</td><td>9</td></tr>
<tr><td></td><td></td><td></td><td></td><td>18</td></tr>
</table>

Mark Scheme 4769
June 2006

Q1				
(i)	$\mathrm{L} = \frac{1}{\sigma_1\sqrt{2\pi}}\mathrm{e}^{-\frac{(W_1-\mu)^2}{2\sigma_1^2}} \cdot \frac{1}{\sigma_2\sqrt{2\pi}}\mathrm{e}^{-\frac{(W_2-\mu)^2}{2\sigma_2^2}}$ $\ln \mathrm{L} = \mathrm{const} - \frac{1}{2\sigma_1^2}(W_1-\mu)^2 - \frac{1}{2\sigma_2^2}(W_2-\mu)^2$ $\frac{\mathrm{d}\ln\mathrm{L}}{\mathrm{d}\mu} = \frac{2}{2\sigma_1^2}(W_1-\mu) + \frac{2}{2\sigma_2^2}(W_2-\mu)$ $= 0 \Rightarrow \sigma_2^2 W_1 - \sigma_2^2\mu + \sigma_1^2 W_2 - \sigma_1^2\mu = 0$ $\Rightarrow \hat{\mu} = \frac{\sigma_2^2 W_1 + \sigma_1^2 W_2}{\sigma_1^2 + \sigma_2^2}$ Check this is a maximum. E.g. $\frac{\mathrm{d}^2\ln\mathrm{L}}{\mathrm{d}\mu^2} = -\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2} < 0$	M1 M1 A1 M1 A1 M1 A1 A1 A1 M1 A1	Product form. Two Normal terms. Fully correct. Differentiate w.r.t. μ. BEWARE PRINTED ANSWER.	11
(ii)	$\mathrm{E}(\hat{\mu}) = \frac{\sigma_2^2\mu + \sigma_1^2\mu}{\sigma_1^2 + \sigma_2^2} = \mu$ $\therefore$ unbiased.	M1 A1		2
(iii)	$\mathrm{Var}(\hat{\mu}) = \left(\frac{1}{\sigma_1^2 + \sigma_1^2}\right)^2 \cdot (\sigma_2^4\sigma_1^2 + \sigma_1^4\sigma_2^2)$ $= \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$	B1 B1	First factor. Second factor. Simplification not required at this point.	2
(iv)	$T = \frac{1}{2}(W_1 + W_2)$ $\mathrm{Var}(T) = \frac{1}{4}(\sigma_1^2 + \sigma_2^2)$ Relative efficiency (y) $= \frac{\mathrm{Var}(\hat{\mu})}{\mathrm{Var}(T)}$ $= \frac{\sigma_2^4\sigma_1^2 + \sigma_1^4\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2} \cdot \frac{4}{\sigma_1^2 + \sigma_2^2}$ $= \frac{4\sigma_1^2\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2}$	B1 M1 M1 A1 A1	Any attempt to compare variances. If correct. BEWARE PRINTED ANSWER.	5
(v)	E.g. consider $\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 = (\sigma_1 - \sigma_2)^2 \geq 0$ $\therefore$ Denominator ≥ numerator, $\therefore$ fraction ≤ 1 [Both $\hat{\mu}$ and T are unbiased,] $\hat{\mu}$ has smaller variance than T and is therefore better.	M1 E1 E1 E1		4
				24

Q2	$\mathrm{f}(x) = \dfrac{\lambda^{k+1} x^k \mathrm{e}^{-\lambda x}}{k!}, \quad [x > 0 \quad (\lambda > 0, k \text{ integer} \geq 0)]$ Given: $\int_0^\infty u^m \mathrm{e}^{-u} \mathrm{d}u = m!$			
(i)	$\mathrm{M}_X(\theta) = \mathrm{E}[\mathrm{e}^{\theta x}]$ $= \int_0^\infty \dfrac{\lambda^{k+1}}{k!} x^k \mathrm{e}^{-(\lambda-\theta)x} \mathrm{d}x$ Put $(\lambda - \theta)x = u$ $= \dfrac{\lambda^{k+1}}{k!(\lambda-\theta)^{k+1}} \int_0^\infty u^k \mathrm{e}^{-u} \mathrm{d}u$ $= \left(\dfrac{\lambda}{\lambda-\theta}\right)^{k+1}$	M1 M1 M1 A1 A1 A1 A1	For obtaining this expression after substitution. Take out constants. (Dep on subst.) Apply “given”: integral = *k*! (Dep on subst.) BEWARE PRINTED ANSWER.	7
(ii)	$Y = X_1 + X_2 + \ldots + X_n$ By convolution theorem:- mgf of *Y* is $\{\mathrm{M}_X(\theta)\}^n$ i.e. $\left(\dfrac{\lambda}{\lambda-\theta}\right)^{nk+n}$ $\mu = \mathrm{M}'(0)$ $\mathrm{M}'(\theta) = \lambda^{nk+n}(-nk-n)(\lambda-\theta)^{-nk-n-1}(-1)$ $\therefore \mu = \dfrac{nk+n}{\lambda}$ $\sigma^2 = \mathrm{M}''(0) - \mu^2$ $\mathrm{M}''(\theta) = (nk+n)\lambda^{nk+n}(-nk-n-1)(\lambda-\theta)^{-nk-n-2}(-1)$ $\therefore \mathrm{M}''(0) = (nk+n)(nk+n+1)/\lambda^2$ $\therefore \sigma^2 = \dfrac{(nk+n)(nk+n+1)}{\lambda^2} - \dfrac{(nk+n)^2}{\lambda^2}$ $= \dfrac{nk+n}{\lambda^2}$	B1 M1 A1 A1 M1 A1 M1 A1		8
(iii)	[Note that $\mathrm{M}_Y(t)$ is of the same functional form as $\mathrm{M}_X(t)$ with *k* + 1 replaced by *nk* + *n*, i.e. *k* replaced by *nk* + *n* –1. This must also be true of the pdf.] Pdf of *Y* is $\dfrac{\lambda^{nk+n}}{(nk+n-1)!} \times y^{nk+n-1} \times \mathrm{e}^{-\lambda y}$ [for *y* > 0]	B1 B1 B1	One mark for each factor of the expression. Mark for third factor shown here depends on at least one of the other two earned.	3
(iv)	$\lambda = 1$, $k = 2$, $n = 5$, Exact P(*Y* > 10) = 0·9165 Use of N(15, 15)	M1 M1	Mean. ft (ii). Variance. ft (ii).	

	P(this > 10) = $\mathrm{P}\left(\mathrm{N}(0,1) > \frac{10-15}{\sqrt{15}} = -1\cdot 291\right)$ = 0·9017 Reasonably good agreement – CLT working for only small *n*.	A1 A1 E2	c.a.o. c.a.o. (E1, E1) [Or other sensible comments.]	6
				24

Q3				
(i)	$\bar{x} = 36.48$ $s = 9.6307$ $s^2 = 92.7507$ $\bar{y} = 45.5$ $s = 14.8129$ $s^2 = 219.4218$	B1	If all correct. [No marks for use of s_n which are 9.1365 and 14.1823 respectively.]	
	Assumptions: Normality of both populations	B1		
	equal variances	B1		
	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A \neq \mu_B$	B1	Do NOT accept $\bar{X} = \bar{Y}$ or similar.	
	Where μ_A, μ_B are the population means.	B1		
	Pooled $s^2 = \frac{9 \times 92.7507 + 11 \times 219.4218}{20}$ $= \frac{834.756 + 24136.64}{20} = 162.4198$	B1	= $(12.7444)^2$	
	Test statistic is $\frac{36.48 - 45.5}{\sqrt{162.4198}\sqrt{\frac{1}{10} + \frac{1}{12}}}$	M1		
	$= \frac{-9.02}{5.4568} = -1.653$	A1		
	Refer to t_{20}.	M1	No ft from here if wrong.	
	Double tailed 5% point is 2·086.	A1	No ft from here if wrong.	
	Not significant.	A1	ft only c's test statistic.	
	No evidence that population mean times differ.	A1	ft only c's test statistic.	12
(ii)	Assumption: Normality of underlying population of differences.	B1		
	$H_0 : \mu_D = 0$ $H_1 : \mu_D > 0$	B1	Do NOT accept $\bar{D} = 0$ or similar.	
	Where μ_D is the population mean of "before – after" differences.	B1	The "direction" of D must be CLEAR. Allow $\mu_A = \mu_B$ etc.	
	Differences are 6.4, 4.4, 3.9, -1.0, 5.6, 8.8, -1.8, 12.1	M1		
	$(\bar{x} = 4.8$ $s = 4.6393)$		[A1 can be awarded here if NOT awarded in part (i)]. Use of s_n (=4.3396) is NOT acceptable, even in a denominator of $\frac{s_n}{\sqrt{n-1}}$	
	Test statistic is $\frac{4.8 - 0}{4.6393 / \sqrt{8}}$	M1		
	=2.92(64)	A1		
	Refer to t_7.	M1	No ft from here if wrong.	
	Single tailed 5% point is 1.895.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean is lowered.	A1	ft only c's test statistic.	10
(iii)	The paired comparison in part (ii) eliminates the variability between workers.	E2	(E1, E1)	2
				24

<table>
<tr><td>Q4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>Latin square.
Layout such as:
<table>
<tr><td></td><td></td><td></td><td></td><td>Locations</td><td></td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td></td><td>I</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>Surf</td><td>II</td><td>B</td><td>C</td><td>D</td><td>E</td><td>A</td></tr>
<tr><td>-aces</td><td>III</td><td>C</td><td>D</td><td>E</td><td>A</td><td>B</td></tr>
<tr><td></td><td>IV</td><td>D</td><td>E</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td></td><td>V</td><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
</table>
</td><td>B1
B1
B1</td><td>(letters = paints)
Correct rows and columns.
A correct arrangement of letters.
SC. For a description instead of an example allow max 1 out of 2.</td><td>3</td></tr>
<tr><td>(ii)</td><td>$X_{ij} = \mu + \alpha_i + e_{ij}$

μ = population
grand mean for whole experiment.

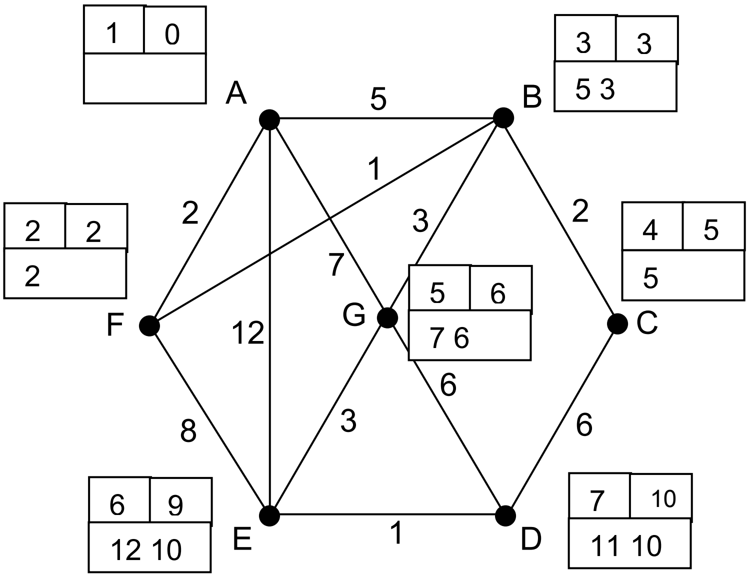
α_i = population
mean amount by which the i^{th} treatment differs from μ.

e_{ij} are experimental errors
~ ind
N(0, σ^2).</td><td>B1
B1
B1
B1
B1
B1
B1
B1
B1</td><td>Allow “uncorrelated”.
Mean.
Variance.</td><td>9</td></tr>
<tr><td>(iii)</td><td>Totals are: 322, 351, 307, 355, 291
(each from sample of size 5)
Grand total: 1626
“Correction factor” CF = $\frac{1626^2}{25} = 105755.04$

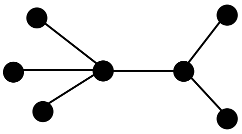
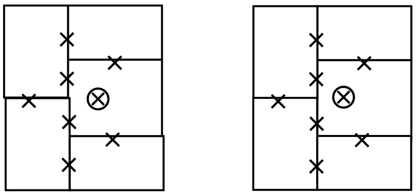
Total SS = 106838 – CF = 1082.96
Between paints SS = $\frac{322^2}{5} + ... + \frac{291^2}{5} - \text{CF}$
= 106368 – CF =612.96
Residual SS (by subtraction) = 1082.96 – 612.96
= 470.00
<table>
<tr><td>Source of variation</td><td>SS</td><td>df</td><td>MS</td></tr>
<tr><td>Between paints</td><td>612.96</td><td>4</td><td>153.24</td></tr>
<tr><td>Residual</td><td>470.00</td><td>20</td><td>23.5</td></tr>
<tr><td>Total</td><td>1082.96</td><td>24</td><td></td></tr>
</table>
MS ratio = $\frac{153.24}{23.5} = 6.52$</td><td>M1
M1
A1
B1
B1
M1
M1
A1</td><td>For correct methods for any two SS.
If each calculated SS is correct.
Degrees of freedom “between paints”.
Degrees of freedom “residual”.
MS column.
Independent of previous M1.
Dep only on this M1.</td><td></td></tr>
</table>

	Refer to $F_{4,\,20}$ Upper 5% point is 2.87 Significant. Seems performances of paints are not all the same.	M1 A1 A1 A1	No ft if wrong. But allow ft of wrong d.o.f. above. No ft if wrong. ft only c's test statistic and d.o.f.'s. ft only c's test statistic and d.o.f.'s.	12
				24

Mark Scheme 4771
June 2006

(i) 	M1 sca Dijkstra A1 labels A1 order of labelling A1 working values
Least weight route: A F B G E D Weight = 10	B1 B1
(ii) 11 From working value. Can't be bettered since new least weight must be bigger than 10.	B1 B1

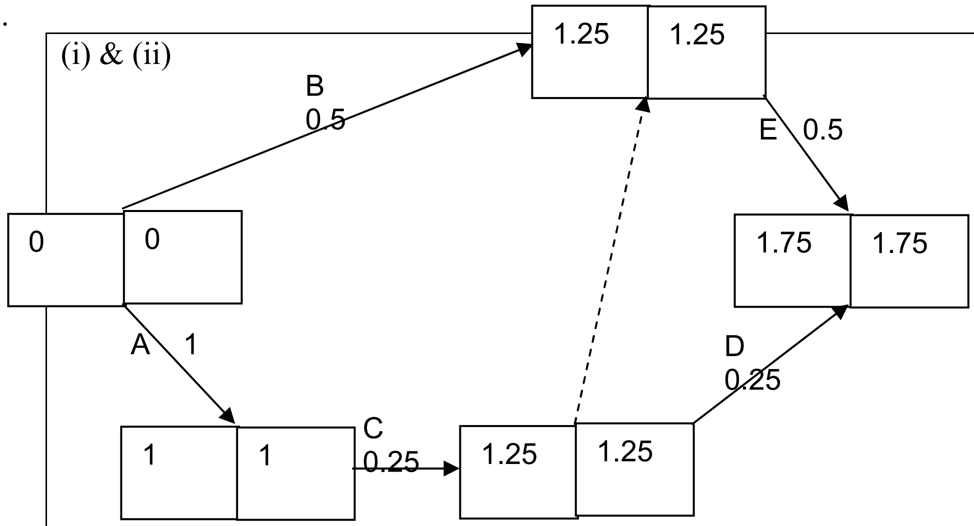
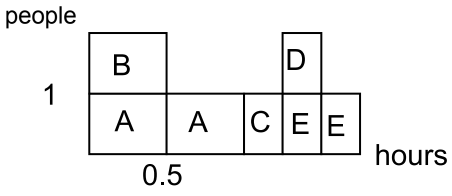
2.

(i) e.g. 	M1 A1
a tree	B1
(ii) 13	B1
(iii) 14	B1
(iv) e.g. 	M1 A1 A1

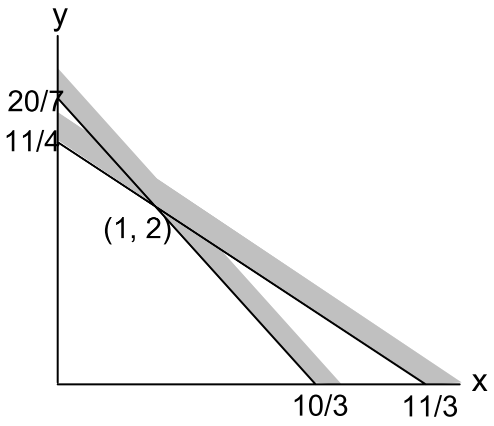
3.

(i) M = 1	B1
f(M) = –1	B1
L = 1	B1
M = 1.5	B1
f(M) = 0.25	B1
R = 1.5	B1
(ii) Solves equations (Allow "Finds root 2".)	B1
(iii) A termination condition	B1

4.

<table>
<tr><td>(i) & (ii)

Critical activities: A, C, E</td><td>M1 sca activity-on-arc
A1 A, B, C
A1 D
A1 E
B1 forward pass (1.25 at end of B/dummy)
B1 backward pass (1.25 at start of dummy/D)
B1</td></tr>
<tr><td>(iii)

(iv) 2 hours (resource smoothing on A/B, but extra time needed for D/E).</td><td>M1
A1
M1
A1</td></tr>
<tr><td>(v) P –
Q –
R –
S Q, R
T Q, R
U R
V S, T, U
W U</td><td>B1
B1
B1
B1
B1</td></tr>
</table>

5.

(i) Let x be the number of hours spent at badminton Let y be the number of hours spent at squash	B1
$3x + 4y \leq 11$	B1
$1.5x + 1.75y \leq 5$	B1
(ii) 	B1 axes labelled and scaled B1 line B1 line B1 shading B1 intercepts B1 (1, 2)
(iii) $x + 2y$	B1
(iv) $22/4 > 5 > 10/3$, so 5.5 at (0, 11/4)	M1 A1
(v) Squash courts sold in whole hours	B1
1 hour badminton and 2 hours squash per week	B1
(vi) 3 hours of badminton and no squash	B1 B1

6.

(i) year 1: 00 – 09 failure, otherwise no failure year 2: 00 – 04 year 3: 00 – 01	M1 A1
year 4: 00 – 19 year 5: 00 – 19 year 6: 00 – 29	A1
(ii)(A)	

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	x		√	√		x	√

	M1 ticks and crosses A1 run 1 A1 runs 2–4 A1 runs 5–7 B1 runs 8–10
(B) 0.6	B1
(iii) (A) if no failure then continue after year 3 – but using rules for yrs 1 to 3	B1 B1
(B)	

	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10
year 1	√	√	√	√	x	√	√	x	√	√
year 2	√	√	√	√		√	√		√	√
year 3	√	√	√	√		√	√		√	√
year 4	√	√	√	√		√	√		x	√

	M1 A1 runs 1–5 A1 runs 6–10
(C) 0.3	B1
(iv) more repetitions	B1

Mark Scheme 4772
June 2006

1.

<table>
<tr>
<td>(i)
<table>
<tr><td>~(</td><td>~</td><td>T</td><td>⇒</td><td>~</td><td>S)</td><td>⇔</td><td>~</td><td>T</td><td>∧</td><td>S</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr>
</table></td>
<td>M1 4 lines
A1 T and S
A1 ~T (twice) and ~S
A1 ⇒
A1 ∧
A1 ~on LHS
M1
A1 result</td>
</tr>
<tr>
<td>(ii)
<table>
<tr><td>A</td><td>⇒</td><td>B</td><td>⇔</td><td>~</td><td>A</td><td>∨</td><td>B</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td></tr>
</table>
or a correct verbal argument</td>
<td>M1
A1</td>
</tr>
<tr>
<td>$\sim(\sim T \Rightarrow \sim S) \Leftrightarrow \sim(T \vee \sim S) \Leftrightarrow \sim T \wedge S$</td>
<td>M1 Boolean
A1 applying result
A1 correct negating</td>
</tr>
<tr>
<td>(iii) Joanna will not try and will succeed</td>
<td>B1 not try
B1 and
B1 succeed</td>
</tr>
</table>

2.

(i)

	1	2	3	4
1	∞	2	6	4
2	2	∞	3	1
3	6	3	∞	1
4	4	1	1	∞

	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

M1 sca Floyd
A1 distance
A1 route

	1	2	3	4
1	∞	2	6	4
2	2	4	3	1
3	6	3	12	1
4	4	1	1	8

	1	2	3	4
1	1	2	3	4
2	1	1	3	4
3	1	2	1	4
4	1	2	3	1

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1

	1	2	3	4
1	4	2	5	3
2	2	4	3	1
3	5	3	6	1
4	3	1	1	2

	1	2	3	4
1	2	2	2	2
2	1	1	3	4
3	2	2	2	4
4	2	2	3	2

A1 no change

	1	2	3	4
1	4	2	4	3
2	2	2	2	1
3	4	2	2	1
4	3	1	1	2

	1	2	3	4
1	2	2	②	2
2	1	4	4	4
3	4	4	4	4
4	2	2	3	2

A1 circled element
A1 rest

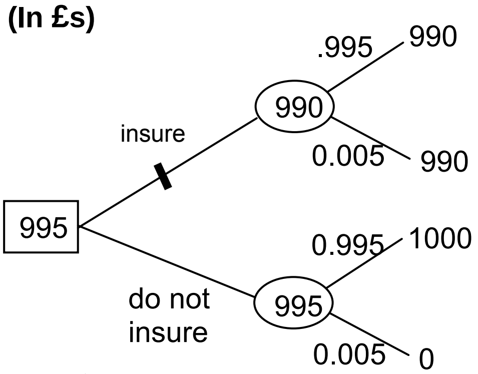
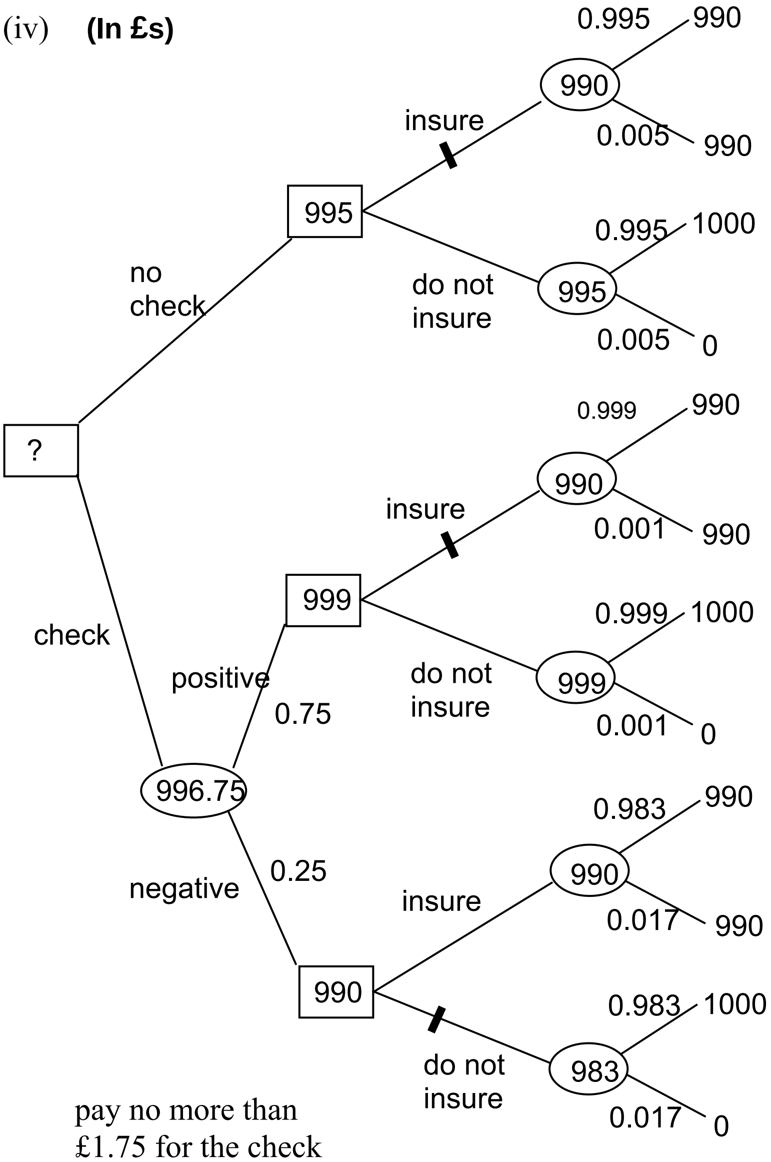
M1 A1
M1 A1

(ii) distance = 4 (row 1, col 3 of dist matrix)
route = 1, 2, 4, 3 (1 – r1c3 – r2c3 – r4c3 of route matrix)

B1
M1 A1
B1

(iii) **1**, **2**, **4**, **3**, **1**
1, **2**, **4**, **3**, **4**, **2**, **1**
8

3.

(i) **(In £s)** 	M1 pay–offs A1 M1 chance nodes A1 M1 decision node A1
(ii) Do not insure. Pay no more than £5 for it.	B1 B1
(iii) Yes $\left(\left(\sqrt[3]{990}\times\left(0.995+0.005\right)\mathrm{v}\left(0.995\times\sqrt[3]{1000}\right)\right)\right.$ $\sqrt[3]{1000-\mathrm{x}}=9.95$ giving $\mathrm{x}=£14.93$	B1 M1 A1
(iv) **(In £s)**  pay no more than £1.75 for the check	M1 check/no check A1 M1 positive/negative A1 M1 insure/not insure A1 M1 go/no go A1 B1

4. (i) a is the number of aardvarks, etc.
First inequality models the furry material constraint
Second inequality models the woolly material constraint
Third inequality models the glass eyes constraint

B1
M1
A1

That would model a "pairs of glass eyes" constraint.

B1

(ii) The problem is an IP, so the number of eyes used will be integer anyway.

B1

(iii) e.g.

P	a	b	c	s1	s2	s3	RHS
1	−3	–5	–2	0	0	0	0
0	0.5	1	1	1	0	0	11
0	2	1.5	1	0	1	0	24
0	2	2	2	0	0	1	30
1	–0.5	0	3	5	0	0	55
0	0.5	1	1	1	0	0	11
0	1.25	0	-0.5	–1.5	1	0	7.5
0	1	0	0	–2	0	1	8
1	0	0	2.8	4.4	0.4	0	58
0	0	1	1.2	1.6	–0.4	0	8
0	1	0	–0.4	–1.2	0.8	0	6

M1
A1

M1 pivot choice
A1 pivot

M1 pivot choice
A1 pivot

Make 6 aardvarks and 8 bears giving £58 profit.
2 eyes are left over.

B1 B1
B1

(iv)

P	a	b	c	s1	s2	s3	su4	a	RHS
1	−3	–5	–(2+M)	0	0	0	M	0	–2M
0	0.5	1	1	1	0	0	0	0	11
0	2	1.5	1	0	1	0	0	0	24
0	2	2	2	0	0	1	0	0	30
0	0	0	1	0	0	0	–1	1	2

B1 new constraint
M1 objective
A1

or

C	P	a	b	c	s1	s2	s3	su4	a	RHS
1	0	0	0	1	0	0	0	–1	0	2
0	1	−3	–5	–2	0	0	0	0	0	0
0	0	0.5	1	1	1	0	0	0	0	11
0	0	2	1.5	1	0	1	0	0	0	24
0	0	2	2	2	0	0	1	0	0	30
0	0	0	0	1	0	0	0	–1	1	2

(v) $8\times0.5 + 2\times1 + 5\times1 = 11$
$8\times2 + 2\times1.5 + 5\times1 = 24$
$8\times2 + 2\times2 + 5\times2 = 30$

B1

$3\times8 + 5\times2 + 2\times5 = 44$ but $3\times6 + 5\times6 + 2\times2 = 52$
1 m^2 of woolly material and 2 eyes left.

B1
B1

Mark Scheme 4773
June 2006

Qu. 1

(i)	Variables	M1	
	ai = amount invested in A in year i, i = 1, 2, 3, 4, 5	A1	a's
	bi = amount invested in B in year i, i = 1, 2, 3	A1	b's
	ci = amount invested in C in year i, i = 3, 4, 5	A1	c's
	Maximise 1.15a5+1.55b3+1.20c5	B1	
	st a1+b1 = 50000	B1	
	a2+b2 = 1.15a1	B1	
	a3+b3+c3 = 1.15a2	B1	
	a4+c4 = 1.15a3+1.55b1+1.20c3	B1	
	a5+c5 = 1.15a4+1.55b2+1.20c4	B1	
(ii)	OBJECTIVE FUNCTION VALUE		

```
1)      114264.0

VARIABLE        VALUE          REDUCED COST
A5                0.000000         0.050000
B3                0.000000         0.178000          M1
C5            95220.000000         0.000000          A1
A1            50000.000000         0.000000
B1                0.000000         0.053280
A2            57500.000000         0.000000
B2                0.000000         0.127200
A3                0.000000         0.072000
C3            66125.000000         0.000000
A4                0.000000         0.060000
C4            79350.000000         0.000000
```

	Invest all in A in year 1. Put all into A in year 2 Thence all into C in years 3, 4 and 5.	B1	
	Gives £114264 at the end of 5 years.	B1	
(iii)	£1.59	M1 A1	(£1.57 to £1.61)
		A1	

Qu. 2

(i) See below – first two columns of s/sheet	M1 A1 A1
(ii) $\mathbf{x^2 - x - 1 = 0}$	M1
$x = \frac{1\pm\sqrt{5}}{2}$	A1
$x = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$	B1
A + B = 1 and $A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1$	B1 B1
giving $u_n = \frac{1}{\sqrt{5}}\frac{\sqrt{5}+1}{2}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{1}{\sqrt{5}}\frac{\sqrt{5}-1}{2}\left(\frac{1-\sqrt{5}}{2}\right)^n$	M1 solving A1 A1
$= \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}$	B1
(iii) =(1/SQRT(5))*(((1+SQRT(5))/2)^(A2+1)-((1-SQRT(5))/2)^(A2+1))	M1 A1
plus printout	M1 A1
(iv) See s/sheet below.	B1
Converges to 1.61803…	
$\left(\frac{1+\sqrt{5}}{2}\right)$	M1 A1

n	F(n)	Formula	Ratios
0	1	1	
1	1	1	1
2	2	2	2
3	3	3	1.5
4	5	5	1.66667
5	8	8	1.6
6	13	13	1.625
7	21	21	1.61538
8	34	34	1.61905
9	55	55	1.61765
10	89	89	1.61818
11	144	144	1.61798
12	233	233	1.61806
13	377	377	1.61803
14	610	610	1.61804
15	987	987	1.61803
16	1597	1597	1.61803
17	2584	2584	1.61803
18	4181	4181	1.61803
19	6765	6765	1.61803

Qu. 3

(i)	Min 2W1S1+2W1S2+W1S3+5W1S4+3W2S1+2W2S2 +2W2S3+4W2S4+5W3S1+5W3S2+W3S3+2W3S4	B1 variables M1 objective A1
	st W1S1+W1S2+W1S3+W1S4<20 W2S1+W2S2+W2S3+W2S4<20 W3S1+W3S2+W3S3+W3S4<20	M1 w/house A1 availabilities
	W1S1+W2S1+W3S1>10 W1S2+W2S2+W3S2>15 W1S3+W2S3+W3S3>12 W1S4+W2S4+W3S4>20	M1 shop A1 requirements

(ii)

```
OBJECTIVE FUNCTION VALUE

1)      104.0000

VARIABLE   VALUE     REDUCED COST
W1S1       8.000000       0.000000
W1S2       0.000000       1.000000
W1S3      12.000000       0.000000
W1S4       0.000000       3.000000
W2S1       2.000000       0.000000
W2S2      15.000000       0.000000
W2S3       0.000000       0.000000
W2S4       0.000000       1.000000
W3S1       0.000000       3.000000
W3S2       0.000000       4.000000
W3S3       0.000000       0.000000
W3S4      20.000000       0.000000
```

B1

M1
A1

Supply shop 1 with 8 from warehouse 1 and 2 from 2 Supply shop 2 from warehouse 2 Supply shop 3 from warehouse 1 Supply shop 4 from warehouse 3 Cost = £104	B1

Qu. 3 (cont)

(iii)	Min	2W1S1+2W1S2+W1S3+5W1S4+3W2S1+2W2S2	B1	new variables
		+2W2S3+4W2S4+5W3S1+5W3S2+W3S3+2W3S4	B1	new objective
		+4S1C1+6S2C1+3S3C1+2S4C1		
		+S1C2+4S2C2+2S3C2+5S4C2	B1	supply constraints
	st	W1S1+W1S2+W1S3+W1S4<20		
		W2S1+W2S2+W2S3+W2S4<20		
		W3S1+W3S2+W3S3+W3S4<20		
		S1C1+S2C1+S3C1+S4C1=30	B1	receipt constraints
		S1C2+S2C2+S3C2+S4C2=27		
		S1C1+S1C2-W1S1-W2S1-W3S1=0		
		S2C1+S2C2-W1S2-W2S2-W3S2=0		
		S3C1+S3C2-W1S3-W2S3-W3S3=0	B1	in/out constraints
		S4C1+S4C2-W1S4-W2S4-W3S4=0		
	A solution is:			
	W1 to S3 20			
	W2 to S3 17		B1	
	W3 to S4 20			
	S3 to C1 10			
	S4 to C1 20		B1	
	S3 to C2 27			

Qu. 4

<table>
<tr><td>(i) 0.22, 0.2325, 0.5475</td><td>M1 A1 A1 A1</td></tr>
<tr><td>(ii) e.g.
look-up tables
<table>
<tr><th></th><th>wet(1)</th><th>showery(2)</th><th>dry(3)</th></tr>
<tr><td>wet</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>showery</td><td>0.2</td><td>0.4</td><td>0.15</td></tr>
<tr><td>dry</td><td>0.5</td><td>0.55</td><td>0.4</td></tr>
</table>
simulation run
<table>
<tr><td>day</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>rand</td><td></td><td>0.14227</td><td>0.43734</td></tr>
<tr><td>weather</td><td>dry</td><td>wet</td><td>dry</td></tr>
</table>
=IF(B8="wet",LOOKUP(C7,$B$2:$B$4,$A$2:$A$4),(IF(B8=" showery", LOOKUP(C7,$C$2:$C$4,$A$2:$A$4),</td><td>M1 probability
A1 distributions
M1 selecting dist.
A1 by weather
M1 sampling from
A1 distribution
B1 two days handled</td></tr>
<tr><td>(iii) repeating and tabulating
calculating experimental probabilities</td><td>B1
M1 A1</td></tr>
<tr><td>(iv) 20 transitions
handling $s
repeating and tabulating
experimental probabilities (theoretical = 0.22, 0.24 and 0.54)</td><td>B1
B1
B1
B1</td></tr>
</table>

Mark Scheme 4776
June 2006

1 Sketch, with explanation. **[M1A1E1]**

f '(x) = 1/(2√x) **[M1A1]**

hence mpe is approx 0.05 / (2√2.5) = 0.015811 (0.016) **[M1A1]**

(or 0.05 / 2√2.45 = 0.015972 or 0.05 / 2√2.55 = 0.015656)

[TOTAL 7]

2

x	0	1				
f(x)	1	-3	change of sign so root			**[M1A1]**

a	b	f(a)	f(b)	x	f(x)	
0	1	1	-3	0.25	-0.24902	**[M1A1]**
0	0.25	1	-0.24902	0.200156	-0.00046	**[A1]**
				0.200 to 3 dp		**[A1]**

x	0.1995	0.2005		
f(x)	0.002816	-0.00218	sign change so root is correct to 3 dp	**[M1A1]**

[TOTAL 8]

3

h	M	T	S	
2	**3.464102**	**3.650282**	**3.526162**	**values**
1	3.510411	**3.557192**	**3.526004**	**[A1A1A1A1A1]**

evidence of efficient formulae for T and S
[M1M1]

3.526(0) appears to be justified **[A1]**

[TOTAL 8]

4

h	0	0.1	0.01	0.001
f(2 + *h*)	1.4427	1.3478	1.4324	1.4416
est f '(2)		-0.949	-1.03	-1.1

[M1A1A1A1]

Clear loss of significant figures as h is reduced **[E1]**

Impossible to know which estimate is most accurate **[E1]**

[TOTAL 6]

5

x	g(*x*)	Δg	Δ^2g		
1	3.2	9.6	6		**table**
2	12.8	15.6	6.2	second differences nearly constant	**[M1A1]**
3	28.4	21.8	5.9	so approximately quadratic	**[E1]**
4	50.2	27.7	6		
5	77.9	33.7			
6	111.6				

g(1.5) = 3.2 + 0.5*9.6 + 0.5*(-0.5)*6/2 = 7.25 **[M1A1A1A1]**

[TOTAL 7]

NB: 3 pi /2 =4.71 (not reqd)

6 (i)

x	4.6	4.7		
x^2-tan(x)	12.29983	-58.6228	change of sign, so root	**[M1A1]**

a	b	sign f(a)	sign f(b)	x	sign f(x)	mpe	
4.6	4.7	1	-1	4.65	1	0.05	**[M1A1]**
4.65	4.7	1	-1	4.675	-1	0.025	**[M1A1]**
4.65	4.675	1	-1	4.6625		0.0125	**[M1A1]**

root is 4.6625 with mpe 0.0125 **[A1]**

[subtotal 9]

(ii)

x	7.7	7.9		
x^2-tan(x)	52.84713	84.12511	no change of sign, so no evidence of root	**[M1A1]**

Sketch showing asymptote for tan(x) at 5pi/2 = 7.854 **[G2]**

So x^2 curve is above tan(x) at both end points **[E1]**

[subtotal 5]

(iii) best possible estimate is 7.8 **[A1]**

x	7.75	7.85		**[M1]**
x^2-tan(x)	50.4801	-189.529	change of sign so 7.8 is correct to 1 dp	**[A1E1]**

[subtotal 4]

[TOTAL 18]

7

(i) D = (36 - 8) / (4 - 2) = 14 **[M1A1]**

I = 0.5 (-3 + 8) + (8 + 36) = 46.5 **[M1A1]**

[subtotal 4]

(ii) q(x) = -3 (x-2)(x-4)/(1-2)(1-4) + 8 (x-1)(x-4)/(2-1)(2-4) + 36 (x-1)(x-2)/(4-1)(4-2) **[M1A1A1A1]**

= - (x^2-6x+8) - 4 (x^2-5x+4) + 6 (x^2-3x+2) **[A1]**

= x^2 + 8x - 12 **[A1]**

q'(x) = 2x + 8 so D = 12 **[M1A1]**

∫ q(x) dx = x^3/3 + 4x^2 - 12x so I = 45 **[M1A1A1]**

[subtotal 11]

(iii) Large relative difference between estimates of D **[E1]**

Small relative difference in estimates of I **[E1]**

To be expected as integration is a more stable process than differentiation **[E1]**

[subtotal 3]

[TOTAL 18]

Mark Scheme 4777
June 2006

MEI Numerical Computation (4777) June 2006 Mark scheme

1

(i) $(x_2 - \alpha) / (x_1 - \alpha) = (x_1 - \alpha) / (x_0 - \alpha)$ to eliminate k **[M1A1A1]**

convincing algebra to required result. **[A1A1]**

[subtotal 5]

(ii)

x	1	1.5		
exp(x) - tan(x)	1.160874	-9.61973	change of sign (and no asymptote)	**[M1A1]**

Examples of divergence:

r	0	1	2	3	4	5	*6*	
x_r	1	0.443023	-0.74554	#NUM!	#NUM!	#NUM!	#NUM!	
x_r	1.25	1.101797	0.679821	-0.21274	#NUM!	#NUM!	#NUM!	
x_r	1.5	2.646275	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	**[M1A1A1]**

x_r	1.25	1.101797	0.679821	α:	1.330227	**[M1A1]**
x_r	1.330227	1.405193	1.788959	α:	1.312029	**[M1A1]**
x_r	1.312029	1.329149	1.40054	α:	1.306628	
x_r	1.306628	1.307521	1.311069	α:	1.306328	
x_r	1.306328	1.30633	1.30634	α:	1.306327	
x_r	1.306327	1.306327	1.306327	α:	1.306328	**[M1]**
					1.30633 to 5 dp	**[A1]**

[subtotal 11]

(iii)

x	3.142	3.2		
exp(-x) - tan(x)	0.042789	-0.01771	change of sign	**[M1A1]**

Examples of divergence:

r	0	1	2	3	4	5	*6*	
x_r	3.142	7.805847	-3.03297	2.215943	#NUM!	#NUM!	#NUM!	
x_r	3.2	2.839176	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	**[M1A1]**

Eg:

x_r	3.18	3.259015	2.13737	α:	3.1852	
x_r	3.1852	3.131898	#NUM!	α:	#NUM!	
x_r	#NUM!	#NUM!	#NUM!	α:	#NUM!	
x_r	#NUM!	#NUM!	#NUM!	α:	#NUM!	**[M1A1]**

but:

x_r	3.184	3.159834	4.003956	α:	3.183327	
x_r	3.183327	3.17584	3.373753	α:	3.183054	
x_r	3.183054	3.182409	3.198122	α:	3.183029	
x_r	3.183029	3.183024	3.183137	α:	3.183029	
					3.18303 to 5 dp	**[M1A1]**

[subtotal 8]

[TOTAL 24]

2

(i) Divided differences do not require data to be equally spaced (as ordinary differences do). Divided differences allow additional data to be added (unlike Lagrange). **[E1E1]**

[subtotal 2]

x	f
1	-3
2	-6.5
2.5	-8.03
3.5	-6.66
4	-2.25
4.5	5.65

(ii)	1	-3	-3.5	0.293333	1.064	-0.01911	**table**
	2	-6.5	-3.06	2.953333	1.006667		**[M1A1A1]**
	2.5	-8.03	1.37	4.966667			
	3.5	-6.66	8.82				
	4	-2.25					**estimates**
	1.5	**-3**	**-4.75**	**-4.82333**	**-4.55733**	**-4.54778**	**[M1A1A1A1A1]**
			linear	quadratic	cubic	quartic	

2nd dp unreliable (from data), 1st dp uncertain: could be -4.5 or -4.6 **[E1E1]**

[subtotal 10]

(iii)	3.5	-6.66	1.37	4.966667	1.006667	-4.4E-16	
	2.5	-8.03	3.853333	3.456667	1.006667		
	4	-2.25	2.125	5.47			**rearrange**
	2	-6.5	4.86				**data and**
	4.5	5.65					**re-run**
	3	**-6.66**	**-7.345**	**-8.58667**	**-8.335**	**-8.335**	**[M1A1]**
			linear	quadratic	cubic	quartic	

2nd dp unreliable (from data), 1st dp seems reliable: -8.3 **[E1]**

4.5	5.65	15.8	6.98	1.006667	-6.2E-16	
4	-2.25	8.82	4.966667	1.006667		
3.5	-6.66	1.37	2.953333			**rearrange**
2.5	-8.03	-3.06				**data and**
2	-6.5					**re-run**
5	**5.65**	**13.55**	**17.04**	**17.795**	**17.795**	**[M1A1]**
		linear	quadratic	cubic	quartic	

2nd dp unreliable (from data), 1st dp seems reliable: 17.8 **[E1]**

[subtotal 6]

(iv)	4	-2.25	15.8	6.98	1.006667	-4.4E-16	
	4.5	5.65	12.31	5.47	1.006667		
	3.5	-6.66	1.37	2.953333			**rearrange data and re-run**
	2.5	-8.03	-3.06				
	2	-6.5					
	4.16	**-2.25**	**0.278**	**-0.10171**	**-0.13786**	**-0.13786**	**[M1A1]**
	4.17	**-2.25**	**0.436**	**0.044422**	**0.006584**	**0.006584**	**[M1A1]**
	4.165	**5.65**	**1.52615**	**0.307571**	**-0.06582**	**-0.06582**	
	4.175	**-2.25**	**0.515**	**0.118012**	**0.079366**	**0.079366**	**[A1]**
			linear	quadratic	cubic	quartic	

Hence root is 4.17 to 2 dp **[A1]**

[subtotal 6]

[TOTAL 24]

3

(i) Substitute central difference formulae for y' and y" to obtain given result (*) **[M1A1]**

Central difference formula for y' at x=0 to show $y_1 = y_{-1}$ **[M1A1]**

Use of (*) to show $y_1 = (2h^2 - (1 + 2h)y_{-1})/(1 - 2h)$ **[M1A1]**

Hence $y_1 = h^2$ as given **[M1]**

h	**x**	**y**	**k**	
0.1	0	0	1	
	0.1	0.01		
	0.2	0.047618		**[M1A1]**
	0.3	0.124458		
	0.4	0.25785		
	0.5	0.473034		
	0.6	0.805379		
	0.7	1.301401		
	0.8	2.015508		
	0.9	2.996344		
	1	**4.253311**	**as required**	**[A1]**

h	**β**	**diffs**	**ratio of diffs**	**extrapolated value**	
0.1	4.253311				
0.05	4.190790	-0.06252			**re-runs**
0.025	4.175759	-0.01503	0.240406		**[A1A1A1]**
0.0125	4.172037	-0.00372	0.247604	4.170797	

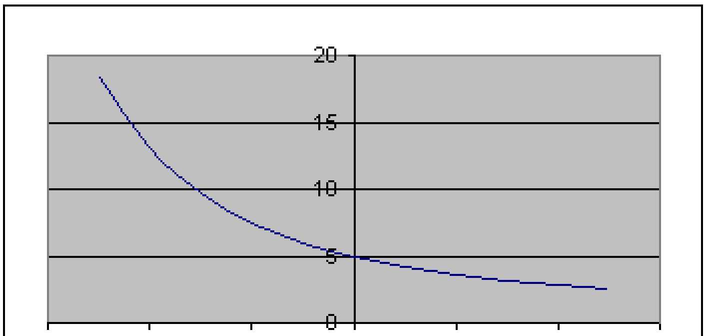
ratio of differences approximately 0.25 so second order **[M1A1E1]**

4.17 to 2 dp is secure **[A1]**

[subtotal 17]

(ii)

k	**β**
-5	18.4
-4	13.1
-3	9.7



mods
[M1A1]

-2	7.4	**values**
-1	6	**[A1A1A1]**
0	4.9	
1	4.2	**graph**
2	3.6	**[G2]**
3	3.2	
4	2.9	
5	2.6	

[subtotal 7]

[TOTAL 24]

4

(i) Diagonal dominance: modulus of diagonal element is greater than or equal to sum of moduli of other elements on the same row. **[E1]**

If diagonal dominance exists (with at least one inequality strict) convergence of Gauss-Seidel is assured. **[E1]**

G-S using the given non-dominant diagonal:

x	y	z	**[M1]**
0	0	0	
0.2	0.028571	-0.01587	**[M1A1]**
0.192381	0.041995	-0.0191	
0.186262	0.047335	-0.01866	
…	…	…	
0.180325	0.04918	-0.01639	
0.180327	0.04918	-0.01639	
0.180328	0.04918	-0.01639	
0.180328	0.04918	-0.01639	**[M1A1]**

[subtotal 7]

(ii)

a=3	**x**	**y**	**z**	**a=4**	**x**	**y**	**z**	
	0	0	0		0	0	0	**mods**
	0.333333	-0.06667	-0.04762		0.5	-0.25	-0.04167	**[M1A1]**
	0.447619	-0.12	-0.09116		0.9375	-0.64583	-0.07639	
	0.54449	-0.16267	-0.12987		1.583333	-1.25694	-0.10532	
	…	…	…		2.543403	-2.18808	-0.12944	**a=3**
	1	-0.33333	-0.33333		3.976273	-3.59684	-0.14953	**[M1A1]**
	1	-0.33333	-0.33333		6.119551	-5.72002	-0.16628	
	1	-0.33333	-0.33333		9.329443	-8.91317	-0.18023	**a=4**
					14.1401	-13.7099	-0.19186	**[M1A1]**
					21.35259	-20.9107	-0.20155	
					…	…	…	
					2.6E+18	-2.6E+18	0	
					3.91E+18	-3.9E+18	0	

5.86E+18	-5.9E+18	0

G-S scheme converges for a=3.3 **[M1A1]**
diverges for a=3.4 **[M1A1]**
(diverges for a=3.35)
So a=3.3 (to 1dp) is required value **[A1]**
[subtotal 11]

(iii) Gauss-Jacobi

	x	y	z	
a=0	0	0	0	
	0.166667	0.125	0.1	**[M1A1]**
	0.054167	-0.00833	-0.04583	
	0.19375	0.120833	0.077083	
	0.067708	-0.01042	-0.05729	
	0.200521	0.119792	0.071354	**[M1A1]**
	…	…	…	
	0.202778	0.119444	0.069444	
	0.072222	-0.01111	-0.06111	
	0.202778	0.119444	0.069444	
	0.072222	-0.01111	-0.06111	
	0.202778	0.119444	0.069444	
	0.072222	-0.01111	-0.06111	**[A1]**

Diverges: diagonal dominance not strict. **[E1]**
[subtotal 6]

[TOTAL 24]

7895-8,3895-3898 AS and A2 MEI Mathematics
June 2006 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	53	45	37	30	23	0
4752	Raw	72	55	48	41	34	27	0
4753	Raw	72	51	44	38	31	24	0
4753/02	Raw	18	14	12	10	9	8	0
4754	Raw	90	57	49	41	33	26	0
4755	Raw	72	58	50	43	36	29	0
4756	Raw	72	52	45	38	31	25	0
4757	Raw	72	51	44	38	32	26	0
4758	Raw	72	62	54	46	37	28	0
4758/02	Raw	18	14	12	10	9	8	0
4761	Raw	72	55	47	40	33	26	0
4762	Raw	72	43	37	31	25	20	0
4763	Raw	72	60	52	44	36	29	0
4764	Raw	72	46	40	35	30	25	0
4766	Raw	72	54	47	40	33	27	0
4767	Raw	72	58	51	44	37	30	0
4768	Raw	72	59	51	43	36	29	0
4769	Raw	72	52	45	38	32	26	0
4771	Raw	72	53	46	39	33	27	0
4772	Raw	72	57	49	41	34	27	0
4773	Raw	72	48	42	36	30	25	0
4776	Raw	72	51	44	37	30	23	0
4776/02	Raw	18	13	11	9	8	7	0
4777	Raw	72	55	47	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	40.4	61.2	77.2	89.2	96.9	100	9024
7896	60.2	77.5	88.7	95.6	99.0	100	1237
7897	70.5	90.9	90.9	93.2	95.5	100	44
7898	100	100	100	100	100	100	5
3895	27.7	43.6	57.9	71.2	82.0	100	11502
3896	50.9	68.6	82.4	90.0	95.6	100	1247
3897	80.7	86.8	94.0	98.8	98.8	100	83
3898	58.8	64.7	76.5	88.2	94.1	100	17

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