OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS 6993

Summer 2005

Monday **20 JUNE 2005** Morning 2 hours

Additional materials:
Answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 Use calculus to show that there is a maximum point at $x = 3$ on the curve $y = 9x^2 - 2x^3$ and find the coordinates of this point. [5]

2 The function f(x) is defined by $f(x) = x^3 - 4x^2 + 5x - 2$.

(i) Find the remainder when f(x) is divided by $(x + 2)$. [2]

(ii) Show that $(x - 1)$ is a factor of f(x). [1]

(iii) Hence solve the equation $f(x) = 0$. [4]

3 A triangle has sides 8 cm, 7 cm and 12 cm. Calculate the largest angle of the triangle, correct to the nearest degree. [5]

4 Find the values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying the equation

$$4\sin\theta = 3\cos\theta.$$

Give your answers to the nearest 0.1 degree. [4]

5 In a large batch of glasses, 14% are defective. From this batch 8 glasses are selected at random. Calculate which is more likely:

(*A*) none of the glasses is defective,

(*B*) at least two of the glasses are defective. [7]

6 **(i)** Expand $\left(x - \frac{1}{x}\right)^4$ using the binomial expansion. Show all your working. [4]

(ii) Explain why the substitution $x = 1$ will help to justify your answer. [1]

7 The gradient function of a curve is given by $\frac{dy}{dx} = a + bx$. Find the values of a and b and the equation of the curve given that it passes through the points $(0, 2)$, $(1, 8)$ and $(-1, 2)$. [7]

8 A car moves in a straight line. Its velocity in metres per second, t seconds after passing a point A, is given by the equation

$$v = 27 - \tfrac{1}{8}t^3.$$

It comes to rest at a point B.

(i) Show that the car is at B when $t = 6$. [1]

(ii) Find the distance AB. [5]

9 **(i)** Using the identity $\cos^2\theta + \sin^2\theta = 1$, show that the equation

$$2\cos^2\theta + \sin\theta = 2$$

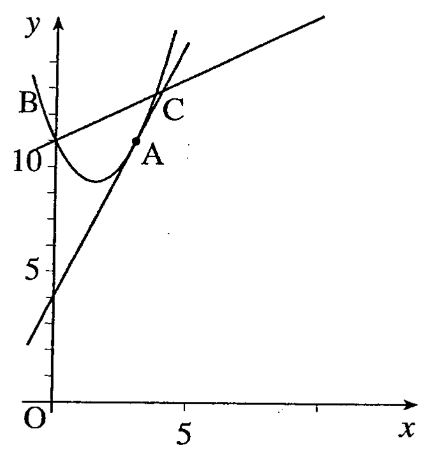
can be written as $2\sin^2\theta - \sin\theta = 0$. [2]

(ii) Hence find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$2\cos^2\theta + \sin\theta = 2.$$ [4]

Section B

10



The curve shown has equation $y = \frac{2}{3}x^2 - 2x + 10$.

(i) Find the equation of the tangent to the curve at A (3, 10). [4]

(ii) Show that the equation of the normal to the curve at B (0, 10) is $2y - x = 20$. [3]

(iii) Find the coordinates of the point C where these two lines intersect. [3]

(iv) Calculate the length BC. [2]

11 A small factory makes two types of components, X and Y. Each component of type X requires materials costing £18 and each component of type Y requires materials costing £11. In each week materials worth £200 are available.

Each component of type X takes 7 man hours to finish and each component of type Y takes 6 man hours to finish. There are 84 man hours available each week.

Components cannot be left part-finished at the end of the week. In addition, in order to satisfy customer demands, at least 2 of each type are to be made each week.

(i) The factory produces x components of type X and y components of type Y each week. Write down four inequalities for x and y. [4]

(ii) On a graph draw suitable lines and shade the region that the inequalities do not allow. (Take 1 cm = 1 component on each axis.) [5]

(iii) If all components made are sold and the profit on each component of type X is £70 and on each component of type Y is £50, find from your graph the optimal number of each that should be made and the total profit per week. [3]

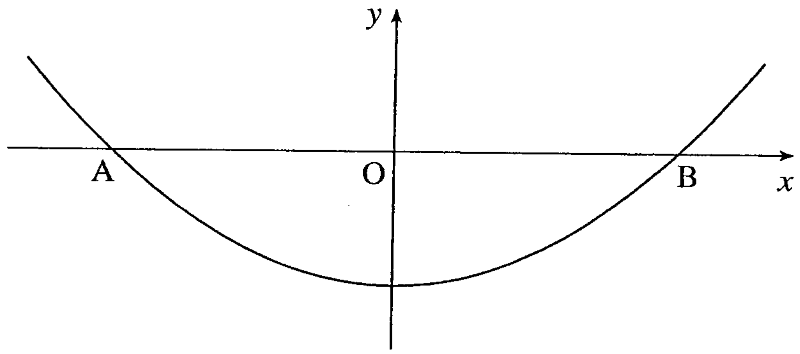
(Do not forget to hand in your graph paper with your answer booklet.)

12 (i) A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$. Find the coordinates of its centre, C, and its radius. [3]

(ii) Find the coordinates of the points, A and B, where the line $y = x + 2$ cuts the circle. [5]

(iii) Find the angle ACB. [4]

13



The shape of the bed of a river is to be modelled mathematically. The diagram represents a cross-section of the river. A and B on the x-axis represent points on opposite banks of the river at water level. (Units are metres.)

The shape of the river bed between A and B is modelled by the equation

$$y = \frac{3}{16}(x^2 - 16).$$

(i) Find the coordinates of A and B and hence state the width of the river represented by the length AB. [2]

(ii) Find the depth of the river at its deepest point. [2]

(iii) Find the area of the cross-section of the river. [5]

(iv) The river flows at 20 metres a minute. You should assume that this rate applies to all points of this cross-section.

Find the volume of water that flows through this cross-section per minute. [1]

(v) Give two reasons why this model may not be a good model. [2]

June 2005

ADVANCED FSMQ

MARK SCHEME

Maximum mark: 100

Syllabus/component:

6993 Additional Mathematics

Paper Date: June 20, 2005

Mark Scheme

Section A

1		$\frac{dy}{dx} = 18x - 6x^2 = 0$ when $x^2 - 3x = 0$	M1	One term with decreasing power
		$\Rightarrow x(x-3) = 0 \Rightarrow x = 3,0$	A1	Equates to 0 and gets 3 or subs 3 and gets 0 Or sub x = 3 to give 0
		$\frac{d^2y}{dx^2} = 18 - 12x$	DM1	Or "before and after" on gradient (or curve)
		When $x = 3$ $\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum	A1	Getting it right and sub 3 and getting max.
		When $x = 3$, $y = 27$	B1	y coordinate
			5	

2	**(i)**	f(–2) = –8 – 16 – 10 – 2 = –36	M1 A1 **2**	Or long division. We need to see $x^3 + 2x^2$ subtracted and x^2 on top.
	(ii)	f(1) = 1–4 + 5 – 2 = 0	B1 **1**	Arithmetic must be seen
	(iii)	$\Rightarrow$ f(x) = (x –1)(x^2–3x + 2) = (x – 1)(x – 1)(x – 2) $\Rightarrow$ x = 1, 1, 2	M1 A1 A1 B1 **4**	To find quadratic Or trial to find other roots Each factor Ans expressed properly

3		Largest angle is opposite longest side.	B1	Or for sum = 180
		$\cos\theta = \frac{8^2 + 7^2 - 12^2}{2 \times 8 \times 7}$ $= -0.2768 = \left(-\frac{31}{112}\right)$ $\Rightarrow \theta = 106^{\circ}$	M1 F1 A1 A1 **5**	Give M1 F1 for cos rule regardless of which angle
		Note 1: Other angles are 34.1 and 39.8 ***Note 2: Scale drawing could get the B1 but no more.***		See note above

4		$4\sin\theta - 3\cos\theta = 0 \Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{3}{4} \Rightarrow \tan\theta = 0.75$	B1	For $\tan\theta = k$
		$\tan\theta = 0.75 \Rightarrow \theta = 36.9,$ Also $\theta = 216.9$	M1 A1 F1	 For 36.9 other value
		Alternatively: $16\sin^2\theta = 9\cos^2\theta \Rightarrow 16 - 16\cos^2\theta = 9\cos^2\theta,$ $\Rightarrow \cos^2\theta = \dfrac{16}{25} \Rightarrow \cos\theta = 0.8$ $\Rightarrow \theta = 36.9,\ 216.9$	**4**	B1 M1 (use of pythag) A1 A1 (ignore extra values)

5		P(all sound) = $(0.86)^8 = 0.299$	B1	Allow 4 d.p.
		P(at least 2 defective) $= 1 - (0.86)^8 - 8(0.86)^7(0.14)$ $= 1 -$ ans above $- 0.3896$ $= 0.311$	B1 M1 A1 M1 A1	powers coeff with term having product (0.3896 implies 8) for 1- 2 terms ans
		i.e. P(at least 2 defective) is greater.	F1 **7**	conclusion (based on their answers providing each is < 1

6	**(i)**	$\left(x - \dfrac{1}{x}\right)^4 = x^4 - 4x^3\dfrac{1}{x} + 6x^2\left(\dfrac{1}{x}\right)^2 - 4x\left(\dfrac{1}{x}\right)^3 + \left(\dfrac{1}{x}\right)^4$ $= x^4 - 4x^2 + 6 - \dfrac{4}{x^2} + \dfrac{1}{x^4}$	B1 B1 B1 B1	powers correct coeffs correct signs outside brackets answer
		For complete expansion by multiplying, give B4, -1 for each mistake.	**4**	
	(ii)	Substituting will give the same value for both (= 0)	B1 **1**	

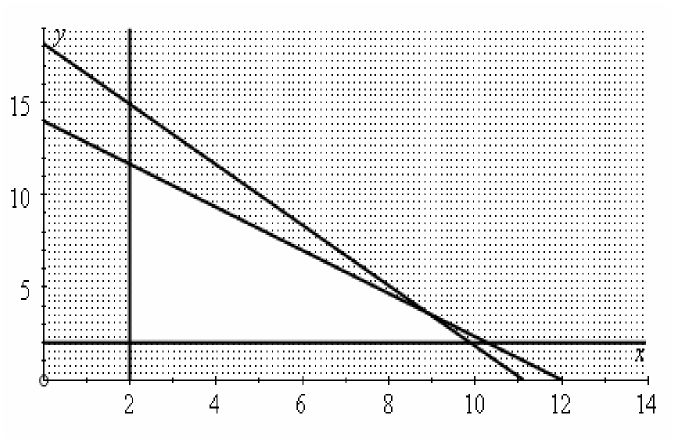
7		$\dfrac{dy}{dx} = a + bx \Rightarrow y = ax + \dfrac{bx^2}{2} + c$	M1 A1	Integrate (ignore c)
		$(0,2) \Rightarrow c = 2$	M1 A1	Add and work out c.
		$(1,8) \Rightarrow 8 = a + \dfrac{b}{2} + 2$	DM1	Attempt to substitute both
		$(-1,2) \Rightarrow 2 = -a + \dfrac{b}{2} + 2$	M1	Solve simultaneously
		Solve: $b = 6, a = 3 \Rightarrow y = 3x + 3x^2 + 2$	A1 **7**	Equation must be given

8	**(i)**	$v = 0$ when $\frac{1}{8}t^3 = 27 \Rightarrow t = 2 \times 3 = 6$	B1 **1**	
	(ii)	$s = \int_0^6 \left(27 - \frac{1}{8}t^3\right) dt$ $= \left[27t - \frac{t^4}{32}\right]_0^6$ $= 162 - 40.5 = 121.5$ Distance = 121.5 metres	M1 A1 M1 A1 B1 **5**	Integrate 2 correct terms Substitute Ans Units dependent on M marks

9	**(i)**	$\cos^2\theta = 1 - \sin^2\theta \Rightarrow 2 - 2\sin^2\theta + \sin\theta = 2$ $\Rightarrow 2\sin^2\theta - \sin\theta = 0$	M1 A1 **2**	N.B. Answer given
	(ii)	$2\sin^2\theta - \sin\theta = 0 \Rightarrow \sin\theta(2\sin\theta - 1) = 0$ $\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$ $\Rightarrow \theta = 0,\ 30, 150, 180$	B1 B1 B1 B1 **4**	B1 for each angle all 4

Section B

10	**(i)**	$\frac{dy}{dx} = \frac{4x}{3} - 2;$ When $x = 3$ $g = 2$ $\Rightarrow (y - 10) = 2(x - 3) \Rightarrow y = 2x + 4$ (*Watch any use of (0,4) - no marks*)	M1 A1 DM1 A1 **4**	Diffn Correct line for their gradient
	(ii)	At (0,10) $g = -2;$ Gradient of Normal = $\frac{1}{2}$ $\Rightarrow y = \frac{1}{2}x + 10 \Rightarrow 2y - x = 20$	M1 A1 B1 **3**	g = –2 must be seen N.B. Answer given
	(iii)	Substitute: $2(2x+4) - x = 20 \Rightarrow 3x = 12$ $\Rightarrow x = 4,$ $y = 12$ (Allow 3 for (4, 12)	M1 A1 F1 **3**	*y* value
	(iv)	Pythagoras for BC = $\sqrt{(12-10)^2 + (4)^2}$ $= \sqrt{20} = 2\sqrt{5} \approx 4.472$	M1 A1 **2**	

11	**(i)**	$18x + 11y \le 200$ $7x + 6y \le 84$ $x \ge 2$ $y \ge 2$	B1 B1 B1 B1 **4**	-1(once only) if = sign missing B0 for any line with inequality the wrong way round.
	(ii)	Graph plus shading 	B4 B1 **5**	B1 for each line correct shading, but only if all lines correct
	(iii)	Maximum profit is at the intersection of the lines: $P = 70x + 50y$. Nearest integer point to it is (9, 3) giving P = 780	B1 B1 B1 **3**	May be implied for any value (Intersection of lines is at (8.77, 3.76) giving 802.4)

12	**(i)**	$(x-1)^2+(y-2)^2=25$; Centre (1, 2), radius 5	M1 A1 F1 **3**	 For stating ans ft from equation
	(ii)	Substitute: $(x-1)^2+x^2=25$ $\Rightarrow 2x^2-2x-24=0$ $\Rightarrow x^2-x-12=0 \Rightarrow (x-4)(x+3)=0$ $\Rightarrow x=4,-3$ $\Rightarrow (4,6)$, $(-3,-1)$	M1 F1 M1 A1 A1 **5**	ft from (i)
	(iii)	AC = BC = 5, AB = $7\sqrt{2}$ $\Rightarrow$ Angle = $2\times\sin^{-1}\left(\dfrac{\frac{7}{2}\sqrt{2}}{5}\right)=2\times\sin^{-1}(0.99)$ $=81.87 \Rightarrow \text{ACB}=163.7$	M1 A1 M1 A1 **4**	 for AB

13	**(i)**	A (-4, 0), B(4, 0) gives width = 8m	B1 B1 **2**	
	(ii)	x = 0, (0, -3) gives depth = 3m	B1 B1 **2**	ignore -ve sign
	(iii)	Cross section area = $\displaystyle\int_{-4}^{4}\frac{3}{16}\left(x^2-16\right)\mathrm{d}x$ $=\displaystyle\frac{3}{8}\int_{0}^{4}\left(x^2-16\right)\mathrm{d}x$ $=\displaystyle\frac{3}{8}\left[\frac{x^3}{3}-16x\right]_0^4=-\frac{3}{8}\left(\frac{128}{3}\right)$ $=16\,\mathrm{m}^2$	M1 M1 A1 A1 A1 **5**	Integration required. Correct expression (his limits from (i)) (ignore $^3/_{16}$) Integration Both terms Working it out - include $^3/_{16}$
	(iv)	$\Rightarrow$ Vol per minute = $16\times20=320\text{m}^3$	B1 **1**	20 × Ans from (iii)
	(v)	Water does not usually flow at a constant rate. The river bed will not be symmetric.	B1 B1 **2**	
		If units are omitted throughout, then subtract one mark. If not all parts have been done then deduct from first answer and bod the rest. ***If any of the answers have units then do not deduct the mark.***		

Examiner’s Report

Free Standing Mathematics Qualification, Advanced Level
6993 Additional Mathematics

Summer, 2005
Chief Examiner's Report

The paper was a little easier this year than last, particularly in Section A. While this meant that a number of good candidates performed even better than the good candidates last year it is still true to say that there are a significant number of candidates who appear to have been entered for a qualification that is not suited to their abilities. For at least one centre no mark achieved reached double figures. The specification states that this is an Advanced FSMQ and that an appropriate starting point is a grade A*, A or B at GCSE with a thorough knowledge of the content of the Higher Tier. We believe that with this starting point a modest mark should be achievable with little or no extra learning. Consequently we believe that those achieving such low marks as single figures were not starting from this point and therefore this specification was not appropriate for them. This could not have been a positive experience for them and Centres might consider seeking advice as to what might be an appropriate course where they could demonstrate positive achievement.
The mean mark was 46.8.

Section A
Q1 (Calculus)
For most candidates this was a straightforward start to the paper, though many missed parts of the question, such as demonstrating that the turning point was a maximum or failing to find the y coordinate of the point.
[Maximum point at (3, 27)]

Q2 (Functions)
Most candidates knew the factor theorem and obtained the second part correctly, while many did not know the remainder theorem for part (i). Instead, long division was carried out, often successfully, but for many the inevitable algebraic errors crept in causing errors.
In part (iii) a few obtained the solution by trial and error, based on their knowledge of what possible roots there could be, given the constant number at the end. The significant majority used division to obtain a quadratic which they then solved correctly. Unfortunately a significant number of candidates failed to finish the question by giving the solution to the cubic equation, being content instead just to give the roots of the quadratic.
[(i) −36, (iii) x = 1, 1, 2]

Q3 (Cosine rule)
This question asked for the largest angle. Those who found it therefore gained full marks, but many of those who did not appreciate that the largest angle was opposite the longest side gave themselves extra work.
[106^0]

Q4 (Trigonometrical equation)
There were a few candidates who plotted the graphs $y = \sin x$ and $y = \cos x$ on their calculators and were able to zoom in on their intersections to find the roots to the required degree of accuracy; quite a number doing it this way, however, either failed to appreciate that there were two roots, or failed to find the values to the correct degree of accuracy, for which they were penalised.
The expected method was to obtain $\tan\theta = 0.75$. There were many errors here, including obtaining $\tan\theta = 1.33$.
The vast majority who failed to get anywhere with the question failed to demonstrate a knowledge of the relationship between the trigonometrical functions.

[36.9 and 216.9^0]

Q5 (Binomial distribution)
A few candidates did not read the question carefully. Some read the question as "exactly 2" rather than "at least 2". Many also failed to give a conclusion at the end.

Q6 (Binomial expansion)
This question was not always well done. The combination of the powers, the coefficients and the signs defeated all but the most able.
The last comment was often fudged and not at all clear, indicating that candidates did not appreciate that to make a substitution of a numerical value in the original and the final expansion should give the same value.
[$x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$]

Q7 (Integration)
This question was not done well and we were surprised by some of the answers. Even the most able candidates were integrating the function $\frac{dy}{dx} = a + bx$ to give $y = \frac{a^2}{2} + b\frac{x^2}{2}$. Even those who got it right then failed to appreciate that there was a non-zero constant of integration.
[$y = 3x + 3x^2 + 2$]

Q8 (Variable acceleration).
The first mark was easy to obtain for all candidates.
Part (ii) separated the candidates into those who knew that integration was required and those who assumed constant acceleration. A few candidates failed to give the answer properly, giving a numerical value for *s* only rather than stating the distance in metres.
[121.5 metres]

Q9 (Trigonometrical equation)
Only a few candidates failed to manipulate the equation using Pythagoras' theorem. However, the solution of the equivalent equation was not done nearly so well. The vast majority failed to appreciate that there are two roots from $\sin\theta = 0$ as well as two from $\sin\theta = 0.5$.
[0^0, 30^0, 150^0, 180^0]

Section B
Q10 (Coordinate geometry)
Part (i) was usually done well but a significant minority took a short cut by making unjustified assumptions from the diagram. In part(ii) there were more assumptions, this time occasionally justified (i.e. that the gradient of the tangent at (0, 10) is −2 given that the gradient at (3, 10) is 2). Candidates need to beware of making assumptions with no justification.
Parts (iii) and (iv) were usually well done and we saw occasionally candidates who were unable to complete parts (i) and (ii) reentering the question here.
The accuracy required for the answer to part (iv) was not given. Candidates should note the rubric of the paper and not simply write down every digit they see on their calculator. In these instances candidates should also be aware of the fact that an exact answer can be given and this was given full credit.
[(i) $y = 2x + 4$, (iii) (4, 12), (iv) $\sqrt{20} = 2\sqrt{5} = 4.472$]

Q11 (Linear programming)
This question was a source of a number of marks for weaker candidates. Practically every candidate realised that the "best" solution was not the intersection of the lines but a point nearest to it; most failed to investigate this with any logical order (by writing, for instance, a table), but many still got the right answer.
[(i) $18x + 11y \le 200$, $7x + 6y \le 84$, $x \ge 2$, $y \ge 2$, (iii) £780 at (9, 3)]

Q12 (Circle)
Those who knew something about the coordinate geometry of the circle got many marks in this question. Unfortunately there were many who were unable to manipulate the equation given into that required (by completing the square) to write down the radius and centre of the circle. Some were able to reenter the question in part (ii) and complete this part successfully. Those who did it most elegantly substituted into their rearranged equation. A few plotted the curve and the line and read off the points of intersections. We felt that this constituted an assumption again, and for full marks we required candidates to demonstrate that the integer points they picked out from their graph did satisfy both equations.
Part (iii) was testing for all candidates but a few were able to complete it successfully.
[(i) Centre (1, 2), radius 5, (ii) (4, 6) and (-3, -1) (iii) 163.7^0]

Q13 (Integration and modelling)
Parts (i) and (ii) were usually done quite well, though again, some good candidates lost marks by failing to answer the question properly. In part (i) they would leave it as A(–4, 0) and B(4, 0) without interpreting that the width was 8 metres. In part (ii) they would simply write $y = -3$ with no interpretation to give the greatest depth.
In part (iii) many got well into this integration but fell down over the manipulation of the fraction and brackets.
Part (iv) was well done even by those who got most of the first three parts wrong.
Many suggestions given in part (v) did not relate to the cross-section and some gave the same explanation in a different way.
Candidates who did not give correct units were penalised - in a modelling question the correct interpretation is required which is usually a little more than the numerical answer.
[(i) Width 8 m, (ii) Depth 3 m, (iii) 16 m^2, (iv) 320 m^3 (v) For e.g. the bed of a river is not usually smooth or symmetric, and the flow of water is not likely to be constant throughout the cross-section.]