GCE

Mathematics (MEI)

Advanced GCE **A2 7850-7, 7895-8**

Advanced Subsidiary GCE **AS 3850-7, 3895-8**

Combined Mark Schemes And Report on the Units

June 2005

3850-7/7850-7/MS/R/05
3895-8/7895-8/MS/R/05

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MARK SCHEME ON THE UNITS

REPORTS FOR THE UNITS

Mark Scheme 2601
June 2005

	Section A			
1	$6x^5 - 12x^3$	B2	B1 if one error	2
2.	150	B2	M1 for $\times 180/\pi$	2
3.	1 792	B3	M1 for ${}^8C_3$ or 56 and M1 for 2^5	3
4.	$y = -5x + 9$	B3	M1 for gradient = $(-6 - 4) \div (3 - 1)$ and M1 for $(y - 4)$ = their $m(x - 1)$ o.e.	3
5.	at least 4 of 1, 0.8, 0.5, 0.3, 0.2 seen or used 1.1	B1 B3	M2 for $0.5/2 \times \{1 + 0.2 + 2(0.8 + 0.5 + 0.3)\}$o.e.; M1 for $k/2 \times \{1 + 0.2 + 2(0.8 + 0.5 + 0.3)\}$o.e , $k \neq 0.5$ or other single error, or for two separate correct traps [0.45, 0.325, 0.2, 0.125]	4
6.	$(5x + 1)(x - 2)$ 2 and $-1/5$ inequalities ft their roots or sketch of quadratic correct way up $x > 2$ or $x < -1/5$ or ft their roots	M1 A1 M1 A1	for attempt at factorisation or quad. formula or B2 or B4	4
7.	$\cos\theta = \frac{\sqrt{2}}{\sqrt{11}}$ or $\sqrt{\frac{2}{11}}$ cosec $\theta = \sqrt{11}/3$	B3 B1	M1 for <u>use</u> of $\sin^2\theta + \cos^2\theta = 1$ or for rt angled triangle with hyp = $\sqrt{11}$ and M1 for $\cos^2\theta = 1 - 9/11$ or $(\text{side})^2 = 2$	4
8.	integral of $\pi(2x)^2$ $[4\pi]x^5/5$ their integral at 2 – their integral at 1 $124/5\pi$ or 24.8π or 77.9(1..)	M1 M1 M1 A1	ft for integral of $2\pi x^4$ or omission of π 0 for original fn or differential used	4
9.	$n - 1$, $n + 1$ seen $n^2 - 2n + 1$ and/or $n^2 + 2n + 1$ seen [sum of squares =] $3n^2 + 2$ e.g. '3 is a factor of $3n^2$ but not of 2' or 'there will always be a remainder of 2' o.e.	B1 B1 B1 E1		4
			Total Section A	30

		Section B			
10	(i)	$(x-2)(x-6)$ A [and B] have x coords 2 [and 6]	M1 A1	correct factors or correct use of formula; marks for A may be earned in (ii)	
		C has x coord (2+6)/2 [=4] C has y coord −4	M1 B1	or M1 for $y'=2x-8$ and $y'=0$ used or for $(x-4)^2-4$	4
	(ii)	$AC^2 = 2^2 + 4^2$ $(x-4)^2 + (y+4)^2 = 20$ o.e.	M1 B2	B1 for one side correct	3
	(iii)	sin (½ ACB) = 2/√20 or cos (½ ACB) = 4/√20 or tan = 2/4 0.4636.. × 2 Area sector = ½ × $(\sqrt{20})^2$ × 0.93 Area tri. = ½ × $(\sqrt{20})^2$ × sin 0.93 Area segment = Area sector – triangle 1.27 to 1.3	M1 M1 M1 M1 M1 A1	or M1 correct use of cos rule and M1 for cos ACB = 12/20 and completion or M1 for 8 = ½ × $(\sqrt{20})^2$ × sin C and M1 for sin C = 0.8 and completion or angle found in degrees and conv or 9.27 to 9.3 or ½ × 4 × 4 or 8 to 8.02	6
	(iv)	$x^3/3 - 4x^2 + 12x$ value at 6 – value at 2 [−] 10.66 to 10.7	M1 M1 A1	condone one error dep on an integral found answer can imply 2nd M1; 0 for answer with no evidence	3
11	(i)	$y = 0$ when $x = -4, 0, 1$ so factors are $(x+4)$, x, $(x-1)$ constructive intermediate step such as $x(x^2+3x-4)$	B1 B1	marks may be earned in either order NB answer given	2
	(ii)	$y' = 3x^2 + 6x - 4$ use of $y' = 0$ attempt at subst in quad formula [x =] 0.53 or −2.53	M1 M1 M1 A2	may be earned in (iii); condone one error 1 each, or A1 for both solns not to 2 dp	5
	(iii)	$(-1)^3 + 3\times(-1)^2 - 4\times -1$ [= 6] subst of −1 in their y' [=−7] grad normal = −1/grad tgt [=1/7] $(y-6) = 1/7(x+1)$ cao and ft	B1 M1 M1 A1	 allow 1/7 bod if prev M1 earned and ans −7 seen there or subst of (−1, 6) in $y = 1/7\,x + c$ or given answer	4
	(iv)	$\frac{1}{7}(x+43) = x^3 + 3x^2 - 4x$ cao correct constructive step in rearranging (not just expanding bracket) $(x+1)$ used as factor to obtain given quadratic	M1 M1 M1	 dep on previous M1 NB answer given	3

Mark Scheme 2602
June 2005

1 (a) $y = \dfrac{x}{1+\ln x}$ $\dfrac{dy}{dx} = \dfrac{(1+\ln x).1 - x.\frac{1}{x}}{(1+\ln x)^2}$ $= \dfrac{\ln x}{(1+\ln x)^2}$.	M1 M1 A1 [3]	d/dx (ln x) = 1/x soi correct expression using product or quotient rule simplified numerator
(b) $y = (1+x^3)^{\frac{1}{2}}$ let $u = 1 + x^3$, $y = u^{1/2}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$ $= ½\, u^{-1/2}.\, 3x^2$ $= \dfrac{3}{2}x^2(1+x^3)^{-\frac{1}{2}}$.	M1 M1 A1cao [3]	chain rule ½ $u^{-1/2}$ soi or equivalent
(c) (i) $y = 1 + x^{1/3}$ $\Rightarrow$ dy/dx = 1/3 $x^{-2/3}$	B1	isw
(ii) dx/dy = $\dfrac{1}{dy/dx}$ $= 3x^{2/3}$	M1 A1ft	1/ their c(i) correct expression in terms of x **If (iii) not done** scB1 for$\Rightarrow$ dx/dy = $3(y-1)^2$ and scB2 for dx/dy =$3x^{2/3}$
(iii) $x^{1/3} = y - 1$ $\Rightarrow x = (y-1)^3$ $\Rightarrow$ dx/dy = $3(y-1)^2$ $= 3\,(x^{1/3})^2 = 3x^{2/3}$ as before	 M1 A1 E1 [6]	 raising to power 3 correctly
(d) $\int_0^{a^2}(1+3\sqrt{x})dx$ $= \left[x + 3\dfrac{x^{3/2}}{3/2}\right]_0^{a^2}$ $= \left[x + 2x^{3/2}\right]_0^{a^2}$ $= a^2 + 2a^3\ \ (-\,0\,)$ $= a^2(1+2a)$*	 M1 M1 E1 [3]	 correctly integrated limits substituted correctly into some attempt at integration www intermediate step (unfactorised form) must be seen for this

2 (i) $a = -8$ $-8 + 19d = 3(-8 + 9d)$ $= -24 + 27d$ $\Rightarrow 16 = 8d$ $\Rightarrow d = 2$	M1 M1 A1 [3]	for either -8 +19d or -8 + 9d their u_{20} = 3 their u_{10} cao (B3 ans without wrong working, verified)
(ii) $\frac{20}{2}(2a+19\times 2) = 3\times\frac{10}{2}(2a+9\times 2)$ $\Rightarrow 10(2a + 38) = 15(2a + 18)$ $\Rightarrow 20a + 380 = 30a + 270$ $\Rightarrow 110 = 10a$ $\Rightarrow a = 11$	M1 M1 A1 A1 [4]	correct expression for either sum their S_{20} = 3 their S_{10} correct equation cao
(iii) $a \times r^{19} = 3 \times a \times r^9$ $\Rightarrow r^{10} = 3$ $\Rightarrow r = 3^{1/10} = 1.12$ (3 s.f.)	M1 A1 B1 [3]	for $a \times r^{19}$ or $a \times r^9$ soi or correct eqn using $a \times r^{19}$ and $a \times r^9$ eg log cao (B3 without working ans 1.12)
(iv) $\frac{a(r^{20}-1)}{r-1} = 3\frac{a(r^{10}-1)}{r-1}$ $\Rightarrow r^{20} - 1 = 3(r^{10} - 1)$ $\Rightarrow r^{20} - 3r^{10} + 2 = 0,\ u = r^{10}$ $\Rightarrow u^2 - 3u + 2 = 0$ * $\Rightarrow (u - 2)(u - 1) = 0$ $\Rightarrow u = 2$ or $u = 1$ $\Rightarrow r^{10} = 2$ or $r^{10} = 1$ $\Rightarrow r = 2^{1/10}$ (= 1.07) ($r \neq 1$)	M1 M1 E1 B1 B1 [5]	for S_{20} or S_{10} their S_{20} = 3 their S_{10} u=2 or $r^{10} = 2$ cao

3 (i) Odd function $f(-x) = -xe^{-(-x)^2/2} = -xe^{-x^2/2}$ =(−f(x))	B1 M1 E1 [3]	f(−x) = −f(x) brackets or comment needed to convince re signs
(ii) $y = e^{-x^2/2}$ let $u = -x^2/2$, du/dx = −2x/2 = −x $y = e^u$, dy/du = e^u $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -x\,e^u = -xe^{-x^2/2}$ $f'(x) = x.(-xe^{-x^2/2}) + 1.\ e^{-x^2/2}$ $= (1 - x^2)\ e^{-x^2/2}$ *	M1 A1 M1 E1 [4]	chain rule s.o.i. product rule (ft) s.o.i. www
(iii) $(1 - x^2)\ e^{-x^2/2} = 0$ $\Rightarrow$ $1 - x^2 = 0$ $\Rightarrow$ $x^2 = 1$ $\Rightarrow$ x = 1 or −1 When x = 1, $y = e^{-1/2}$ When x = −1, $y = -e^{-1/2}$	M1 A1 A1 A1 [4]	$1 - x^2 = 0$ or first line =0 x = 1 $y = e^{-1/2}$ $(-1, -e^{-1/2})$ SC A1 for both y-coords decimal only, 0.61 or better
(iv) $A = \int_0^1 xe^{-\frac{1}{2}x^2}\,dx$ let $u = \frac{1}{2}x^2$, du/dx = x $\Rightarrow$ du = xdx When x = 0, u = 0 When x = 1, u = ½ $\Rightarrow A = \int_0^{1/2} e^{-u}\,du$ * $= \left[-e^{-u}\right]_0^{1/2}$ $= -e^{-1/2} + 1 = 1 - e^{-1/2}$.	M1 M1 E1 M1 A1 [5]	correct integral (condone missing limits and *dx)* dealing with dx change of limits shown (convincing recovery needed from no *dx*) $\left[-e^{-u}\right]$ or equivalent (no decimals)

4 (i) $y = a \times b^x$ $\Rightarrow$ $\ln y = \ln a + x \ln b$ c.f. $y = c + x\ m$ gradient = $\ln b$, $\ln y$ – intercept = $\ln a$	M1 B1 B1 [3]	condone log allow m= if M1 scored allow c = if M1 scored otherwise need gradient and intercept
(ii) $b = e^{0.7} = 2.01 \approx 2$ Intercept = 0.7	B1 B1 [2]	ln 2 = 0.69 "which fits" or indication of checking with graph
(iii) Gradient = $-\frac{1.1}{0.68} = -1.62 = -\ln c$ $\Rightarrow$ $c = e^{1.6} = 5$ (to nearest whole no)	M1 M1 A1 A1cao [4] M1 M1 A1 A1 M1 M1 A1 A1	gradient = +/- ln c soi using graph values to obtain gradient +/–1.6(2) **OR** lny = ln3 – x ln c substituting a point from graph lnc = numerical expression cao **OR** manipulating equation without lns substituting a point from graph and calculating value of y from value of lny c = numerical expression cao
(iv) 0.15	B1 [1]	
(v) $2 \times 3^x = 3 \times c^{-x}$ $\Rightarrow$ $\ln 2 + x \ln 3 = \ln 3 - x \ln c$ $\Rightarrow$ $x \ln 3 + x \ln c = \ln 3 - \ln 2$ $\Rightarrow$ $x(\ln 3 + \ln c) = \ln 3 - \ln 2$ $\Rightarrow$ $x = \frac{\ln 3 - \ln 2}{\ln c + \ln 3}$ * $= \frac{\ln 3 - \ln 2}{\ln 5 + \ln 3}$ = 0.1497	M1 M1 E1 B1ft [4]	taking lns at any stage collecting x's at any stage factorisation seen ft integer value of c accept 0.15

Mark Scheme 2603
June 2005

2603
MEI P3 June 2005 Mark Scheme post-coordination
General Instructions

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
(b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
(c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓̸
   - For error in follow through work, use ✗̸

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not be earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:
   c.a.o. : correct answer only
   b.o.d. : benefit of doubt (where full work is not shown)
   X
   | : work of no mark value between crosses
   X
   s.o.i. : seen or implied
   s.c. : special case (as defined in the mark scheme)
   w.w.w : without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting prior to the batch 1 date, send a further sample of about 40 scripts, from complete centres. You should record the marks for these scripts on your marksheets. They will not be returned to you, but you will receive feedback on them. If all is well, you will then be given clearance to send your batch 1 scripts and marksheets to Cambridge.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

1 (a) $5\cos x - 6\sin x = R\cos(x+\alpha)$ $= R(\cos x\cos\alpha - \sin x\sin\alpha)$ $\Rightarrow R\cos\alpha = 5,\ R\sin\alpha = 6$ $\Rightarrow R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 6^2 = 61$ $\Rightarrow R^2 = 61,\ R = \sqrt{61} = 7.81$ $\tan\alpha = R\sin\alpha / R\cos\alpha = 6/5$ $\Rightarrow \alpha = 0.876$ Accept 0.88 or 0.279π but not $50.2°$	M1 A1 M1 A1cso [4]	Expansion or equations in R and α, soi NB $R(\cos x\cos\alpha + \sin x\sin\alpha)$ gains M1 but final A1 cso will be lost. $R = 7.81$ or $\sqrt{61}$. Accept 7.8 $\tan\alpha = 6/5$ soi Note, $\cos\alpha = 5$, $\sin\alpha = 6 \Rightarrow \tan\alpha = 6/5$ is M0.
(b) $\int_0^{\frac{1}{4}\pi} x\sin 2x\,dx$ let $u = x$, $dv/dx = \sin 2x$ $\Rightarrow v = -\frac{1}{2}\cos 2x$ $= \left[-\frac{1}{2}x\cos 2x\right]_0^{\frac{1}{4}\pi} - \int_0^{\frac{1}{4}\pi}(-\frac{1}{2}\cos 2x).1.dx$ $= 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{1}{4}\pi}$ $= \frac{1}{4}$	M1 A1 A1ft DM1 A1cao [5]	$u = x$, $dv/dx = \sin 2x$ $v = -\frac{1}{2}\cos 2x$ $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x$ ft their v substituting limits
(c) $1 + \ln 1 = 1$, $\sin\pi/2 = 1$, so $(\pi/2, 1)$ lies on curve. $\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = \cos x$ $\Rightarrow \frac{dy}{dx}(1 + \frac{1}{y}) = \cos x$ $\Rightarrow \frac{dy}{dx} = \frac{\cos x}{1 + 1/y} = \frac{y\cos x}{y+1}$ * Allow verification: Separating the variables M1 $y + \ln y = \sin x + C$ A1 $x = \pi/2$, $y = 1 \Rightarrow C = 0$ E1 When $x = \pi/2$, $y = 1$, $dy/dx = 0$	E1 M1 B1 E1 B1 [5] [14]	$1 + \ln 1 = 1$ or $1 + 0 = 1$ $\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx}$ .Some evidence of implicit differentiation. Ignore spurious dy/dx. Allow $\frac{dy}{dx} + y\frac{dy}{dx} =$, or $1 + \frac{1}{y}\frac{dy}{dx} = ...$ but not $1 + y\frac{dy}{dx} = \ldots$ $\cos x$ An intermediate step must be seen. www.

2 (i) $\frac{5}{(2+x)(1-2x)} = \frac{A}{2+x} + \frac{B}{1-2x}$ $\Rightarrow \quad 5 = A(1-2x) + B(2+x)$ $x = -2 \Rightarrow 5 = 5A \;, \Rightarrow A = 1$ $x = ½ \Rightarrow 5 = 2½\, B \;\Rightarrow B = 2$ $\Rightarrow \quad \frac{5}{(2+x)(1-2x)} = \frac{1}{2+x} + \frac{2}{1-2x}$	M1 A1 A1 [3]	Equating numerators soi $A = 1/(2+x)$ $B = 2/(1-2x)$ SC If no working seen , one fraction correct B2, the other correct B1.
(ii) $\frac{dy}{dx} = \frac{5y}{(2+x)(1-2x)}$ $\Rightarrow \quad \int \frac{1}{y} dy = \int \frac{5}{(2+x)(1-2x)} dx$ $= \int (\frac{1}{2+x} + \frac{2}{1-2x}) dx$ $\Rightarrow \quad \ln y \;= \ln\|2+x\| - \ln\|1-2x\| + c$ When $x = 0$, $\ln 2 = \ln 2 + c \;\Rightarrow c = 0$ $\Rightarrow \quad \ln y = \ln(2+x) - \ln(1-2x) = \ln(\frac{2+x}{1-2x})$ $\Rightarrow \quad y = \frac{2+x}{1-2x}$ *	M1 B1 B1 B1 DM1 E1 [6]	Re-arranging. Allow eg $\int y\, dy = \ldots$ $\ln y$) $\ln(2+x)$) Condone no C $-\ln(1-2x)$) calculating c without incorrect log work eg $y = \frac{2+x}{1-2x} + e^c \Rightarrow c = 0$ is DM0 www.
(iii) $y = \frac{2+x}{1-2x} = (2+x)(1-2x)^{-1}$ $= (2+x)(1+2x+4x^2+8x^3+\ldots)$ $= \;2+4x+8x^2+16x^3$ $+\; x+2x^2+4x^3+\ldots$ $= \;2+5x+10x^2+20x^3+\ldots$ * Valid for $\|x\| < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$	M1 M1 E1 B1 [4]	Binomial expansion : evidence of correct binomial coefficients and correct use of $(-2x)^r$ multiplying out given answer
(iv) $f'(x) = 5 + 20x + 60x^2$ $f'(0.01) = 5.206$ $y = \frac{2.01}{0.98} = 2.051\ldots$ $\frac{dy}{dx} = \frac{5 \times 2.051...}{0.98 \times 2.01} = 5.206\cdot\cdot$ So accurate to 3 decimal places (OR $\frac{dy}{dx} = \frac{5}{(1-2x)^2} = \frac{5}{(1-0.02)^2}$ using the quotient rule, (must be correct), and substitution, = 5.206)	B1 M1 A1 [3] [16]	5.206 substituting for x in y,and then x and y in dy/dx 5.206

3 (i) At A, $\cos 2\theta = -1$, $\Rightarrow$ A is (2, 0) At B, $\sin 2\theta = 1$ $\Rightarrow$ B is (1, 2).	M1 A1 M1 A1 [4]	Or, $y = 0 \Rightarrow \sin 2\theta = 0$ $\theta = \pi/2, x=2$ or, $x = 1 \Rightarrow \cos 2\theta = 0$ $\theta = \pi/4, y=2$ SC Allow B2, B2 for correct answers without working
(ii) $y = x \Rightarrow 1 - \cos 2\theta = 2\sin 2\theta$ $\Rightarrow 2\sin^2\theta = 4\sin\theta\cos\theta$ $\Rightarrow 2\sin\theta(\sin\theta - 2\cos\theta) = 0$ $\Rightarrow (\sin\theta = 0)$ or $\sin\theta = 2\cos\theta$ $\Rightarrow \tan\theta = 2$ *	M1 M1 A1 E1 [4]	Equating Attempt to use the double angle formulae to obtain an equation in θ. Any correct equation in θ
(iii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{4\cos 2\theta}{2\sin 2\theta}$ $= 2\cot 2\theta$* $\Rightarrow$ gradient at C = $2.\frac{1-\tan^2\theta}{2\tan\theta}$ $= 2\times\frac{(-3)}{4}$ $= -1.5$	M1 A1 E1 M1 A1cao [5]	$\frac{dy}{dx} = \frac{their dy/d\theta}{their dx/d\theta}$ $\frac{4\cos 2\theta}{2\sin 2\theta}$ (Allow these marks if seen in part (i).) Use of tan 2θ formula or $\theta = 63.43\ldots°$ or 1.107… rads
(iv) $\cos 2\theta = 1 - x$, $\sin 2\theta = y/2$ $\Rightarrow \cos^2 2\theta + \sin^2 2\theta = (1-x)^2 + \frac{y^2}{4} = 1$ or eg $y^2 + 4x^2 - 8x = 0$	M1 A1 [2] [15]	These equations plus a valid method of eliminating 2θ

4 (i) $\mathbf{a} = \begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$ $\Rightarrow \cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert} = \dfrac{(-3)\times 2 + 4\times 4 + 12\times 4}{\sqrt{169}\times\sqrt{36}}$ $= \dfrac{58}{78} \Rightarrow \theta = 41.96°$ (Accept 42° or 0.73 radians) Area of triangle OAB = $\dfrac{1}{2}\times 13\times 6\sin\theta$ = 26.1 (units ^{2}) (Accept 26)	M1 A1 A1 A1 M1 A1 ft [6]	use of scalar product (allow one slip) correct numerator correct denominator ft their θ
(ii) $\mathbf{r} = \begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix} + \lambda\begin{pmatrix} 8 \\ -9 \\ 5 \end{pmatrix}$	B1 B1 [2]	**r** = $\begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix} + \ldots$ or, using D, **r** = $\begin{pmatrix} 5 \\ -5 \\ 17 \end{pmatrix} + \ldots$ $\ldots + \lambda\begin{pmatrix} 8 \\ -9 \\ 5 \end{pmatrix}$ o.e.
(iii) $\mathbf{c}.\overrightarrow{OA} = \begin{pmatrix} 8 \\ -9 \\ 5 \end{pmatrix}.\begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix} = -24 - 36 + 60 = 0$ $\mathbf{c}.\overrightarrow{OB} = \begin{pmatrix} 8 \\ -9 \\ 5 \end{pmatrix}.\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = 16 - 36 + 20 = 0$ $\Rightarrow$ **c** is perpendicular to the plane OAB OAB: $8x - 9y + 5z = 0$ CDE: $8x - 9y + 5z = d$ At C, $x = 8$, $y = -9$, $z = 5$ $\Rightarrow$ $d = 64 + 81 + 25 = 170$ $\Rightarrow$ $8x - 9y + 5z = 170$	B1 B1 B1 M1 A1 [5]	Working must be seen
(iv) OC = √170 Volume of prism = 26.1 × √170 = 340 (units^3)	M1 A1cao [2] [15]	

Section B 1. The masses are measured in units. The ratio is dimensionless	B1 B1 [2]	'units cancel out' $\Rightarrow$ B2
2. Converting from base 5, $3.03232 = 3 + \frac{0}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5}$ = 3.14144	M1 A1 [2]	Allow this M1 for misreading and $3.03232 = 3 + \frac{0}{6} + \frac{3}{6^2} + \frac{2}{6^3} + \frac{3}{6^4} + \frac{2}{6^5}$ then A0
3. n x_n 0 0.5 1 0.8 2 0.512 3 **0.7995392** 4 **0.5128840565** 5 **0.7994688035**	B1 [1]	Ignore the ninth and tenth d.p.
4. $\frac{\phi}{1} = \frac{1}{\phi - 1}$ $\Rightarrow \phi^2 - \phi = 1 \Rightarrow \phi^2 - \phi - 1 = 0$ **M1** Using the quadratic formula gives $\phi = \frac{1 \pm \sqrt{5}}{2}$	M1 E1 [2]	A Q.E. in ϕ, (either form), and an attempt to solve- formula must be used correctly. SC Allow B2 for verification if completely correct.
5. Let $r = \frac{a_{n+1}}{a_n} = \frac{a_n}{a_{n-1}}$ $a_{n+1} = 2a_n + 3a_{n-1}$ dividing through by a_n. $\Rightarrow r = 2 + \frac{3}{r}$ $\Rightarrow r^2 - 2r - 3 = 0$ $\Rightarrow (r-3)(r+1) = 0$ $\Rightarrow$ r = 3 (discounting −1)	M1 M1 A1 A1 [4]	Either ratio Forming the equation in r, $r = 2 + \frac{3}{r}$ Correct equation in standard QE form Correct solution by factorisation or formula and rejecting the –ve root SC Allow B2 for calculation at least as far as a_8 = 1093, a_9=3281, a_{10}=9841 $\Rightarrow a_9/a_8$ =3.00…, a_{10}/a_9 = 2.99…
6. The length of the next interval = l, where $\frac{0.0952...}{l} = 4.669...$ $\Rightarrow$ l = 0.0203… So next bifurcation at 3.5437… + 0.0203… ≈ 3.564	M1 A1 M1 A1 [4] [15]	

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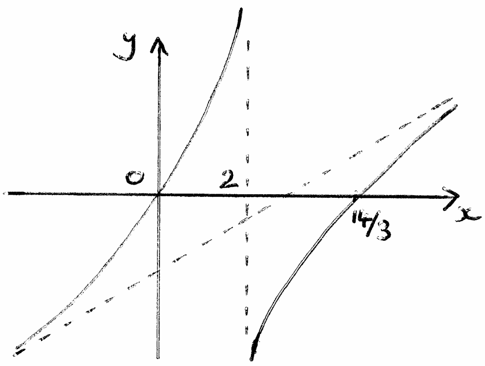
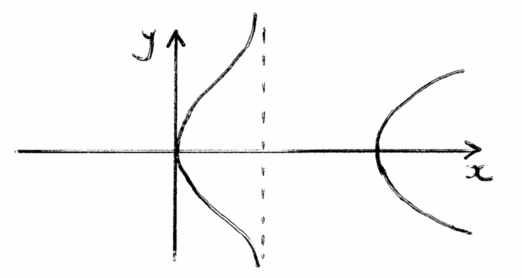
1 (i)	$x=2$ $y=\dfrac{(x-2)(3x-8)-16}{x-2}$ $=3x-8-\dfrac{16}{x-2}$ Oblique asymptote is $y=3x-8$	B1 M1 A1 A1	**4**	 Dividing by $(x-2)$ to obtain a linear quotient $ax+b$ with $a\neq 0,\ b\neq 0$ For quotient $3x-8$
(ii)	$\dfrac{dy}{dx}=3+\dfrac{16}{(x-2)^2}$	M1 A1		Differentiation (at most one error) *or* $\dfrac{(x-2)(6x-14)-(3x^2-14x)}{(x-2)^2}$ Any correct form
	$\dfrac{dy}{dx}>3$ so gradient is always positive	M1 A1	**4**	*or* $\dfrac{16}{(x-2)^2}>0$ Correctly shown *These 2 marks can be earned in (iii) but only if linked to 'positive gradient'*
	OR $3x^2-12x+28=0$ has no solutions, since $12^2-4\times 3\times 28=-192<0$ M1 so $\dfrac{3x^2-12x+28}{(x-2)^2}>0$ for all *x* A1			*or* $x=2\pm 2.3\mathrm{j}$ *or* $3(x-2)^2+16$ Correctly shown *SR* $(3x-2)^2+24$ scores M1A0
(iii)		B1 B1 B1	**3**	LH section: positive gradient, through O RH section: positive gradient, through $(\frac{14}{3},\ 0)$ *(Accept 4.6 to 4.7 on an accurate graph)* Fully correct shape, approaching asymptotes correctly
(iv)	$\dfrac{x(3x-14)}{x-2}=20$ when $3x^2-34x+40=0$ $x=\frac{4}{3},\ 10$ $\dfrac{x(3x-14)}{x-2}<20$ when $x<\frac{4}{3},\quad 2<x<10$	M1 M1 A1 M1 A1A1	**6**	Obtaining quadratic equation *(condone inequality)* Solving to obtain 2 values of *x* or factors $(3x-4)(x-10)$ Considering intervals defined by critical values $\frac{4}{3},\ 2,\ 10$ (ft) *Condone 1.33 but not 1.3*

<table>
<tr><td>(v)</td><td>
</td><td>B1 ft

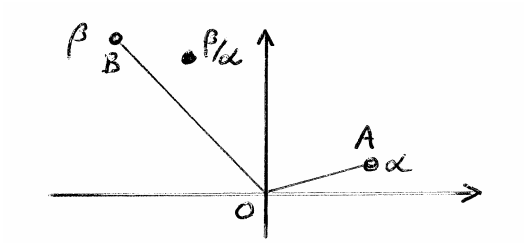
B1 ft

B1
3</td><td>No curve in ‘negative regions’ and curve in ‘positive regions’

Symmetry in x-axis

Fully correct shape, including infinite gradients when crossing x-axis (condone one ‘doubtful’ case)</td></tr>
</table>

2 (a)	$\sum_1^n (5r^3+9r^2)$ $= \frac{5}{4}n^2(n+1)^2 + \frac{9}{6}n(n+1)(2n+1)$ $= \frac{1}{4}n(n+1)(5n^2+17n+6)$ $= \frac{1}{4}n(n+1)(n+3)(5n+2)$	M1 A1A1 M1 A1 **5**	Multiplying out and using formulae
(b)	$\frac{3^{r+1}}{r+1} - \frac{3^r}{r} = \frac{3^{r+1}r - 3^r(r+1)}{r(r+1)}$ $= \frac{3^r(3r-r-1)}{r(r+1)} = \frac{3^r(2r-1)}{r(r+1)}$ $\sum_1^n \frac{3^r(2r-1)}{r(r+1)} = \sum_1^n \left(\frac{3^{r+1}}{r+1} - \frac{3^r}{r}\right)$ $= \left(\frac{3^2}{2} - \frac{3^1}{1}\right) + \left(\frac{3^3}{3} - \frac{3^2}{2}\right) + \ldots + \left(\frac{3^{n+1}}{n+1} - \frac{3^n}{n}\right)$ $= \frac{3^{n+1}}{n+1} - 3$	M1 A1 (ag) M1 A1 M1 A1 **6**	Three terms correct Cancelling to leave one fraction at the beginning and one fraction at the end
(c)	When $n=1$, LHS $= \frac{2-1}{1^2 \times 2^2} = \frac{1}{4}$ RHS $= \frac{1^2}{2^2} = \frac{1}{4} =$ LHS Assuming it is true for $n=k$, $\sum_1^{k+1} = \frac{k^2}{(k+1)^2} + \frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ $= \frac{k^2(k^2+4k+4)+2k^2+4k+1}{(k+1)^2(k+2)^2}$ $= \frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2}$ $= \frac{(k+1)^4}{(k+1)^2(k+2)^2} = \frac{(k+1)^2}{(k+2)^2}$ True for $n=k \Rightarrow \text{True for } n=k+1$ Hence true for all positive integers *n*	B1 M1 A2 M1 M1 A1 A1 A1 **9**	Attempt at $S_k + (k+1)$ st term Give A1 if one slip Correctly obtained Stated or clearly implied *Dependent on previous 7 marks*

3 (a)(i)	$\lvert\alpha\rvert=\sqrt{2}$, $\arg\alpha=\frac{1}{12}\pi$	B1	
	$\lvert\beta\rvert=4\sqrt{2}$, $\arg\beta=\frac{3}{4}\pi$	B1B1	
	$\left\lvert\dfrac{\beta}{\alpha}\right\rvert=4$	B1 ft	*SR* Just $4(\cos\frac{2}{3}\pi+\mathrm{j}\sin\frac{2}{3}\pi)$: B1 only
	$\arg\dfrac{\beta}{\alpha}=\frac{3}{4}\pi-\frac{1}{12}\pi=\frac{2}{3}\pi$	B1 ft	*For each of the following, withhold the first B1 so earned but award subsequent marks:* *Non-exact values for modulus* *Non-exact values for argument* *Arguments given in degrees*
	 	B1	α in first quadrant
		B1	β in second quadrant
		B1 **8**	β/α in second quadrant with smaller argument than β *(Dependent on β in second quadrant)* Maximum B2 for diagram if points are not labelled
(ii)	$\dfrac{\beta}{\alpha}=4(\cos\frac{2}{3}\pi+\mathrm{j}\sin\frac{2}{3}\pi)$	M1	A complete exact method is required
	$=-2+2\sqrt{3}\mathrm{j}$	A1 **2**	(Just $-2+2\sqrt{3}\mathrm{j}$ with no working scores M0)
(iii)	Line AB	B1 ft	
	$\mathrm{AB}^2=(\sqrt{2})^2+(4\sqrt{2})^2-2(\sqrt{2})(4\sqrt{2})\cos \mathrm{A\hat{O}B}$	M1 A1 ft	Use of cos rule A0 if not exact
	$=2+32-16\cos\frac{2}{3}\pi$	A1	for $\mathrm{A\hat{O}B}=\frac{2}{3}\pi$
	$=42$		
	$\lvert\alpha-\beta\rvert=\sqrt{42}$	A1 **5**	
	OR $\lvert\alpha-\beta\rvert^2=(\sqrt{2}\cos\frac{1}{12}\pi+4)^2+(\sqrt{2}\sin\frac{1}{12}\pi-4)^2$ M1A1		A0 if not exact
	$=34+8\sqrt{2}(\cos\frac{1}{12}\pi-\sin\frac{1}{12}\pi)$ A1		
	$=34+8\sqrt{2}\sqrt{2}\cos(\frac{1}{12}\pi+\frac{1}{4}\pi)$		
	$\lvert\alpha-\beta\rvert=\sqrt{42}$ A1		Correct intermediate step required

(b)	Let $z = a + b\text{j}$, $z^* = a - b\text{j}$ $\text{j}(a + b\text{j}) + (2 + \text{j})(a - b\text{j}) = 10 - 2\text{j}$ Real parts: $-b + 2a + b = 10$ Imaginary parts: $a + a - 2b = -2$ $a = 5$, $b = 6$ $z = 5 + 6\text{j}$	M1 M1 A1 A1 A1 **5**	 Equating real or imaginary parts *Correct answer always scores 5 marks*

4 (a)(i)	$x = 3 + 10\lambda = 1 + 9\mu$ (1) $y = 4 + 5\lambda = 2 + (k-2)\mu$ (2) $z = 7 - 5\lambda = 3 + 3\mu$ (3) Solving (1) and (3), $\lambda = \frac{2}{5}$, $\mu = \frac{2}{3}$ Substitute into (2), $4 + 2 = 2 + \frac{2}{3}(k-2)$ $k = 8$	M1 A1A1 M1 M1 A1 **6**	Equating (at least two) components *using different parameters* Two correct equations Finding λ *or* μ Equation for *k*
(ii)	$x = 7,\ y = 6,\ z = 5$ P is $(7,\ 6,\ 5)$	M1 A1 **2**	Finding *x, y, z*
	Alternative for (i) and (ii) Solving $\dfrac{x-3}{10} = \dfrac{z-7}{-5}$ and $\dfrac{x-1}{9} = \dfrac{z-3}{3}$ M2A1A1 $x = 7,\ z = 5$ $\dfrac{x-3}{10} = \dfrac{y-4}{5} \Rightarrow y = 6$ M1 P is $(7,\ 6,\ 5)$ A1 $\dfrac{x-1}{9} = \dfrac{y-2}{k-2} \Rightarrow \dfrac{7-1}{9} = \dfrac{6-2}{k-2}$ M1 $k = 8$ A1		For M2, must obtain a value for *x* or *z*
(iii)	Normal is $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$ Equation is $3x - 5y + z = 3\times3 - 5\times4 + 7$ $3x - 5y + z + 4 = 0$	M1A1 M1 A1 **4**	or other method for finding normal Using a point to find the constant
	OR $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ Eliminating λ and μ, M2 $3x - 5y + z + 4 = 0$ A2		Give A1 for $3x - 5y + z$
(b)(i)	Rotation Centre O $\cos\theta = 0.8,\ \sin\theta = -0.6$ Through 0.64 rad ($37°$) clockwise	M1 A1 M1 A1 **4**	either one (or $\tan\theta = -0.75$) *Allow* through $-37°$, *etc*
(ii)	Suppose $(x,\ y)$ is on *L* $\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.8x + 0.6y \\ -0.6x + 0.8y \end{pmatrix}$ $-0.6x + 0.8y = 0.8x + 0.6y - 2$ $y = 7x - 10$	M1 A1 M1 A1 **4**	For $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$
	OR $\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{T}^{-1}\begin{pmatrix} t \\ t-2 \end{pmatrix} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}\begin{pmatrix} t \\ t-2 \end{pmatrix}$ M1 $x = 0.2t + 1.2,\ y = 1.4t - 1.6$ A1		Using $\mathbf{T}^{-1}$ to transform general point on $y = x - 2$ or *two* particular points

	Eliminating t, $y = 7x - 10$	M1 A1		*or* images of two points correct Obtaining equation in x, y

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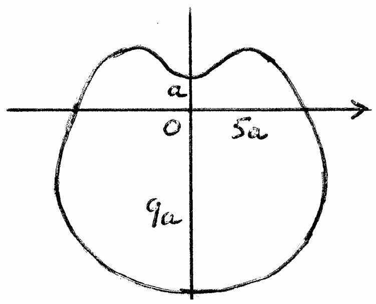
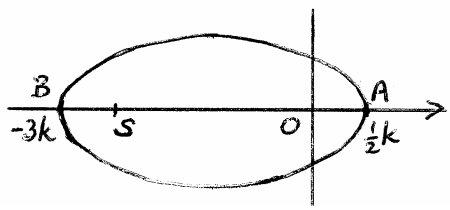
1 (i)	$\sum\alpha=-3,\quad \sum\alpha\beta=8,\quad \alpha\beta\gamma=-10$	B1B1B1 **3**	
(ii)	$\sum\alpha^2=\left(\sum\alpha\right)^2-2\sum\alpha\beta$ $=(-3)^2-2(8)$ $=-7$	M1 A1 **2**	For correct formula
(iii)	Roots cannot all be real One real, two complex (conjugate) roots	M1 A1 **2**	*Or* there are complex roots *Accept 'imaginary' for 'complex'* *When* $\sum\alpha^2>0$*, M1 cannot be awarded, but give B2 for 'one real and two complex'*
(iv)	$\sum\alpha^2\beta=\left(\sum\alpha\right)\left(\sum\alpha\beta\right)-3\alpha\beta\gamma$ $=(-3)(8)-3(-10)$ $=6$	M1A1 M1 A1 (ag) **4**	or $\sum\alpha^2\beta=\left(\sum\alpha\right)\left(\sum\alpha^2\right)-\sum\alpha^3$ *and* $\sum\alpha^3+3\sum\alpha^2+8\sum\alpha+30=0$ *Dependent on previous M1*
(v)	$\beta\gamma(\gamma\alpha+\gamma\beta+\alpha^2+\alpha\beta)=\beta\gamma(8+\alpha^2)$ $=8\beta\gamma+\alpha(\alpha\beta\gamma)$ $=8\beta\gamma-10\alpha$ $\sum\beta\gamma(\gamma+\alpha)(\alpha+\beta)=8\sum\alpha\beta-10\sum\alpha$ $=8(8)-10(-3)$ $=94$	M1 A1 (ag) M1 A1 **4**	
(vi)	Sum $=2\sum\alpha\beta\quad(=16)$ Sum in pairs = 94 Product $=\alpha\beta\gamma(\sum\alpha^2\beta+2\alpha\beta\gamma)\quad(=140)$ Equation is $y^3-16y^2+94y-140=0$	B1 M1A1 M1 A1 **5**	 Forming cubic equation (with numerical coefficients) A0 if =0 omitted
	OR Let $y=8+\dfrac{10}{x},\quad x=\dfrac{10}{y-8}$ M1 $\left(\dfrac{10}{y-8}\right)^3+3\left(\dfrac{10}{y-8}\right)^2+8\left(\dfrac{10}{y-8}\right)+10=0$ A1 $1000+300(y-8)+80(y^2-16y+64)$ $+10(y^3-24y^2+192y-512)=0$ M1 $y^3-16y^2+94y-140=0$ A2		Give A1 if just one slip made

2(a)(i)	$\frac{1}{2}(e^x + e^{-x}) = c$ $e^{2x} - 2ce^x + 1 = 0$ $e^x = \dfrac{2c \pm \sqrt{4c^2 - 4}}{2}$ $= c \pm \sqrt{c^2 - 1}$ $x = \ln(c \pm \sqrt{c^2 - 1})$ $(c + \sqrt{c^2 - 1})(c - \sqrt{c^2 - 1}) = c^2 - (c^2 - 1) = 1$ Hence $\ln(c - \sqrt{c^2 - 1}) = \ln(c + \sqrt{c^2 - 1})^{-1}$ $= -\ln(c + \sqrt{c^2 - 1})$	M1 M1 M1 A1 M1 A1 **6**	or M3 for a complete alternative method
(ii)	$\cosh^2 x - 1 + 3\cosh x = 9$ $\cosh^2 x + 3\cosh x - 10 = 0$ $(\cosh x - 2)(\cosh x + 5) = 0$ $\cosh x = 2$ $x = \pm\ln(2 + \sqrt{3})$	M1 M1 A1 M1A1 **5**	Using $\cosh^2 x - \sinh^2 x = 1$ *(using wrong identity is M0)* Solving quadratic *Or* $\ln(2 \pm \sqrt{3})$ (A0 if any other solutions given)
	OR Writing in exponential form and obtaining quadratic factors M2 $(e^{2x} - 4e^x + 1)(e^{2x} + 10e^x + 1) = 0$ A1 $e^x = 2 \pm \sqrt{3}$ M1 $x = \ln(2 \pm \sqrt{3})$ A1		for $(e^{2x} - 4e^x + 1)$ Obtaining a value for *x* in log form
(b)(i)	$f'(x) = \dfrac{1}{\sqrt{1 - (\frac{3}{5} + x)^2}}$ $f''(x) = \left(\frac{3}{5} + x\right)\left(1 - (\frac{3}{5} + x)^2\right)^{-\frac{3}{2}}$	B1 M1A1 **3**	
(ii)	$f(0) = \arcsin\frac{3}{5}$ $f'(0) = \dfrac{5}{4}, \quad f''(0) = \dfrac{75}{64}$ $f(0) + x f'(0) + \dfrac{x^2}{2!} f''(0) + \ldots$ $= \arcsin\dfrac{3}{5} + \dfrac{5}{4}x + \dfrac{75}{128}x^2 + \ldots$	M1 A1A1 ft **3**	Evaluating $f'(0)$ *or* $f''(0)$ For $p = \dfrac{5}{4}$ and $q = \dfrac{75}{128}$ *(ft requires non-zero values)*
(iii)	$\left[x\arcsin\dfrac{3}{5} + \dfrac{5}{8}x^2 + \dfrac{25}{128}x^3 + \ldots \right]_0^{0.1}$ $= 0.064350 + 0.00625 + 0.000195 + \ldots$ $= 0.0708$	B1 ft M1 A1 **3**	ft requires three non-zero terms Evaluating three non-zero terms

3 (i)	(*A*) $(\cos\theta + \mathrm{j}\sin\theta) + (\cos\theta - \mathrm{j}\sin\theta)$ $= 2\cos\theta$ (*B*) $1 - 3(\mathrm{e}^{\mathrm{j}\theta} + \mathrm{e}^{-\mathrm{j}\theta}) + 9$ $= 10 - 6\cos\theta$	M1 A1 M1A1 A1 **5**	For $\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + \mathrm{j}\sin\theta$ or $(1 - 3\cos\theta)^2 + 9\sin^2\theta$
(ii)	$C + \mathrm{j}S = \mathrm{e}^{\mathrm{j}\theta} + 3\mathrm{e}^{2\mathrm{j}\theta} + 9\mathrm{e}^{3\mathrm{j}\theta} + \ldots$ $= \dfrac{\mathrm{e}^{\mathrm{j}\theta}(1 - [3\mathrm{e}^{\mathrm{j}\theta}]^n)}{1 - 3\mathrm{e}^{\mathrm{j}\theta}}$ $= \dfrac{\mathrm{e}^{\mathrm{j}\theta}(1 - 3^n\mathrm{e}^{\mathrm{j}n\theta})(1 - 3\mathrm{e}^{-\mathrm{j}\theta})}{(1 - 3\mathrm{e}^{\mathrm{j}\theta})(1 - 3\mathrm{e}^{-\mathrm{j}\theta})}$ $= \dfrac{\mathrm{e}^{\mathrm{j}\theta} - 3 + 3^{n+1}\mathrm{e}^{\mathrm{j}n\theta} - 3^n\mathrm{e}^{\mathrm{j}(n+1)\theta}}{10 - 6\cos\theta}$ $C = \dfrac{\cos\theta - 3 + 3^{n+1}\cos n\theta - 3^n\cos(n+1)\theta}{10 - 6\cos\theta}$ $S = \dfrac{\sin\theta + 3^{n+1}\sin n\theta - 3^n\sin(n+1)\theta}{10 - 6\cos\theta}$	M1 M1 A1 M1 A1 M1 A1 (ag) A1 **8**	Obtaining a geometric series Summing a geometric series Using conjugate of denominator Expression with real denominator and numerator multiplied out Equating real or imaginary parts Correctly obtained *Summing to infinity can earn all the M marks but no A marks*
(iii)	$3\mathrm{e}^{\mathrm{j}\frac{\pi}{6}}$, $3\mathrm{e}^{\mathrm{j}\frac{5\pi}{6}}$, $3\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}$	B1B1B1 **3**	If B0, give B2 for 3 arguments correct B1 for 2 arguments correct
(iv)	$w = 3\mathrm{e}^{\mathrm{j}\theta}$ where $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, -\dfrac{\pi}{2}$ $(1 - w)(1 - w^*) = (1 - 3\mathrm{e}^{\mathrm{j}\theta})(1 - 3\mathrm{e}^{-\mathrm{j}\theta})$ $= 10 - 6\cos\theta$ $= 10 - 3\sqrt{3},\ 10 + 3\sqrt{3},\ 10$	M1 B1B1B1 **4**	Implied if next B3 earned Accept 4.8, 15.2

4(a)(i)	$y = r\sin\theta = a(5\sin\theta - 4\sin^2\theta)$ $\frac{dy}{d\theta} = a(5\cos\theta - 8\sin\theta\cos\theta)$ $= 0$ when $\cos\theta = 0$ or $\sin\theta = \frac{5}{8}$ When $\sin\theta = \frac{5}{8}$, maximum $y = a(5\times\frac{5}{8} - 4\times\frac{25}{64})$ $= \frac{25}{16}a$	M1 A1 M1 A1 A1 (ag) **5**	Differentiating $r\sin\theta$ Solving $\frac{dy}{d\theta} = 0$ For $\sin\theta = \frac{5}{8}$
	OR $y = r\sin\theta = a(5\sin\theta - 4\sin^2\theta)$ M1 $= a\left[\frac{25}{16} - \left(2\sin\theta - \frac{5}{4}\right)^2\right]$ A1A1 $\le \frac{25}{16}a$ M1A1		Completing the square
(ii)		B1 B1 B1 **3**	Correct shape in 1st or 2nd quadrant Correct shape in 3rd or 4th quadrant Fully correct, with *a* , 5*a* , 9*a* shown, and zero gradient when crossing the *y*-axis
(iii)	Area $= \int_0^{\frac{1}{2}\pi} \frac{1}{2}a^2(5 - 4\sin\theta)^2\, d\theta$ $= \int_0^{\frac{1}{2}\pi} \frac{1}{2}a^2(25 - 40\sin\theta + 8 - 8\cos 2\theta)\, d\theta$ $= \frac{1}{2}a^2\Big[\, 33\theta + 40\cos\theta - 4\sin 2\theta \,\Big]_0^{\frac{1}{2}\pi}$ $= \frac{1}{4}a^2(33\pi - 80)$	M1 A1 B1 B1B1 ft B1 **6**	Integral of r^2 Correct integral expression For $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ Integrating $a + b\sin\theta$ and $c\cos 2\theta$ Accept $5.92a^2$
(b)(i)		B1 B1 **2**	For any ellipse Ellipse with O as RH focus
(ii)	BS = OA S is $\left(\frac{5}{2}k,\ \pi\right)$	M1 A1 **2**	

(iii)	OP + PS = length of major axis $= \frac{7}{2}k$	M1 A1 **2**	

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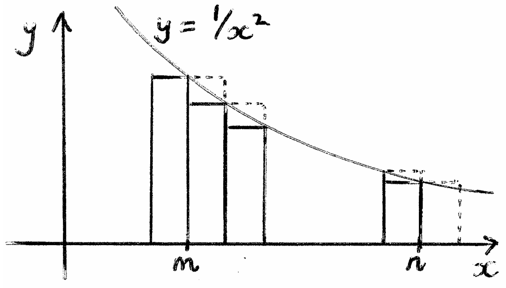
1 (i)	$\begin{pmatrix} 2 & 6 & 8 \\ 2 & 3 & 5 \\ -6 & 8 & 2 \end{pmatrix}\begin{pmatrix} 19 \\ 13 \\ -5 \end{pmatrix} = \begin{pmatrix} 76 \\ 52 \\ -20 \end{pmatrix}$ $= 4\begin{pmatrix} 19 \\ 13 \\ -5 \end{pmatrix}$ hence it is an eigenvector with eigenvalue 4	M1A1 A1 A1 **4**	If done as part of (ii), B2 for eigenvalue 4 correctly obtained B2 for $\begin{pmatrix} 19 \\ 13 \\ -5 \end{pmatrix}$ correctly obtained
(ii)	$\det(\mathbf{M} - \lambda\mathbf{I}) = -\lambda^3 + 7\lambda^2 - 12\lambda$ For eigenvalues, $-\lambda^3 + 7\lambda^2 - 12\lambda = 0$ $-\lambda(\lambda - 3)(\lambda - 4) = 0$ Other eigenvalues are $\lambda = 0,\ 3$ If $\lambda = 0$, $2x + 6y + 8z = 0$ $2x + 3y + 5z = 0$ $x = -z,\quad y = -z$ Eigenvector is $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ If $\lambda = 3$, $2x + 6y + 8z = 3x$ $2x + 3y + 5z = 3y$ $x = -\frac{5}{2}z,\quad y = -\frac{7}{4}z$ Eigenvector is $\begin{pmatrix} -10 \\ -7 \\ 4 \end{pmatrix}$	M1A1 M1 A1 M1 A1 M1 A1 **8**	Solving characteristic equation Or $\begin{pmatrix} 2 \\ 6 \\ 8 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ $\left[= \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} \right]$ Or $\begin{pmatrix} -1 \\ 6 \\ 8 \end{pmatrix} \times \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ $\left[= \begin{pmatrix} 30 \\ 21 \\ -12 \end{pmatrix} \right]$
(iii)	$\mathbf{P} = \begin{pmatrix} 19 & -1 & -10 \\ 13 & -1 & -7 \\ -5 & 1 & 4 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}^4 = \begin{pmatrix} 256 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 81 \end{pmatrix}$	B1 ft M1A1 ft **3**	B0 if **P** is clearly singular
(iv)	Characteristic eqn is $-\lambda^3 + 7\lambda^2 - 12\lambda = 0$ By CHT, $-\mathbf{M}^3 + 7\mathbf{M}^2 - 12\mathbf{M} = \mathbf{0}$ $\mathbf{M}^3 = 7\mathbf{M}^2 - 12\mathbf{M}$	M1 A1 (ag) **2**	
(v)	$\mathbf{M}^4 = 7\mathbf{M}^3 - 12\mathbf{M}^2$ $= 7(7\mathbf{M}^2 - 12\mathbf{M}) - 12\mathbf{M}^2$ $= 37\mathbf{M}^2 - 84\mathbf{M}$	M1 M1 A1 **3**	

2(a)(i)	$\mathrm{G}'(x)=\sqrt{1+x^3}$	B2 **2**	Give B1 for $\sqrt{1+u^3}$
(ii)	$\lim\limits_{x\to 2}\dfrac{\mathrm{G}'(x)}{\frac{2x}{x^2-3}}$ $=\dfrac{3}{4}$	M1 B1 M1 A1 **4**	Differentiating num and demon For $\dfrac{2x}{x^2-3}$ Putting $x=2$ *Dep on previous M1*
(b)(i)	 $\sum\limits_{r=m}^{n}\dfrac{1}{r^2}$ is the total area of the rectangles; under the curve from $x=m-1$ to $x=n$ or above the curve from $x=m$ to $x=n+1$ Integrals give the area under the curve	B1 B1 B1 B1 **4**	Diagram showing curve $y=\dfrac{1}{x^2}$ and rectangles of width 1
(ii)	$\sum\limits_{r=m}^{n}\dfrac{1}{r^2}<\left[-\dfrac{1}{x}\right]_{m-1}^{n}=\dfrac{1}{m-1}-\dfrac{1}{n}$ $<\dfrac{1}{m-1}$ for all n Hence it is convergent and $\sum\limits_{r=m}^{\infty}\dfrac{1}{r^2}<\dfrac{1}{m-1}$ $\sum\limits_{r=m}^{n}\dfrac{1}{r^2}>\dfrac{1}{m}-\dfrac{1}{n+1}$ As $n\to\infty$, $\sum\limits_{r=m}^{\infty}\dfrac{1}{r^2}>\dfrac{1}{m}$	M1 A1 A1 (ag) M1 A1 (ag) **5**	Evaluating integral (may have ∞ as upper limit) Requires conclusion that series is convergent Evaluating integral
(iii)	$\sum\limits_{r=1}^{\infty}\dfrac{1}{r^2}=\sum\limits_{r=1}^{60}\dfrac{1}{r^2}+\sum\limits_{r=61}^{\infty}\dfrac{1}{r^2}$ $\sum\limits_{r=1}^{\infty}\dfrac{1}{r^2}>1.62835+\dfrac{1}{61}$ $=1.64474...>1.6447$ $\sum\limits_{r=1}^{\infty}\dfrac{1}{r^2}<1.62845+\dfrac{1}{60}$ $=1.64511...<1.6452$	M1 M1 A1 (ag) M1 A1 (ag) **5**	Condone 1.6284 Condone 1.6284 If A0, give A1 for 1.64479… and 1.64506…

3 (i)	$\frac{\partial z}{\partial x} = y(2x+y)$ $\frac{\partial z}{\partial y} = x^2 + 2xy + 3y^2 - 9$	B1 B2 **3**	 Give B1 for two terms correct
(ii)	$y(2x+y)=0$ and $x^2+2xy+3y^2-9=0$ $y=0$ and $x^2-9=0$ $x=\pm 3$ $y=-2x$ and $x^2-4x^2+12x^2-9=0$ $x=\pm 1$ Stationary points are $(3,\ 0,\ 0)$ $(-3,\ 0,\ 0)$ $(1,\ -2,\ 12)$ $(-1,\ 2,\ -12)$	M1 M1 M1 A1 A1 A1 A1 **7**	 Condone omission of $z=0$ Condone omission of $z=0$
(iii)	At $(2,\ 1,\ -2)$, $\frac{\partial z}{\partial x}=5$, $\frac{\partial z}{\partial y}=2$ Normal vector is $\begin{pmatrix}5\\2\\-1\end{pmatrix}$ Normal line is $\mathbf{r}=\begin{pmatrix}2\\1\\-2\end{pmatrix}+\lambda\begin{pmatrix}5\\2\\-1\end{pmatrix}$	B1 M1 A1 ft **3**	 Accept any form, but it must be a proper equation
(iv)	$y(2x+y)=0$ and $x^2+2xy+3y^2-9=27$ $y=0$ (and $x^2-9=27$, $x=\pm 6$) $z=0$ $k=27y-z=0$ $y=-2x$ and $x^2-4x^2+12x^2-9=27$ $x=2$ or -2 $y=-4$ or 4 $z=-12$ or 12 $k=27y-z=-96,\ 96$	M1 M1 A1 M1 M1 A1A1 **7**	Condone $=-27$ Finding y and z for at least one point (in the $y=-2x$ case)

4 (a)	Arc length is $\int_0^a \sqrt{1+\left(\frac{2x}{a}\right)^2}\,\mathrm{d}x$	M1 A1	For $1+\left(\frac{2x}{a}\right)^2$ Correct integral expression (limits required)
	Putting $u=\frac{2x}{a},\ \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{2}{a}$ Arc length is $\int_0^2 \sqrt{1+u^2}\left(\frac{a}{2}\right)\mathrm{d}u$ $=\frac{1}{2}a\int_0^2 \sqrt{1+u^2}\,\mathrm{d}u \quad =\frac{1}{2}ak$	M1 A1 A1 (ag) **5**	 Including change of limits
(b)	CSA is $\int_0^{\frac{1}{2}\pi a} 2\pi\left(2a\sin\frac{x}{a}\right)\sqrt{1+\left(2\cos\frac{x}{a}\right)^2}\,\mathrm{d}x$	M1 A1	For $y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$ Correct integral expression (limits required)
	Putting $u=2\cos\frac{x}{a},\ \frac{\mathrm{d}u}{\mathrm{d}x}=-\frac{2}{a}\sin\frac{x}{a}$ CSA is $\int_2^0 2\pi\sqrt{1+u^2}\left(-a^2\right)\mathrm{d}u$ $=2\pi a^2\int_0^2 \sqrt{1+u^2}\,\mathrm{d}u \quad =2\pi a^2 k$	M1 A1 A1 (ag) **5**	 Including change of limits
(c)(i)	At $(\frac{1}{6}\pi a,\ a)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos\frac{x}{a}=\sqrt{3}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{2}{a}\sin\frac{x}{a}=-\frac{1}{a}$ $\rho=\dfrac{\left(1+(\sqrt{3})^2\right)^{\frac{3}{2}}}{\frac{1}{a}}$ $=8a$	B1 M1A1 M1 A1 **5**	*In (i) and (ii) general expressions in terms of x can earn the M marks* Condone omission of $-$ For ρ or κ Condone $-8a$
(ii)	$\hat{\mathbf{n}}=\begin{pmatrix}\frac{1}{2}\sqrt{3}\\ -\frac{1}{2}\end{pmatrix}$ $\mathbf{c}=\begin{pmatrix}\frac{1}{6}\pi a\\ a\end{pmatrix}+8a\begin{pmatrix}\frac{1}{2}\sqrt{3}\\ -\frac{1}{2}\end{pmatrix}$ $=\begin{pmatrix}a(\frac{1}{6}\pi+4\sqrt{3})\\ -3a\end{pmatrix}$	M1 A1 M1 A1 A1 **5**	Normal has gradient $-\frac{1}{\sqrt{3}}$ Condone opposite direction Accept 7.45*a*
	OR $\tan\psi=\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{3}$ M1 $\sin\psi=-\frac{1}{2}\sqrt{3},\ \cos\psi=-\frac{1}{2}$ A1 M1		Finding $\sin\psi$ or $\cos\psi$ Condone both positive For $x\pm\rho\sin\psi$ or $y\pm\rho\cos\psi$

	$x - \rho\sin\psi = \frac{1}{6}\pi a + 4a\sqrt{3}$ $y + \rho\cos\psi = -3a$	A1 A1		
5(a)(i)	Element a b c d e f g h Inverse c b a d e h g f	B2	**2**	Give B1 for five correct
(ii)	Element a b c d e f g h Order 4 2 4 2 1 4 2 4	B3	**3**	Give B2 for six correct B1 for three correct
(iii)	$\{e,\ b\},\ \{e,\ d\},\ \{e,\ g\}$ $\{e,\ a,\ b,\ c\}$ $\{e,\ b,\ f,\ h\}$ $\{e,\ b,\ d,\ g\}$	B2 B1 B1 B1	**5**	Give B1 for two correct If more than 6 subgroups given, deduct B1 from total for each in excess of 6 (but ignore {*e*} and *G*)
(b)(i)	A set of vectors which are linearly independent and which span *V*	B1 B1	**2**	
(ii)	If $\begin{pmatrix} x \\ y \end{pmatrix} = \lambda\mathbf{e}_1 + \mu\mathbf{e}_2$, $\quad 4\lambda + 6\mu = x$, $\ 3\lambda + 5\mu = y$ $\lambda = \frac{1}{2}(5x - 6y),\quad \mu = \frac{1}{2}(4y - 3x)$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(5x - 6y)\mathbf{e}_1 + \frac{1}{2}(4y - 3x)\mathbf{e}_2$	B1 M1 A1	**3**	Both equations correct Obtaining λ or μ
(iii)	$\begin{pmatrix} 4 \\ 1 \end{pmatrix} = 7\mathbf{e}_1 - 4\mathbf{e}_2$ $\begin{pmatrix} 1 & 3 \\ 6 & 10 \end{pmatrix}\begin{pmatrix} 7 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\mathrm{T}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = -5\mathbf{e}_1 + 2\mathbf{e}_2 = \begin{pmatrix} -8 \\ -5 \end{pmatrix}$	M1 M1A1 M1A1	**5**	
	OR $\begin{pmatrix} 4 \\ 1 \end{pmatrix} = 7\mathbf{e}_1 - 4\mathbf{e}_2$ M1 $\mathrm{T}\mathbf{e}_1 = \mathbf{e}_1 + 6\mathbf{e}_2 = \begin{pmatrix} 40 \\ 33 \end{pmatrix}$ M1 $\mathrm{T}\mathbf{e}_2 = 3\mathbf{e}_1 + 10\mathbf{e}_2 = \begin{pmatrix} 72 \\ 59 \end{pmatrix}$ A1 $\mathrm{T}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = 7\begin{pmatrix} 40 \\ 33 \end{pmatrix} - 4\begin{pmatrix} 72 \\ 59 \end{pmatrix} = \begin{pmatrix} -8 \\ -5 \end{pmatrix}$ M1A1			
	OR Matrix of T wrt the standard basis is $\mathbf{Q}\begin{pmatrix} 1 & 3 \\ 6 & 10 \end{pmatrix}\mathbf{Q}^{-1}$ where $\mathbf{Q} = \begin{pmatrix} 4 & 6 \\ 3 & 5 \end{pmatrix}$ M2 $= \begin{pmatrix} -8 & 24 \\ -6 & 19 \end{pmatrix}$ A1 $\mathrm{T}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 & 24 \\ -6 & 19 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -8 \\ -5 \end{pmatrix}$ M1A1			Give M1 if order wrong

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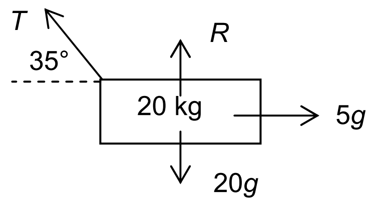
Q 1		mark		sub
(i)	2000 = 1000a so a = 2 so 2 m s^{-2}	B1		1
(ii)	$2000 - R = 1000 \times 1.4$ R = 600 so 600 N (AG)	M1 E1	N2L. Accept $F = mga$. Accept sign errors. Both forces present. Must use a = 1.4	2
(iii)	$2000 - 600 - S = 1800 \times 0.7$ S = 140 so 140 N (AG)	M1 A1 E1	N2L overall or 2 paired equations. $F = ma$ and use 0.7. Mass must be correct. Allow sign errors and 600 omitted. All correct Clearly shown	3
(iv)	$T - 140 = 800 \times 0.7$ T = 700 so 700 N	M1 B1 A1	N2L on trailer (or car). F = 800a (or 1000a). Condone missing resistance otherwise all forces present. Condone sign errors Use of 140 (or 2000 – 600) and 0.7	3
(v)	N2L in direction of motion car and trailer $-600 - 140 - 610 = 1800a$ a = - 0.75 For trailer $T - 140 = -0.75 \times 800$ so T = -460 so 460 thrust	 M1 A1 A1 M1 A1 F1	 Use of F = 1800a to find new accn. Condone 2000 included but not T. Allow missing forces. All forces present; no extra ones. Allow sign errors. Accept $\pm$. cao. N2Lwith their a ($\neq 0.7$) on trailer or car. Must have correct mass and forces. Accept sign errors cao. Accept ± 460 Dep on M1. Take tension as +ve unless clear other convention	6
				15

Q 2		mark		sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$	B1	Accept any accuracy 2 s. f. or better	
	$\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3 s. f.)	M1	Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** 15.62cosθ = 10 or **their** 15.62sinθ = 12)	
		A1	[FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u\cos\theta = 10$ and $u\sin\theta = 12$ seen]	3
(ii)	vert $12t - 0.5 \times 10t^2 + 9$	M1	Use of $s = ut + 0.5at^2$, $a = \pm 9.8$ or ± 10 and u = 12 or 15.62.. Condone $-9 = 12t - 0.5 \times 10t^2$, condone $y = 9 + 12t - 0.5 \times 10t^2$. Condone g.	
		A1	All correct with origin of u = 12 clear; accept 9 omitted	
	= $12t - 5t^2 + 9$ (AG)	E1	Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	
	horiz $10t$	B1		4
(iii)	$0 = 12^2 - 20s$	M1	Use of $v^2 = u^2 + 2as$ or equiv with u = 12, v = 0. Condone $u \leftrightarrow v$	
	s = 7.2 so 7.2 m	A1	From CWO. Accept 16.2.	2
(iv)	Horiz displacement of B: $20\cos 60t$ = 10t	B1	Condone unsimplified expression. Award for 20cos60 = 10	
	Comparison with Horiz displacement of A	E1	Comparison clear, must show 10t for each or explain.	2
(v)	vertical height is $20\sin 60t - 0.5 \times 10t^2$ = $10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and 20sin60 = $10\sqrt{3}$ not explained.	1
(vi)	Need $10\sqrt{3}t - 5t^2$ = $12t - 5t^2 + 9$	M1	Equating the **given** expressions	
	$\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$	A1	Expression for t obtained in any form	
	t = 1.6915… so 1.7 s (2 s. f.) (AG)	E1	Clearly shown. Accept 3 s. f. or better as evidence Award M1 A1 E0 for 1.7 sub in each ht	3
				15

Q 3		mark		sub
(a) (i)	$5g$ (=49) N	B1	[If MR of 5N B0 then FT for remainder of (a)]	1
(ii)		B1 B1	All forces present with labels. No extras. Accept 49 N, *mg*, *T* and *w* without duplication. Angle not required. All forces on diagram with correct arrows	2
(iii)	$T\cos 35 = 49$ *T* = 59.81795… so 59.8 N (3 s. f.)	M1 A1	Resolve horizontally. Condone *T*sin35 used. No extra forces. Any reasonable accuracy	2
(iv)	$R + T\sin 35 = 20g$ *R* = 161.6898… so 162 N (3 s. f.)	M1 B1 A1	Resolve vertically. All forces present. Condone *T*cos35 used and sign errors. No extra forces. *T* sin35 (FT their *T*) in an equation Any reasonable accuracy. FT their *T.*	3
(b) (i)	**R** + $\begin{pmatrix}-3\\4\end{pmatrix}+\begin{pmatrix}21\\-7\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ **R** = $\begin{pmatrix}-18\\3\end{pmatrix}$	M1 A1	Sum to zero Award if seen here or in (ii) or used in (ii). [SC 1 for $\begin{pmatrix}18\\-3\end{pmatrix}$]	2
(ii)	$\lvert$ **R** $\rvert$ = $\sqrt{18^2+3^2}$ = 18.248… so 18.2 N (3 s. f.) angle is $180-\arctan\left(\frac{3}{18}\right)$ = 170.53…° so 171° (3 s. f.)	M1 A1 M1 A1	Use of Pythagoras Any reasonable accuracy. FT **R** (with 2 non-zero cpts). Allow arctan $\left(\frac{\pm 3}{\pm 18}\right)$ or arctan $\left(\frac{\pm 18}{\pm 3}\right)$ Any reasonable accuracy. FT **R** provided their angle is obtuse but not 180°	4
				1 4

Q 4		mark		sub
(i)	Acceleration is 8 m s^{-2} speed is $0+0.5\times4\times8$ = 16 m s^{-1}	B1 B1		2
(ii)	t = 7 a > 0 for t < 7 and a < 0 for t > 7	B1 E1	 Full reason required	2
(iii)	Area under graph $0.5\times2\times8-0.5\times1\times4=6$ so 6 m s^{-1} Increase	M1 B1 E1	Both areas under graph attempted. Accept both positive areas. If 2×3 seen accept ONLY IF reference to average accn has been made. Award for $v=-2t^2+28t+c$ seen or 24 and 30 seen Award if 6 seen. Accept '24 to 30'. This must be clear. Mark dept. on award of M1	3
(iv)	a = 2t $v=\int 2t\,\mathrm{d}t$ $=t^2+C$ v = 0 when t = 0 so C = 0 (AG)	B1 M1 E1	 Integration. No arb const required Must be explicit	3
(v)	1st part $s=\int_1^4 t^2\mathrm{d}t=\left[\frac{t^3}{3}\right]_1^4$ $=\frac{64}{3}-\frac{1}{3}=21$ 2nd part **either** $16\times1+0.5\times8\times1^2$ =20 **or** $s=\int_0^1 8t+16\,\mathrm{d}t$ = 20 $s=\int_1^4 t^2\,\mathrm{d}t$ + distance (t = 4 to t = 5) total 21 + 20 = 41 m	 M1 F1 M1 A1 M1 A1 M1 A1	 Integrate. No arb const or limits required FT limits only if there has been integration Use of constant accn results with u = 16 and a = 8. v = 8t + c (c non-zero) and integrate (ignore limits) Both parts of motion considered and results added cao	6
				16

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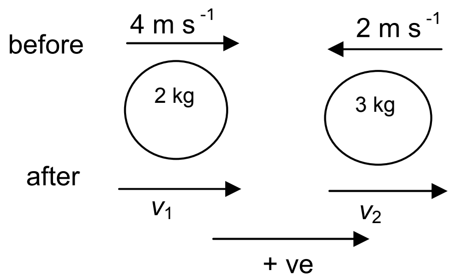
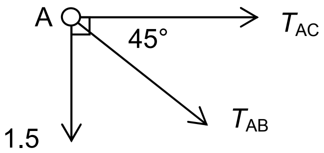
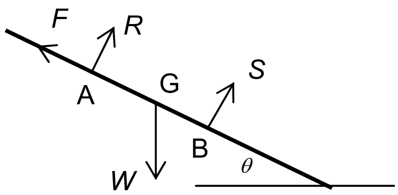
Q 1		mark		Sub
(a) (i)	240 **i** N s $\rightarrow$	B1		1
(ii)		M1	Equating to **their** 240 **i** in this part	
(A)	240 **i** = 70u **i** – 50u **i**	M1	Must have u in both RHS terms and opposite signs	
	u = 12 so **v** = – 12 **i** m s^{-1}	A1	FT 240 **i**	
(B)	240 **i** = 280(**i** + **j**) +50$\mathbf{v}_B$	M1	FT 240 **i** Must have all terms present	
	so $\mathbf{v}_B$ = (-0.8 **i** – 5.6 **j**) m s^{-1}	A1	cao	5
(b) (i)				
	NEL $\dfrac{v_2 - v_1}{-2-4} = -0.5$	M1	NEL	
	so $v_2 - v_1 = 3$	A1	Any form	
	PCLM $8-6 = 2v_1 + 3v_2$	M1	PCLM	
		A1	Any form	
	Solving v_2 = 1.6 so 1.6 m s^{-1} $\rightarrow$	A1	Direction must be clear (accept diagram)	5
(ii)	1.6 m s^{-1}	B1	FT **their** 1.6	
	at 60° to the wall (glancing angles both 60°)	B1		
	No change in the velocity component parallel to the wall as no impulse	E1	Must give reason	
	No change in the velocity component perpendicular to the wall as perfectly elastic	E1	Must give reason	4
	total	15		

Q 2		mark		Sub
(i)	We need $\frac{mgh}{t} = \frac{850\times 9.8\times 60}{20}$ = 24 990	M1	Use of $\frac{mgh}{t}$	
	so approx 25 kW	E1	Shown	2
(ii)	Driving force – resistance = 0	B1	May be implied	
		M1	Use of $P = Fv$	
	$25000 = 800v$ so v = 31.25 and speed is 31.25 m s^{-1}	A1	cao	3
(iii)	$0.5\times 850\times 20^2 = 0.5\times 850\times 15^2$	M1	W-E equation with KE and power term	
		B1	One KE term correct	
	$+25000\times 6.90$	B1	Use of Pt .Accept wrong sign	
	$-800x$	B1	WD against resistance. Accept wrong sign	
		A1	All correct	
	x = 122.6562… so 123 m (3 s. f.)	A1	cao	6
(iv)	**either**			
	$0.5\times 850\times v^2 = 0.5\times 850\times 20^2$	M1	W-E equation inc KE, GPE and WD	
	$-850\times 9.8\times \frac{105}{20}$	M1	GPE term with attempt at resolution	
		A1	Correct. Accept expression. Condone wrong sign.	
	-800×105	B1	WD term. Neglect sign.	
	$v^2 = 99.452...$ so 9.97 m s^{-1}	A1	cao	
	or			
	N2L + ve up plane			
	$-(800 + 850g\times 0.05) = 850a$	M1	N2L. All terms present. Allow sign errors.	
	a = -1.43117…	A1	Accept ±	
	$v^2 = 20^2 + 2\times(-1.43117...)\times 105$	M1	Appropriate *uvast*. Neglect signs.	
		A1	All correct including consistent signs. Need not follow sign of a above.	
	$v^2 = 99.452...$ so 9.97 m s^{-1}	A1	cao	5
	total	16		

Q 3		mark		Sub
(i)	$28\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=16\begin{pmatrix}2\\2\end{pmatrix}+2\begin{pmatrix}5\\0\end{pmatrix}+2\begin{pmatrix}6\\1\end{pmatrix}+2\begin{pmatrix}5\\2\end{pmatrix}$	M1	Complete method	
	$+2\begin{pmatrix}0\\5\end{pmatrix}+2\begin{pmatrix}1\\6\end{pmatrix}+2\begin{pmatrix}2\\5\end{pmatrix}$	B1	Total mass correct	
		B1	3 c. m. correct (or 4 *x*- or *y*-values correct)	
	$\bar{x}=2.5$	A1		
	$\bar{y}=2.5$	A1		
			[Allow A0 A1 if only error is in total mass] [If $\bar{x}=\bar{y}$ claimed by symmetry and only one component worked replace final A1, A1 by B1 explicit claim of symmetry A1 for the 2.5]	5
(ii)	$\bar{x}=\bar{y}$	B1	Or by direct calculation	
		M1	Dealing with 'folded' parts for $\bar{x}$ or for $\bar{z}$	
	$28\bar{x}=16\times2+6\times4+2\times0+2\times1+2\times2$	A1	At least 3 terms correct for $\bar{x}$	
	$\bar{x}=\frac{31}{14}$ (2.21428…)	A1		
	$\bar{z}=\frac{8\times(-1)+4\times(-2)}{28}=-\frac{4}{7}$ (- 0.57142…)	A1	All terms correct allowing sign errors	
		A1		
	Distance is $\sqrt{\left(\frac{31}{14}\right)^2+\left(\frac{31}{14}\right)^2+\left(\frac{4}{7}\right)^2}$	M1	Use of Pythagoras in 3D on **their** c.m.	
	= 3.18318.. so 3.18 m (3 s. f.)	F1		8
	total	13		

Q 4		mark		Sub
(a) (i)	Moments c.w. about A $2R = 5L$ so $R = 2.5L$ Resolve $\rightarrow$ $U = 0$ Resolve $\uparrow$ $V + R = L$ so $V = -1.5L$	 E1 E1 M1 E1	 Resolve vertically or take moments about B (or C)	4
(ii)	 For equilibrium at A $\uparrow$ $T_{AB}\cos 45 + 1.5L = 0$ so $T_{AB} = -\frac{3\sqrt{2}L}{2}$ so $\frac{3\sqrt{2}L}{2}$ N (C) in AB $\rightarrow$ $T_{AC} + T_{AB}\cos 45 = 0$ so $T_{AC} = \frac{3L}{2}$ so $\frac{3L}{2}$ N (T) in AC	M1 M1 A1 F1 F1	Equilibrium at a pin-joint Attempt at equilibrium at A including resolution using 45° (2.12L (3 s. f.)) (1.5L) Award for T/C correct from their internal forces. Do not award without calcs	5
(b) (i)		B1	All forces present with arrows and labels. Angles and distances not required.	1
(ii)	c.w.moments about B $R \times 3 - W \times 1\cos\theta = 0$ so $R = \frac{1}{3}W\cos\theta$	 M1 A1 A1	 If moments about other than B, then need to resolve perp to plank as well Correct	3
(iii)	Resolve parallel to plank $F = W\sin\theta$ $\mu = \frac{F}{R} = \frac{W\sin\theta}{\frac{1}{3}W\cos\theta} = 3\tan\theta$	 B1 M1 A1	 Use of $F = \mu R$ and their F and R Accept any form.	3
	total	16		

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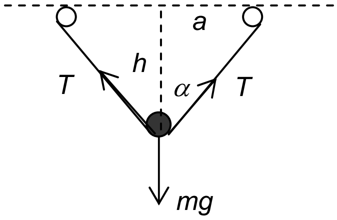
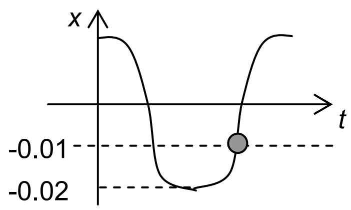
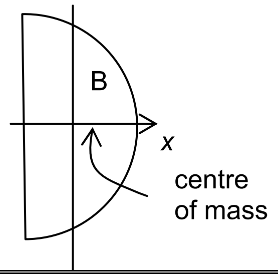
Q 1		mark		sub
(i)	$T = \frac{mg}{a}(8a - 6a) = 2mg$	B1	Use of HL	
	$E = \frac{mg}{2a}(2a)^2$	M1	Use of $0.5k\varepsilon^2$	
	$= 2mga$	F1	FT extension used in calculation of T	3
(ii)				
	Extension is $2\sqrt{a^2+h^2} + 6a - 6a = 2\sqrt{a^2+h^2}$	E1	Need some reference to +6a – 6a (accept words).	
	$\uparrow \quad 2T\cos\alpha = mg$ (1)	B1	Resolve vertically	
	$T = \frac{mg}{a} \times 2\sqrt{a^2+h^2}$	B1	Or equivalent	
	$\cos\alpha = \frac{h}{\sqrt{a^2+h^2}}$	B1		
	Substituting in (1) $2 \times \frac{2mg}{a}\sqrt{a^2+h^2} \times \frac{h}{\sqrt{a^2+h^2}} = mg$	M1	Eliminating **their** T .	
	so $h = \frac{a}{4}$	A1		6
(iii)	**either**			
	Put $h = \frac{a}{4}$ gives extensions		h need not be substituted	
	at equilib pos $2\sqrt{\frac{17a^2}{16}}$, at bottom $2\sqrt{\frac{5a^2}{4}}$	M1	Attempt to find extension and energy in terms of a and/or h	
	EPE at equilib pos $\frac{17mga}{8}$, $\left[\frac{2mg}{a}\left(a^2+h^2\right)\right]$	F1	Equilib EPE. If h substituted FT from (ii)	
	EPE at bottom $\frac{5mga}{2}$, $\left[\frac{2mg}{a}\left(a^2+4h^2\right)\right]$	F1	Bottom EPE. If h substituted FT from (ii)	
	$\frac{5mga}{2} = \frac{mga}{4} + \frac{17mga}{8} + \frac{1}{2}mv^2$	M1	W-E. Accept EPE at equilib and GPE missing. Accept EPE using Δext	
		A1	All present. Accept sign errors and FT **their** values	
		B1	GPE term. Accept sign error. If h subst FT from (ii)	
	so $v^2 = \frac{1}{4}ag$ and $v = \frac{1}{2}\sqrt{ag}$ $\left(\approx 1.57\sqrt{a}\right)$	A1	cao. Accept either form.	

	$\left[\sqrt{\frac{2hg}{a}(6h-a)}\right]$			7

Q 1	continued	mark		sub
	or Consider the SHM ext from equilib *y* ↓ $mg - 2T\cos\alpha = m\ddot{y}$	 M1 F1 A1	 *h* need not be substituted Attempt at an equation of motion in the vertical direction. Must have weight. *T* need not be resolved. *T* correct and resolved All correct	
	giving $\ddot{y} + \frac{4g}{a}y = 0$	A1	cao . Accept any form.	
	$v = a\omega$ giving $\frac{a}{4}\times 2\sqrt{\frac{a}{g}} = \frac{1}{2}\sqrt{ag}$	M1 F1 A1	Attempt to find *v*. Must be from SHM equation. Use of **their** ω cao	
			total	16

Q 2		mark		sub
(a) (i)	[a] = [x] = L [v] = L T^{-1}	B1		
	L^2 T^{-2} = [ω^2] L^2	M1	Equating	
	so [ω] = T^{-1}	E1	[Award max of 2 if any units used instead of dimensions]	3
(ii)	angular speed frequency	B1	cao. Accept ang vel. Accept examples e.g. pulse rate	1
(b) (i)	$F = ky \Rightarrow 0.015 \times 9.8 = 0.098k$	M1	$F = mg$ (0.147) and y = 0.098 in $F = ky$. Give for $k = \dfrac{0.015g}{0.098}$ seen or implied.	
	$\Rightarrow k = 1.5$	E1	Fully explained. Accept $F = ky$ not explained.	2
(ii)	N2L $\downarrow$ $0.015 \times 9.8 - (0.098 + x) \times 1.5 = 0.015\ddot{x}$	M1	N2L applied including attempts at weight and upthrust	
		B1	Upthrust term. Allow $\pm(0.098 + x) \times 1.5$.	
		A1	All correct including signs. Accept $x \uparrow$ and using y	
	$\Rightarrow \ddot{x} + 100x = 0$	E1	Clearly shown with given x.	4
(iii)	$v^2 = 100(0.02^2 - 0.01^2)$	M1	Use of this result or differentiation etc	
	$\Rightarrow v = 0.1732...$ so 0.173 m s^{-1} $(\sqrt{0.03})$	A1	[Using a W-E approach M1 for GPE, KE and WD against F term attempted. A1] [If time found first,: M1 for $\dot{x}$ (**their** time) used. A1]	
	$x = 0.02\cos 10t$	B1	Or equivalent	
	we need $-0.01 = 0.02\cos 10t$ $\Rightarrow \cos 10t = -0.5$	M1	Equating in **their** expression for x. Allow sign error.	
	$\Rightarrow 10t = \dfrac{4\pi}{3}$ so $t = \dfrac{2\pi}{15}$ (= 0.418879… so 0.419 s (3 s. f.)	A1	cao	
			[If graphical method used: B1 shape; B1 x = -0.01 line; B1 cao]	5
			total	15

Q 3		mark		sub
(i)	No tangential acceleration	E1		1
(ii)	$R = mr\omega^2 = 0.3 \times 0.4 \times 100$	M1	N2L radially. (Award for v^2/r with $v = 10$)	
	= 12 so 12 N	A1		2
(iii)	$\uparrow \quad F = 0.3g$ (2.94)	B1	Accept inequality	
	$F \le \mu R$	M1	Accept =. Only vertical F considered. Use **their** R from (ii).	
	so $\mu \ge \frac{g}{40}$ so least value is $\frac{g}{40}$ (0.245)	A1	Accept inequality inc strict. FT value of R from (ii).	3
(iv)	$R = mr\omega^2 = 0.12(10+5t)^2$	M1	N2L radially and substitute for ω. (Not v^2/r with $v = (10 + 5t)$)	
	$= 3(2+t)^2$	E1	Clearly shown	2
(v)	$F = mr\dot{\omega} = 0.3 \times 2 = 0.6$	B1	N2L transverse direction	
	$F \le \mu R \Rightarrow \mu \ge \frac{0.6}{3(2+t)^2} = \frac{1}{5(2+t)^2}$	M1	Condone = and $\le$. R must be correct. F must be attempted from consideration of transverse motion.	
		E1		
	We need the greatest value μ can take which is when $t = 0$.	E1	Clear explanation required	
	This gives $\mu \ge 0.05$.	E1	Accept =. Dependent on correct reason given.	5
			total	13

Q 4		mark		sub
(i)	$V = \int_0^h \pi\left(r^2 - x^2\right)dx$	M1	Accept π omitted.	
	$= \pi\left[r^2 x - \frac{1}{3}x^3\right]_0^h$	A1	At least one term correct. Limits not required. Accept any multiple	
	$= \frac{\pi h}{3}\left(3r^2 - h^2\right)$	A1	Any form	
	$V\bar{x} = \int_0^h \pi x\left(r^2 - x^2\right)dx$	M1	Accept π omitted.	
		B1	RHS correct	
	$= \pi\left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^h$	B1	RHS has at least one term correct. Limits not required. Accept any multiple	
	$= \frac{\pi h^2}{4}\left(2r^2 - h^2\right)$	B1	RHS correct in any form. Accept any multiple.	
	$\bar{x} = \dfrac{\frac{\pi h^2}{4}\left(2r^2 - h^2\right)}{\frac{\pi h}{3}\left(3r^2 - h^2\right)} = \frac{3h}{4}\left(\frac{2r^2 - h^2}{3r^2 - h^2}\right)$	E1	Clearly shown	8
(ii)	Put $h = r$ and $\bar{x} = \frac{3r}{4} \times \frac{1}{2} = \frac{3r}{8}$	E1	Must see $h = r$ implicit or explicit. Accept no comment on result	1
(iii)	$V = \frac{11\pi r^3}{24}$	E1	Must see full substitution and working	
	$\bar{x} = \frac{21r}{88}$	E1	Must see full substitution and working	
			[SC 1: Both substituted, neither worked]	2
(iv)	$V_B = \frac{2\pi r^3}{3} - \frac{11\pi r^3}{24} = \frac{5\pi r^3}{24}$	B1		
	$\frac{3r}{8} \times \frac{2\pi r^3}{3} = \frac{11\pi r^3}{24} \times \frac{21r}{88} + \frac{5\pi r^3}{24}\bar{x}$	M1	Expression involving $\bar{x}$	
		A1	LHS or 1st term RHS correct	
	$\Rightarrow \frac{1}{4}r^4 = \frac{7}{64}r^4 + \frac{5r^3}{24}\bar{x}$ so $\bar{x} = \frac{27}{40}r$	A1	Or equivalent. cao.	
	distance is $\frac{27r}{40} - \frac{r}{2} = \frac{7r}{40}$	F1	FT subtraction of 0.5r or use of $\bar{x} + 0.5r$ above.	
			[If fresh calculation started. B1 obtaining $V_B = 5\pi r^3/24$ M1 A1 obtaining $\int \pi y^2 x\, dx = 9\pi r^4/64$ A1 $\bar{x} = 27r/40$ cao F1 $\bar{x} = 7r/40$ FT subtraction of 0.5r or use of $\bar{x} + 0.5r$ above.]	5
			total	16

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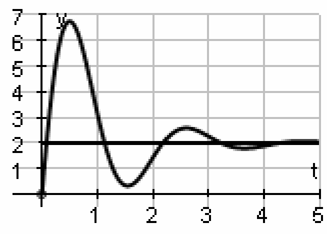
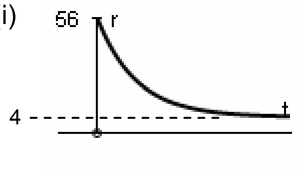
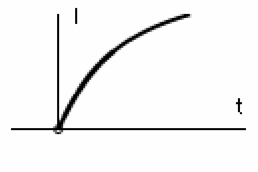
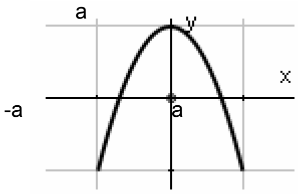
Question	Working	Mark	Comment	Total
1(i)	$\frac{d^2x}{dt^2} = -4\frac{dx}{dt} + 6\frac{dy}{dt}$	M1	differentiate	
	$= -4\frac{dx}{dt} + 6(-3x + 2y + 26)$	M1	substitute for $\frac{dy}{dt}$	
	$= -4\frac{dx}{dt} + -18x + \frac{12}{6}\left(\frac{dx}{dt} + 4x - 28\right) + 156$	M1	substitute for y	
	$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 100$	E1		
				4
(ii)	$\alpha^2 + 2\alpha + 10 = 0$	M1	auxiliary equation	
	$\alpha = -1 \pm 3j$	A1		
	CF $x = e^{-t}(A\cos 3t + B\sin 3t)$	F1	CF for their solutions	
	PI $x = \frac{100}{10} = 10$	B1		
	GS $x = 10 + e^{-t}(A\cos 3t + B\sin 3t)$	F1	their CF + their PI	
	$\frac{dx}{dt} = e^{-t}(-A\cos 3t - B\sin 3t - 3A\sin 3t + 3B\cos 3t)$	M1	differentiate	
	$y = \frac{1}{6}(\dot{x} + 4x - 28)$	M1	rearrange and substitute	
	$y = 2 + \frac{1}{2}e^{-t}\left((A+B)\cos 3t + (B-A)\sin 3t\right)$	A1	cao	
				8
(iii)	$10 + A = 0 \Rightarrow A = -10$	M1	use t = 0 with their x = 0	
	$2 + \frac{1}{2}(A+B) = 0 \Rightarrow B = 6$	M1	use t = 0 with their y = 0 (or their $\dot{x} = 28$)	
	$x = 10 + e^{-t}(6\sin 3t - 10\cos 3t)$ $y = 2 + e^{-t}(8\sin 3t - 2\cos 3t)$	A1	both (cao)	
				3
(iv)		B1	initial condition and asymptote for one graph	
		B1	both generally correct (must start at origin)	
		B1	long-term values	
	long-term values can be found by setting $\dot{x} = \dot{y} = 0$ in DE's	M1		
	and solving the resulting equations	A1		
				5

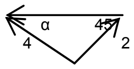
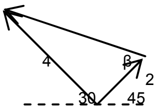
2(i)	 	B1 B1	*r* starts at 56 and decreases, tending to 4 *I* starts at 0, gradient is positive but decreases	
				2
(ii)	$\frac{dr}{dt} = -k(r-4)$ $\int \frac{dr}{r-4} = -k\int dt$ $\ln\|r-4\| = -kt + c_1$ $r = 4 + Ae^{-kt}$ $t = 0, r = 56 \Rightarrow A = 52$ $r = 4\left(1+13e^{-kt}\right)$	M1 A1 M1 A1 M1 M1 E1	 separate and integrate all correct rearranging use initial condition 	
				7
(iii)	$I = \int r\,dt = 4\left(t - \frac{13}{k}e^{-kt}\right) + c_2$ $t = 0, I = 0 \Rightarrow c_2 = \frac{52}{k}$ $I = 4t + \frac{52}{k}\left(1 - e^{-kt}\right)$ $3000 = 4\times 620 + \frac{52}{k}\left(1 - e^{-620k}\right)$ $\Rightarrow 10k = 1 - e^{-620k}$ unless *k* very small, $e^{-620k} \approx 0 \Rightarrow 10k \approx 1 \Rightarrow k \approx 0.1$	M1 M1 A1 M1 E1 E1	integrate *r* use initial condition cao condition on their *I* must follow correct *I* independent of other marks	
				6
(iv)	$\alpha + 0.1 = 0 \Rightarrow \alpha = -0.1$ so CF $r = Be^{-0.1t}$ for given PI $\frac{dr}{dt} = -2b\sin 2t + 2c\cos 2t$ $0.1a + (0.1c - 2b)\sin 2t + (0.1b + 2c)\cos 2t = 0.4 + 0.2\cos 2t$ $0.1a = 0.4$ $0.1c - 2b = 0$ $0.1b + 2c = 0.2$ $a = 4, b = \frac{2}{401}, c = \frac{40}{401}$ conditions $\Rightarrow B = 51.995$, so $r = 51.995e^{-0.1t} + 4 + \frac{2}{401}(\cos 2t + 20\sin 2t)$	B1 M1 M1 A1 A1	 differentiate and substitute compare at least two coefficients and solve 	
				5

3(a)(i)	$\frac{dy}{dx} = \frac{\cos x - 2\sin y}{x+1}$	M1	may be implied	
	0.16 0.625629	M1	use of algorithm	
	0.18 0.143637 0.591150	A1	$y(0.18)$	
	0.20 0.155460	M1	use of algorithm	
		A1	$y(0.20)$	5
(a)(ii)	dy/dx decreases	B1		
		B1	sketch showing curve and step by step solution	
	For each step, gradient used is greater than dy/dx over interval, hence overestimates y.	E1	convincing argument	3
(b)(i)	$\frac{dy}{dx} + \frac{2}{x+1}y = \frac{\cos x}{x+1}$	M1	rearrange	
	$I = \exp\left(\int \frac{2}{x+1}dx\right) = (x+1)^2$	M1 A1	attempt integrating factor	
	$(x+1)^2\frac{dy}{dx} + 2(x+1)y = (x+1)\cos x$	M1	multiply	
	$(x+1)^2 y = \int (x+1)\cos x dx = (x+1)\sin x - \int \sin x dx$	M1	attempt integration by parts	
	$= (x+1)\sin x + \cos x + A$	A1		
	$x = 0, y = 0 \Rightarrow A = -1$	M1		
	$y = \frac{(x+1)\sin x + \cos x - 1}{(x+1)^2}$	E1		
	$x = 0.2 \Rightarrow y = 0.1517$	B1		9
(b)(ii)	Since sin y < y, replacing sin y by y in $\frac{dy}{dx} = \frac{\cos x - 2\sin y}{x+1}$	M1	consider effect on dy/dx	
	gives an underestimate for dy/dx.	A1		
	Hence (while y > 0), the approx. DE will underestimate y.	E1		3

4(i)	$\alpha^3 + 2\alpha^2 - \alpha - 2 = 0$	M1	auxiliary equation	
	$(\alpha - 1)(\alpha^2 + 3\alpha + 2) = 0$	M1	factorise or demonstrate 1 is a root	
	$\alpha = 1, -1, -2$	A1		
	CF $y = A\,e^{-t} + B\,e^{-2t} + C\,e^{t}$	F1	CF for their roots (must have 3 constants)	
	PI $y = a\,e^{-3t}$	B1	correct form	
	$\dot{y} = -3a\,e^{-3t}, \ddot{y} = 9a\,e^{-3t}, \dddot{y} = -27a\,e^{-3t}$	M1	differentiate and substitute	
	$-27a + 18a + 3a - 2a = 4$	M1	compare coefficients	
	$a = -\frac{1}{2}$	A1		
	$y = A\,e^{-t} + B\,e^{-2t} + C\,e^{t} - \frac{1}{2}e^{-3t}$	F1	their CF + their PI	9
(ii)	decays $\Rightarrow C = 0$	B1		
	$t = 0, y = -\frac{3}{2} \Rightarrow -\frac{3}{2} = A + B - \frac{1}{2}$	M1	condition on *y*	
	$\dot{y} = -A\,e^{-t} - 2B\,e^{-2t} + \frac{3}{2}e^{-3t}$	M1	differentiate	
	$t = 0, \dot{y} = \frac{3}{2} \Rightarrow \frac{3}{2} = -A - 2B + \frac{3}{2}$	M1	condition	
	$A = -2, B = 1$ so $y = -2\,e^{-t} + e^{-2t} - \frac{1}{2}e^{-3t}$	E1		5
(iii)	let $u = e^{-t}$ so $y = -2u + u^2 - \frac{1}{2}u^3 = -\frac{1}{2}u(u^2 - 2u + 4)$			
	$u = e^{-t} \neq 0$ and $u^2 - 2u + 4 = (u-1)^2 + 3 > 0$	M1	consider quadratic (any valid method)	
	hence $y \neq 0$ for all *t*	E1	if discriminant used, value or working must be shown must indicate *u* non-zero	
	$\dot{y} = 2\,e^{-t} - 2\,e^{-2t} + \frac{3}{2}e^{-3t} = \frac{3}{2}u\left(u^2 - \frac{4}{3}u + \frac{4}{3}\right)$			
	$= \frac{3}{2}u\left(\left(u - \frac{2}{3}\right)^2 + \frac{8}{9}\right) > 0$	M1	consider quadratic (any valid method)	
	hence no turning points	E1		
		B1 B1	starts at $-\frac{3}{2}$ and asymptote $y = 0$ Shape (increasing)	6

Mark Scheme 2611
June 2005

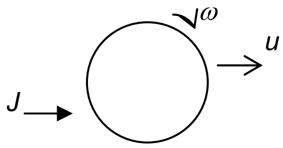
Question	Working	Mark	Guidance	Total
1(i)	$m\ddot{\mathbf{r}} = -ma\omega^2\left(\sin\omega t\mathbf{i} + 4\cos 2\omega t\mathbf{j}\right)$	M1	use of N2L	
	$\dot{\mathbf{r}} = -a\omega\left(-\cos\omega t\mathbf{i} + 2\sin 2\omega t\mathbf{j}\right) + \mathbf{c}_1$	M1 A1	integrating	
	$t = 0, \dot{\mathbf{r}} = a\omega\mathbf{i} \Rightarrow \mathbf{c}_1 = \mathbf{0}$	M1	condition	
	$\dot{\mathbf{r}} = a\omega\left(\cos\omega t\mathbf{i} - 2\sin 2\omega t\mathbf{j}\right)$	A1		
	$\mathbf{r} = a\left(\sin\omega t\mathbf{i} + \cos 2\omega t\mathbf{j}\right) + \mathbf{c}_2$	M1	integrating	
	$t = 0, \mathbf{r} = a\mathbf{j} \Rightarrow \mathbf{c}_2 = \mathbf{0} \Rightarrow \mathbf{r} = a\left(\sin\omega t\mathbf{i} + \cos 2\omega t\mathbf{j}\right)$	E1		
				7
(ii)	$x = a\sin\omega t, y = a\cos 2\omega t$	M1	components and eliminate t	
	$y = a(1 - 2\sin^2\omega t)$	B1	trig identity	
	$y = a - \dfrac{2x^2}{a}$	A1		
		B1 B1	parabola $(\cap)$ through (0,a) endpoints indicated	
				5
(iii)	$P = \mathbf{F}\cdot\mathbf{v} = (-ma\omega^2)(-a\omega)\begin{pmatrix}\sin\omega t\\ 4\cos 2\omega t\end{pmatrix}\cdot\begin{pmatrix}-\cos\omega t\\ 2\sin 2\omega t\end{pmatrix}$	M1		
	$= ma^2\omega^3(8\sin 2\omega t\cos 2\omega t - \sin\omega t\cos\omega t)$	A1		
				2
(iv)	$\text{WD} = \int_0^{\pi/2\omega} P\,\mathrm{d}t = ma^2\omega^3\int_0^{\pi/2\omega}(4\sin 4\omega t - \tfrac{1}{2}\sin 2\omega t)\mathrm{d}t$	M1	attempt integral of P	
	$= ma^2\omega^3\left[-\tfrac{1}{\omega}\cos 4\omega t + \tfrac{1}{4\omega}\cos 2\omega t\right]_0^{\pi/2\omega}$	M1 A1	trig identity and integrate [ ] or constant multiple	
	$= ma^2\omega^2(-\tfrac{1}{4} - \tfrac{1}{4}) = -\tfrac{1}{2}ma^2\omega^2$	A1		
	$\Delta\text{KE} = \tfrac{1}{2}m\left(\lvert\mathbf{0}\rvert^2 - \lvert a\omega\mathbf{i}\rvert^2\right)$	M1		
	$= -\tfrac{1}{2}ma^2\omega^2$	E1	both results	
				6

2(a)(i)	${}_B\mathbf{v}_A$ must be due West		B1	may be implied
		or ${}_B\mathbf{v}_A = \begin{pmatrix} -4\cos\alpha - \sqrt{2} \\ 4\sin\alpha - \sqrt{2} \end{pmatrix}$	M1	diagram or relative velocity vector
	$\dfrac{2}{\sin\alpha} = \dfrac{4}{\sin 45°}$	or $4\sin\alpha - \sqrt{2} = 0$	M1	
	$\alpha = 20.7°$ hence bearing $290.7°$		A1	
				4
(ii)		or ${}_B\mathbf{v}_A = \begin{pmatrix} -2\sqrt{3} - \sqrt{2} \\ 2 - \sqrt{2} \end{pmatrix}$	M1	diagram or relative velocity vector
	$\lvert {}_B\mathbf{v}_A \rvert^2 = 4^2 + 2^2 - 2\cdot 4\cdot 2\cos 105°$	or $\lvert {}_B\mathbf{v}_A \rvert^2 = \left(-2\sqrt{3} - \sqrt{2}\right)^2 + \left(2 - \sqrt{2}\right)^2$	M1	calculate magnitude
	$\lvert {}_B\mathbf{v}_A \rvert = 4.913 \text{ km h}^{-1}$		A1	
	$\dfrac{4}{\sin\beta} = \dfrac{4.913\ldots}{\sin 105°}$	or $\tan^{-1}\left(\dfrac{-2\sqrt{3} - \sqrt{2}}{2 - \sqrt{2}}\right)$	M1	calculate angle
	$\beta = 51.8\ldots°$ hence bearing $276.8°$		A1	
	$d = 10\sin(\beta - 45°)$		M1	complete method
	= 1.19 km		A1	
				7
(b)	${}_Y\mathbf{v}_X = \begin{pmatrix} 20 \\ -100 \\ -10 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 0 \end{pmatrix} = \begin{pmatrix} -50 \\ -300 \\ -10 \end{pmatrix}$		M1 A1	
	$\overrightarrow{XY} = \begin{pmatrix} 100 \\ 600 \\ 100 \end{pmatrix} - \begin{pmatrix} 50 \\ 300 \\ 10 \end{pmatrix} t$		A1	
	$\lvert\overrightarrow{XY}\rvert^2 = (100 - 50t)^2 + (600 - 300t)^2 + (100 - 10t)^2$ $= 200(463t^2 - 1860t + 1900)$		A1	
	min. when $t = \dfrac{930}{463} = 2.0086\ldots$		M1 A1	any valid method for minimum
	vertically above $\Rightarrow \overrightarrow{XY}$ parallel to $\mathbf{k} \Rightarrow t = 2$		M1	
	hence closest at t = 2.0086… i.e. just after vertical		E1	
	least distance = 79.96		A1	
				9

3(i)	$m\ddot{\mathbf{r}} = -\dfrac{km\mathbf{r}}{r^3}$			
	$m \cdot \dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\theta}\right) = 0$	M1		
	$\Rightarrow r^2\dot{\theta} =$ constant	E1		
	$m(\ddot{r} - r\dot{\theta}^2) = -\dfrac{kmr}{r^3}$	M1 A1		
	$\dot{\theta} = \dfrac{h}{r^2} \Rightarrow \ddot{r} - r\left(\dfrac{h}{r^2}\right)^2 = -\dfrac{k}{r^2}$	M1		
	$\Rightarrow \ddot{r} - \dfrac{h}{r^3}^2 = -\dfrac{k}{r^2}$	E1		
				6
(ii)	$\dot{r} = -\dfrac{1}{u^2}\dot{u} = -\dfrac{1}{u^2}\dfrac{\mathrm{d}u}{\mathrm{d}\theta}\dot{\theta}$	M1		
	$= -\dfrac{1}{u^2}\dfrac{\mathrm{d}u}{\mathrm{d}\theta}hu^2$	M1		
	$= -h\dfrac{\mathrm{d}u}{\mathrm{d}\theta}$	A1		
	$\ddot{r} = -h\dfrac{\mathrm{d}^2u}{\mathrm{d}\theta^2}\dot{\theta} = -h\dfrac{\mathrm{d}^2u}{\mathrm{d}\theta^2}hu^2$	M1		
	$= -h^2u^2\dfrac{\mathrm{d}^2u}{\mathrm{d}\theta^2}$	A1		
	$-h^2u^2\dfrac{\mathrm{d}^2u}{\mathrm{d}\theta^2} - h^2u^3 = -ku^2$	M1		
	$\dfrac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = \dfrac{k}{h^2}$	A1		
				7
(iii)	CF $u = A\sin\theta + B\cos\theta$	M1		
	PI $u = \dfrac{k}{h^2}$	B1		
	$u = \dfrac{k}{h^2} + A\sin\theta + B\cos\theta$	A1		
	r min. $\Rightarrow u$ max. at $\theta = 0 \Rightarrow A = 0$	B1		
	$\theta = 0, r = r_0 \Rightarrow \dfrac{1}{r_0} = \dfrac{k}{h^2} + B \Rightarrow B = \dfrac{1}{r_0} - \dfrac{k}{h^2}$	M1		
	at $\theta = 0,$ rad. vel. $= 0 \Rightarrow$ trans. vel., $r\dot{\theta} = v_0 \Rightarrow v_0 = \dfrac{h}{r_0}$	M1	relating *h* to r_0, v_0	
	$r = \dfrac{{v_0}^2{r_0}^2}{k + ({v_0}^2 r_0 - k)\cos\theta}$	A1		
				7

4(i)	Taking a semicircular 'strip' of radius r and width δr			
	area $\approx \pi r \delta r$ so mass $\approx \dfrac{m\pi r}{\frac{1}{2}\pi a^2}\delta r$	M1		
	mom. of inertia $\approx \dfrac{2mr}{a^2}\delta r \times r^2$	M1		
	$I = \dfrac{2m}{a^2}\int_0^a r^3 \mathrm{d}r = \dfrac{2m}{a^2}\left[\frac{1}{4}r^4\right]_0^a$	M1		
	$= \frac{1}{2}ma^2$	E1		4
(ii)	$I_{\text{square}} = \frac{1}{3}\lambda m(a^2 + a^2) + \lambda ma^2$	M1		
	$I_{\text{O}} = \frac{1}{2}ma^2 + I_{\text{square}}$	B1		
	$I_{\text{O}} = \frac{1}{2}ma^2 + \frac{5}{3}\lambda ma^2 = \frac{1}{6}ma^2(3+10\lambda)$	E1		3
(iii)	$\lambda m \cdot a = m \cdot \dfrac{4a}{3\pi} \Rightarrow \lambda = \dfrac{4}{3\pi}$	B1		
	$I_{\text{O}} = \frac{1}{6}\left(\dfrac{M}{1+\lambda}\right)a^2(3+10\lambda)$	M1		
	$= \dfrac{3+10\lambda}{6(1+\lambda)}Ma^2$	A1		
	$k = \dfrac{3+\frac{40}{3\pi}}{6(1+\frac{4}{3\pi})} = \dfrac{9\pi+40}{18\pi+24}$	E1		4
(iv)	$I_{\text{X}} = kMa^2 + M(2a)^2$	M1	parallel axis theorem	
	$= (k+4)Ma^2$	A1		2
(v)	$\frac{1}{2}I_{\text{X}}\dot{\theta}^2 - Mg\cdot 2a\cos\theta = -Mg\cdot 2a\cos\alpha$	M1 A1	energy	
	$\Rightarrow \dot{\theta}^2 = \dfrac{4g}{a(k+4)}(\cos\theta - \cos\alpha)$	A1	aef	3
(vi)	$2\dot{\theta}\ddot{\theta} = \dfrac{4g}{a(k+4)}(-\sin\theta\,\dot{\theta})$	M1 A1	differentiate or use equation of rotation	
	θ small $\Rightarrow \sin\theta \approx \theta$	M1	approximation	
	$\ddot{\theta} \approx -\dfrac{2g}{a(k+4)}\theta$ i.e. SHM	A1		4

Mark Scheme 2612
June 2005

1(i)		$J = mu$	B1		
		$-J \cdot \frac{1}{2}a = \frac{2}{5}ma^2\omega$	M1		
			A1		
		$-\frac{1}{2}mau = \frac{2}{5}ma^2\omega$	M1	eliminate *J*, *m*	
		$\omega = -\dfrac{5u}{4a}$ so ang.speed $= \dfrac{5u}{4a}$	E1		
	$u \neq a\omega \Rightarrow$ slipping occurs		E1		
					6
(ii)	$m\ddot{x} = -F$		M1	N2L	
	$= -\mu mg$		B1	$F = \mu mg$ stated or used	
	$\dot{x} = u - \mu gt$		A1		
	$\frac{2}{5}ma^2\ddot{\theta} = aF = \mu mga$		M1 A1	equation of rotation (ignore sign)	
	$\Rightarrow \ddot{\theta} = \dfrac{5\mu g}{2a}$		M1		
	$\dot{\theta} = -\dfrac{5u}{4a} + \dfrac{5\mu g}{2a}t$		A1	cao	
					7
(iii)	stops slipping when $\dot{x} = a\dot{\theta}$		B1		
	$u - \mu gt = -\dfrac{5u}{4} + \dfrac{5\mu g}{2}t$		M1	use condition	
	$t = \dfrac{9u}{14\mu g}$		A1	cao	
					3
(iv)	$\dot{x} = a\dot{\theta} \Rightarrow \ddot{x} = a\ddot{\theta}$		M1		
	$F \neq 0 \Rightarrow \ddot{x}$ and $\ddot{\theta}$ have opposite signs so $\ddot{x} = \ddot{\theta} = 0$		E1		
	therefore constant velocity of $u - \mu g\left(\frac{9u}{14\mu g}\right) = \frac{5}{14}u$		F1	follow their *t* in either $\dot{x}$ or $a\dot{\theta}$	
	$F = -m\ddot{x} = 0$		B1	must be justified	
					4

2(a)	$\Sigma\mathbf{F} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}+\begin{pmatrix}2\\-1\\0\end{pmatrix}+\begin{pmatrix}-1\\-1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	M1 A1	evidence of working needed
	$\mathbf{C} = \Sigma\mathbf{r}\times\mathbf{F}$	M1	
	$= \begin{vmatrix}\mathbf{i} & 1 & -1\\ \mathbf{j} & 1 & 2\\ \mathbf{k} & 0 & -1\end{vmatrix} + \begin{vmatrix}\mathbf{i} & 0 & 2\\ \mathbf{j} & 2 & -1\\ \mathbf{k} & 0 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & 0 & -1\\ \mathbf{j} & 1 & -1\\ \mathbf{k} & \lambda & 1\end{vmatrix}$	M1	attempt vector product
	$= \begin{pmatrix}-1\\1\\3\end{pmatrix}+\begin{pmatrix}0\\0\\-4\end{pmatrix}+\begin{pmatrix}\lambda+1\\-\lambda\\1\end{pmatrix}=\begin{pmatrix}\lambda\\1-\lambda\\0\end{pmatrix}$	A1	
	which cannot be zero hence a couple	E1	must observe that **C** is non-zero
	$\lvert\mathbf{C}\rvert^2 = \lambda^2 + (1-\lambda)^2 = 2\left(\lambda-\tfrac{1}{2}\right)^2 + \tfrac{1}{2}$	M1	or alternative method to find minimum
	hence minimum magnitude of couple = $\tfrac{1}{\sqrt{2}}$	A1	must be magnitude
	when $\lambda = \tfrac{1}{2}$	A1	
			9
(b)(i)	$\mathbf{v} = \begin{pmatrix}-2\omega\sin\omega t\\ -\omega\cos\omega t\\ 2\end{pmatrix}$	M1 A1	differentiate **r**
	$\mathbf{L} = \mathbf{r}\times m\mathbf{v}$	M1	
	$= m\begin{pmatrix}-2\sin\omega t + 2\omega t\cos\omega t\\ -4\omega t\sin\omega t - 4\cos\omega t\\ -2\omega\end{pmatrix}$	M1 A1 A1	attempt vector product one correct component all correct (aef)
			6
(ii)	torque $= \dfrac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$	M1	
	$= m\begin{pmatrix}-2\omega\cos\omega t + 2\omega\cos\omega t - 2\omega^2 t\sin\omega t\\ -4\omega^2 t\cos\omega t - 4\omega\sin\omega t + 4\omega\sin\omega t\\ 0\end{pmatrix}$	M1	differentiate
	$= -2m\omega^2 t\begin{pmatrix}\sin\omega t\\ 2\cos\omega t\\ 0\end{pmatrix}$	A1	aef
	if $t > 0$, torque = 0 $\Rightarrow \sin\omega t = \cos\omega t = 0$ but $\sin\omega t = 0 \Rightarrow \cos\omega t = \pm 1$ so torque not zero	M1 E1	attempt to show vector non-zero convincing argument
			5

3(i)	$V = mga\cos\theta + \frac{\lambda}{2a}\left(2a\sin\frac{1}{2}\theta - a\right)^2$	M1 A1 A1	attempt V in terms of θ GPE ($\pm$ constant) EPE
	$V'(\theta) = -mga\sin\theta + \frac{\lambda}{2a}\cdot 2\left(2a\sin\frac{1}{2}\theta - a\right)\left(a\cos\frac{1}{2}\theta\right)$	M1 M1	differentiate good attempt at both terms
	$= -2mga\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta + \lambda a\cos\frac{1}{2}\theta\left(2\sin\frac{1}{2}\theta - 1\right)$		
	$= \lambda a\cos\frac{1}{2}\theta\left(-1 + 2\left(1 - \frac{mg}{\lambda}\right)\sin\frac{1}{2}\theta\right)$	E1	
			6
(ii)	$\theta = \pi \Rightarrow \cos\frac{1}{2}\theta = 0 \Rightarrow V'(\theta) = 0$	M1	
	$\Rightarrow$ equilibrium	E1	
	$V''(\theta) = -\frac{1}{2}\lambda a\sin\frac{1}{2}\theta\left(-1 + 2\left(1 - \frac{mg}{\lambda}\right)\sin\frac{1}{2}\theta\right)$ $+\lambda a\cos\frac{1}{2}\theta\cdot\left(1 - \frac{mg}{\lambda}\right)\cos\frac{1}{2}\theta$	M1	differentiate again
	$V''(\pi) = -\frac{1}{2}\lambda a\left(1 - \frac{2mg}{\lambda}\right)$		
	$\lambda < 2mg \Rightarrow V''(\pi) > 0 \Rightarrow$ stable	M1 E1	consider sign of $V''(\pi)$
	$\lambda > 2mg \Rightarrow V''(\pi) < 0 \Rightarrow$ unstable	B1	
	$\lambda = 2mg \Rightarrow V'(\theta) = 2mga\cos\frac{1}{2}\theta(-1 + \sin\frac{1}{2}\theta)$		
	$V'(\pi - \varepsilon) = (+)(-) < 0, \quad V'(\pi + \varepsilon) = (-)(-) > 0$	M1	any valid method
	$\Rightarrow$ minimum, hence stable	A1	
			8
(iii)	$\cos\frac{1}{2}\theta \neq 0, V'(\theta) = 0 \Rightarrow -1 + 2\left(1 - \frac{mg}{\lambda}\right)\sin\frac{1}{2}\theta = 0$	M1	
	$\Rightarrow \sin\frac{1}{2}\theta = \frac{1}{2\left(1 - \frac{mg}{\lambda}\right)}$	A1	or equivalent
	$\lambda > 2mg \Rightarrow 0 < \frac{mg}{\lambda} < \frac{1}{2} \Rightarrow \left(\frac{1}{2} <\right)\sin\frac{1}{2}\theta < 1$	M1	only required to establish $\sin\frac{1}{2}\theta < 1$
	$\Rightarrow \theta < \pi$	E1	
	$\lambda = 2mg \Rightarrow \sin\frac{1}{2}\theta = 1 \Rightarrow \theta = \pi$ as before	E1	
	$\lambda < 2mg \Rightarrow \sin\frac{1}{2}\theta > 1 \Rightarrow$ no solutions, so only $\theta = \pi$ as before	E1	
			6

4(i)	If δm is mass lost in time δt PCLM $mv = (m-\delta m)(v+\delta v) - \delta m(u-v)$	M1	change in momentum over time δt	
	$m\frac{\delta v}{\delta t} = u\frac{\delta m}{\delta t} + \delta v\frac{\delta m}{\delta t} \Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t} = -u\frac{\mathrm{d}m}{\mathrm{d}t}$ $\left(\text{NB using } \delta m > 0 \text{ but } \frac{\mathrm{d}m}{\mathrm{d}t} < 0\right)$	A1	accept sign error	
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -k \Rightarrow m = m_0 - kt$	M1	get m in terms of t	
	$\Rightarrow (m_0 - kt)\frac{\mathrm{d}v}{\mathrm{d}t} = uk$	E1		
	$v = \int \frac{uk}{m_0 - kt}\mathrm{d}t$	M1	separate and integrate	
	$= -u\ln(m_0 - kt) + c$	A1	multiple of $u\ln(m_0 - kt)$	
	$t = 0, v = 0 \Rightarrow c = u\ln m_0$	M1	use initial condition	
	$v = u\ln\left(\frac{m_0}{m_0 - kt}\right)$	A1	aef	
				8
(ii)	matter all ejected when $kt = \frac{1}{2}m_0$	M1		
	$t = \frac{m_0}{2k}$	A1	cao	
	distance = $\int_0^{\frac{m_0}{2k}} u\ln\left(\frac{m_0}{m_0 - kt}\right)\mathrm{d}t$	M1 M1	integral limits (0 to their t)	
	$= \int_0^{\frac{m_0}{2k}} -u\ln\left(1 - \frac{k}{m_0}t\right)\mathrm{d}t$	M1	rearrange into any suitable form for integrating	
	$= \frac{um_0}{k}\left[\left(1-\frac{k}{m_0}t\right)\ln\left(1-\frac{k}{m_0}t\right) - \left(1-\frac{k}{m_0}t\right)\right]_0^{\frac{m_0}{2k}}$	M1 A1	reasonable attempt at integral	
	$= \frac{um_0}{k}\left(\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2} - -1\right) = \frac{um_0}{2k}(1 - \ln 2)$	E1		
				8
(iii)	speed when fuel runs out $= u\ln\left(\frac{m_0}{\frac{1}{2}m_0}\right) = u\ln 2$	M1	use $kt = \frac{1}{2}m_0$ or their t from (ii) in their v	
	distance remaining $= a - \frac{um_0}{2k}(1 - \ln 2) = \frac{um_0}{2k}\ln 2$	B1		
	time after fuel runs out $\frac{\frac{um_0}{2k}\ln 2}{u\ln 2} = \frac{m_0}{2k}$	M1	their distance/their speed	
	$\text{total time} = \frac{m_0}{2k} + \frac{m_0}{2k} = \frac{m_0}{k}$	E1	must be all correct	
				4

Mark Scheme 2613
June 2005

Question 1

(i)	Mid f mf m^2f 4 4 16 64 7 6 42 294 10 12 120 1200 14 14 196 2744 19 14 266 5054 26 10 260 6760 60 900 16116 mean = 900/60 = 15 chapters variance = $\frac{1}{60}\left(16116-\frac{900^2}{60}\right)$ = 43.6 $\rightarrow$ standard deviation = 6.60(3) with n-1 divisor = 6.65(9)	B1 mid points B1 correct m^2f A1 mean M1 sd method A1 correct sd	5
(ii)	Exact values not given	E1	1
(iii)	1: Definitely false 14.7 – 2(6.1) = 2.5 There are no values below 2.5 2: Possibly true 14.7 + 2(6.1) = 26.9 There may be values above 26.9	A1 answer M1 for 2.5 E1 for comment A1 answer M1 for 26.9 E1 for comment	6
(iv)	Mean = 20(14.7) + 15 = 309 pages Sd = 20(6.1) = 122 pages	B1 M1 A1	3
			15

Question 2

(i)	P(M win 1 – 0) = (0.3)(0.4) = 0.12	B1	1
(ii)	P(Game ends 2 – 0) = (0.35)(0.4) + (0.2)(0.1) = 0.16	M1 sum of 2 pairs A1	2
(iii)	P(neither team wins) = (0.2)(0.4)+(0.3)(0.5)+(0.35)(0.1) or (0.2)(0.6) + (0.3)(0.1) = 0.265	M1 for 1 pair M1 for all pairs A1	3
(iv)	P(S scores more goals) = (0.1)(0.2)+(0.1)(0.3)+(0.5)(0.2) = 0.15	M1 for 1 pair M1 for 3 pairs A1	3
(v)	P(M scores 0 given S wins) $= \frac{(0.1)(0.2)+(0.5)(0.2)}{0.15}$ = 0.8	M1 numerator M1 denominator A1	3
(vi)	$0.4^k > 0.01$ $0.4^5 = 0.01024$ and $0.4^6 = 0.004096$ $\rightarrow$ maximum value of k is 5	M1 for inequality M1 for 0.01024 and 0.004096 A1	3
			15

Question 3

(i)	Randomly select start value 1 - 10 Then select every 10^{th} component until 20 have been selected.	E1 E1	2
(ii)	Advantage – cheaper/simpler to sample on just one day. Disadvantage – any problem could be missed for several days. Or any other sensible suggestion.	E1 E1	2
(iii)	If number generated is 001-200, select that component. If number generated is 201-000, subtract any whole 200's, or any correct strategy for numbers outside 001 – 200. Discard any repeated numbers.	E1 E1 E1	3
(iv)	$\binom{15}{5} = 3003$	M1 A1	2
(v)	(*A*) Prob = $\dfrac{\binom{13}{5}}{\binom{15}{5}}$ or $\dfrac{13.12.11.10.9}{15.14.13.12.11}$ = 1287/3003 = 3/7 (0.429) OR (A) $\binom{5}{0}\times\binom{10}{2}/\binom{15}{2} = \dfrac{3}{7}$ OR (A) $\dfrac{10}{15}\times\dfrac{9}{14}=\dfrac{3}{7}$	M1 numerator M1 denominator A1 M1 numerator M1 denominator A1 cao M1 for 1^{st} fraction M1 for 2^{nd} fraction A1 cao	

	(B) Prob = $\dfrac{\binom{13}{4}\binom{2}{1}}{\binom{15}{5}}$ or $\dfrac{5.13.12.11.10.2}{15.14.13.12.11}$ $= \dfrac{(2)(715)}{3003}$ = 0.476 = 10/21	M1 numerator M1 denominator A1	
	OR (B) $\binom{5}{1}\times\binom{10}{1}/\binom{15}{2} = \dfrac{10}{21}$	M1 numerator M1 denominator A1 cao	
	OR (B) $\dfrac{5}{15}\times\dfrac{10}{14}\times 2=\dfrac{10}{21}$	M1 for 1st fraction M1 for 2nd fraction A1 cao	6
			15

Question 4

(i)	$P(X = 13) = P(X \le 13) - P(X \le 12)$ $= 0.8701 - 0.7480$ $= 0.1221$	M1 M1 A1	3
(ii)	$P(X \ge 8) = 1 - P(X \le 7)$ $= 1 - 0.0580$ $= 0.942$	M1 A1	2
(iii)	Expected number pupils $= 20(0.55)$ $= 11$	M1 A1	2
(iv)	Let p be the probability of a pupil achieving a grade C or better $H_0: p = 0.55$ $H_1: p > 0.55$ Because dept. looking for improvement	 B1 B1 E1	3
(v)	$X \sim B(20, 0.55)$ $P(X \ge 16) = 1 - P(X \le 15)$ $= 1 - 0.9811$ $= 0.0189$ This is less than 5% so reject H_0 Conclude proportion with C or better has increased.	 M1 for correct tail M1 for method A1 M1 comparison with 5% E1 comment in context	5
			15

Mark Scheme 2614
June 2005

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

B marks are independent of all others. They are usually awarded for a single correct answer. Typically they are available for correct quotation of points such as 1.96 from tables.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.
- Please indicate incorrect working by ringing or underlining as appropriate.
- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.
- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.
- Probabilities should be given as fractions, decimals or percentages.
- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".
- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.
- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
ISW	Ignore subsequent working

2614 MEI Statistics 2

Question 1

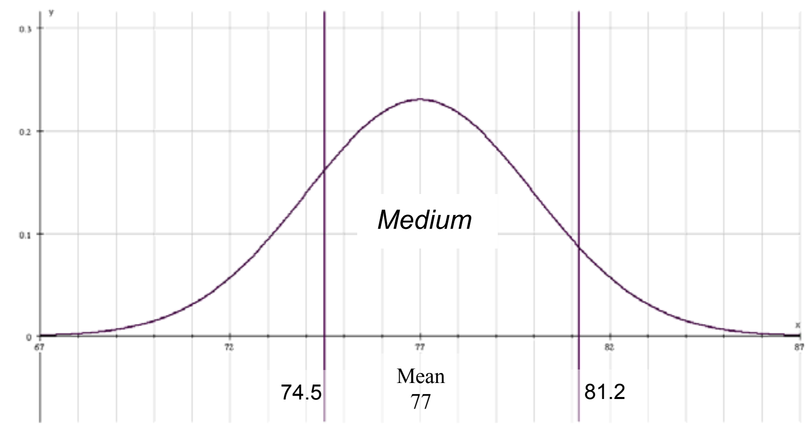
(i)	*Shop* A B C D E F G H I J; *Judge* 1: 44 28 12 24 16 32 48 8 20 36; *Judge* 2: 41 44 36 38 40 47 42 35 45 50; *Rank* 1: 2 5 9 6 8 4 1 10 7 3; *Rank* 2: 6 4 9 8 7 2 5 10 3 1; *d*: -4 1 0 -2 1 2 -4 0 4 2; d^2: 16 1 0 4 1 4 16 0 16 4	B2 for ranks (B1 for $\leq$ 3 errors)	
		B1 for d^2 f.t. their ranks	
	$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 62}{10 \times 99} = 1 - \frac{372}{990} = 1 - 0.3758$	M1 for r_s	
	$= 0.62$ (to 2 s.f.) [*or* 0.624 to 3 s.f.]	A1 f.t. for $\lvert r_s \rvert < 1$ *Allow use of pmcc on ranks*	5
(ii)	H_0: 'Independence' or $\rho = 0$;	B1 for H_0 ;	
	H_1: Positive 'association' or $\rho > 0$ [one tailed test]	B1 for H_1	
	Looking for positive association: critical value at 5% level is 0.5636	B1 for ± 0.5636 [f.t. from their H_1]	
	Since 0.624 > 0.5636, there is sufficient evidence to reject H_0, i.e. there is sufficient evidence to conclude that there is positive association between the judges' marks.	M1 for comparison with c.v., provided $\lvert r_s \rvert < 1$ A1 for conclusion in context	5
(iii)	A test based on the pmcc requires a *bivariate Normal* parent population.	B1	
	Sensible comment for/against the use of pmcc with a reason, relating to *ellipticity* of the scatter diagram.	B1 for conclusion with reason	2
(iv)	• Shop G is best according to judge 1 and shop J is best according to judge 2.	E1	
	• Shop J is the best overall, (because)	E1	
	• Shop J has the best overall pair of ranks (first and third)	E1	
	Or	*Or*	
	• Shop G has the highest total score of 90.	E1	
	• Shop J has the best overall pair of ranks (first and third)	E1	
	• Basing this judgement on position may be fairer, since the marks are more subjective than the rankings, (particularly since there is such a large discrepancy in the dispersion of the marks awarded by each judge).	E1	3
			15

Data table for (i):

Shop	*Judge* 1	*Judge* 2	*Rank* 1	*Rank* 2	*d*	d^2
A	44	41	2	6	-4	16
B	28	44	5	4	1	1
C	12	36	9	9	0	0
D	24	38	6	8	-2	4
E	16	40	8	7	1	1
F	32	47	4	2	2	4
G	48	42	1	5	-4	16
H	8	35	10	10	0	0
I	20	45	7	3	4	16
J	36	50	3	1	2	4

2614 MEI Statistics 2

Question 2

(i)		G1 for shape and mean = 77 G1 for 'Medium' and limits 74.5 and 81.2	**2**
(ii)	P(74.5 < X < 81.2) = $\mathrm{P}\left(\frac{74.5-77}{3} < Z < \frac{81.2-77}{3}\right)$ = P(–0.8333 < Z < 1.4) = 0.9192 – (1 – 0.7976) = 0.7168 *or* 0.717 (to 3 s.f.) *or* 0.72 (to 2 s.f.) so the proportion is 72% (to 2 s.f.)	M1 for $74.5 < X < 81.2$ M1 for standardizing M1 for prob. calc. A1	**4**
(iii)	P(14 out of 20 are medium) = $\binom{20}{14} \times 0.7168^{14} \times 0.2832^{6}$ = 0.189 (to 3 s.f.) *or* $\binom{20}{14}$ x 0.72 x 0.28 = 0.188 (to 3 s.f.) The 20 men must form an independent random sample.	M1 for $\binom{20}{14} \times p^{14} \times q^{6}$ [where $q = 1 - p$] A1 B1 for 'random' *or* 'independent'	**3**
(iv)	From tables $\Phi^{-1}(0.98) = 2.054$ $\frac{x-77}{3} = 2.054$ $\Rightarrow$ $x = 77 + 3 \times 2.054$ $\Rightarrow$ x = 83.2 cm	B1 for 2.054 seen M1 for equation in x with sensible positive z-value A1 **cao**	**3**
(v)	$1 - 0.98^n > 0.9$ $\Rightarrow 0.98^n < 0.1$ $\Rightarrow n \log 0.98 < \log 0.1$ $\Rightarrow n > \log 0.1 / \log 0.98$ $\Rightarrow n > 113.974$ $\Rightarrow$ Min. value of n is 114 *Or* $1 - e^{-0.02n} > 0.9$ $\Rightarrow e^{-0.02n} < 0.1$ $\Rightarrow -0.02n < \ln 0.1$ $\Rightarrow n > \ln 0.1 / (-0.02)$ $\Rightarrow n > 115.129$ $\Rightarrow$ Min. value of n is 116	B1 for inequality M1 for attempt to solve by logs (including Poisson approximation) or by trial and improvement A1 **cao**	**3**
			15

2614 MEI Statistics 2

Question 3

(i)	*Uniform (average) rate of occurrence* Junk mail is likely to arrive *randomly* and/or *independently*	E1 for suitable reason E1 for suitable reason in context	**2**
(ii)	Mean = $\frac{\Sigma xf}{n} = \frac{31+50+33+16}{100} = \frac{130}{100}$ = 1.3 Variance = $\frac{1}{n}\left(\Sigma x^2 f - \frac{(\Sigma xf)^2}{n}\right)$ = $\frac{1}{100}\left(294 - \frac{130^2}{100}\right)$ = 1.25 NB *Answer is 1.263 with divisor* n – *1* Or Variance = $\frac{\Sigma x^2 f}{n} - \overline{x}^2 = \frac{31+100+99+64}{100} - 1.3^2$ = $\frac{294}{100} - 1.3^2$ = 1.25	B1 for mean M1 for calculation A1 *Or* M1 for calculation A1	**3**
(iii)	Yes, since mean is close to variance	B1	**1**
(iv)	(*A*) P(X = 2) = $e^{-1.3}\frac{1.3^2}{2!}$ = 0.230 (to 3 s.f.) = 0.23 (to 2 s.f.) (*B*) $\lambda = 6 \times 1.3 = 7.8$ Using tables: P(X > 10) = 1 – P($X \le$ 10) = 1 – 0.8352 = 0.1648	M1 for probability calculation A1 **cao** B1 for mean (SOI) **cao** M1 for probability A1 **cao**	**5**
(v)	Mean no. of items in 50 days = 50 × 1.3 = 65 Using Normal approx. to the Poisson X ~ N(65, 65): P($X \ge$ 79.5) = P$\left(Z > \frac{79.5-65}{\sqrt{65}}\right)$ = P(Z > 1.799) = 1 – P($Z \le$ 1.799) = 1 – 0.9641 = 0.0359 (to 3 s.f.)	B1 for Normal approx. (SOI) B1 for continuity corr. M1 for probability A1 **cao**	**4**
			15

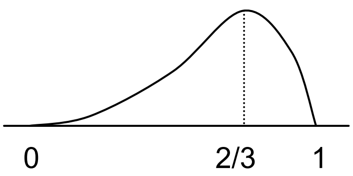
2614 MEI Statistics 2

Question 4

(i)	(*A*) If 3 out of 4 are correctly matched then the fourth must also be correct. (*B*) $9k + 8k + 6k + k = 1 \Rightarrow 24k = 1$ $\Rightarrow k = \frac{1}{24}$ (*C*) There are 4! = 24 different arrangements of which just one has all four correctly matched. (All are equally likely), so $P(X = 4) = \frac{1}{24}$ *Or* $P(X = 4) = \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times 1 = \frac{1}{24}$	E1 M1 for forming equation A1 E1 for 4! arrangements E1 for "just one has all 4 correctly matched" *Or* E1 for $\frac{1}{4} \times p \times q \times r$ [r = 1 may be implied] E1 dep. for correct p, q, r	5
(ii)	$E(X) = \Sigma\, r\, P(X = r)$ $= 0 \times \frac{9}{24} + 1 \times \frac{8}{24} + 2 \times \frac{6}{24} + 4 \times \frac{1}{24} = \frac{24}{24} = 1$ $Var(X) = E(X^2) - [E(X)]^2$ $= 0 \times \frac{9}{24} + 1 \times \frac{8}{24} + 4 \times \frac{6}{24} + 16 \times \frac{1}{24} - 1^2$ $= \frac{48}{24} - 1 = 1$	M1 for E(X) A1 **cao** M1 for E(X^2) A1	4
(iii)	Mean prize money = £100 x 1 = £100 Variance of prize money = 100^2 x 1 = 10000	B1 for mean B1 for variance	2
(iv)	(*A*) P(just one correct in three out of 5 rounds) $= \binom{5}{3} \times \left(\frac{8}{24}\right)^3 \times \left(\frac{16}{24}\right)^2$ $= \frac{40}{243} = 0.1646 = 0.165$ (to 3 s.f.) (*B*) Expected prize money in the five extra rounds = 5 × £1000 x 1 = £5000 So total expected money = £5000 + £400 = £5400	M1 for $\binom{5}{3} \times (8k)^3 \times (1 - 8k)^2$ A1 **cao** M1 for "5000×E(X) + ..." A1 **cao**	4
			15

Mark Scheme 2615
June 2005

2615 Statistics 3

Q1	$f(x) = 12x^2(1 - x),\ 0 \le x \le 1.$			
(i)	Mode given by $f'(x) = 0$. $f'(x) = 24x - 36x^2$ Which = 0 (at $x = 0$ and) at $x = 2/3$. Mode is 2/3.	M1 A1	For attempting to find $f'(x)$ and set =0. c.a.o. No need to explicitly confirm maximum. Do NOT allow if it happens to ft from an incorrect $f'(x)$.	 2
(ii)	 	G1 G1 G1	Correct general shape (anything continuous, smooth and unimodal, in [0, 1]). Maximum at $x = 2/3$ (ft candidate's mode). Slope 0 at $x = 0$ and steeply descending at $x = 1$.	 3
(iii)	$\mathrm{E}(X) = \int_0^1 12x^3(1-x)\mathrm{d}x$ $= \left[12\frac{x^4}{4} - 12\frac{x^5}{5}\right]_0^1 = 3 - \frac{12}{5} = \frac{3}{5}$ $\mathrm{E}(X^2) = \int_0^1 12x^4(1-x)\mathrm{d}x$ $= \left[12\frac{x^5}{5} - 12\frac{x^6}{6}\right]_0^1 = \frac{12}{5} - \frac{12}{6} = \frac{2}{5}$ $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2 = \frac{2}{5} - \left(\frac{3}{5}\right)^2 = \frac{1}{25}$	M1 A1 M1 A1 A1	Integral for E(X) including limits (which may appear later). Integral for $E(X^2)$ including limits (which may appear later). ft from candidate's values unless Var ≤ 0.	 5
(iv)	$X_1 + X_2 + ... + X_{52} \sim$ approx $\mathrm{N}(31 \cdot 2,\ 2 \cdot 08)$ $\mathrm{P}(\text{this} > 30) = \mathrm{P}\left(Z > \frac{30 - 31 \cdot 2}{\sqrt{2 \cdot 08}} = -0 \cdot 832(05)\right)$ = 0·797(3)	B1 B1 F B1 F M1 A1	Normal. Mean; f.t. candidate's mean × 52. Variance; f.t. candidate's variance (>0) × 52. Accept sd if indicated clearly as such. If the name of the distribution is wrong or missing then allow the marks for the parameters either if they are the conventional parameters for the named distribution or they are named explicitly. For an attempt to standardise a reasonable Normal distribution. c.a.o. Accept 0·8, 0·80 if clearly correctly obtained.	 5
				15

Q2	$X \sim N(12{\cdot}6, \sigma = 2{\cdot}2)$ $Y \sim N(8{\cdot}8, \sigma = 1{\cdot}6)$ $Z \sim N(20{\cdot}4, \sigma = 3{\cdot}2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$X + Y + Z \sim N(41{\cdot}8,$ $\sigma^2 = 2{\cdot}2^2 + 1{\cdot}6^2 + 3{\cdot}2^2 = 17{\cdot}64)$ $P(X+Y+Z<40)$ $= P\left(N(0,1) < \frac{40-41\cdot8}{4\cdot2} = -0\cdot4286\right)$ $= 1 - 0\cdot6659 = 0\cdot334(1)$	B1 B1 B1	Mean. Variance. Accept $\sigma = 4{\cdot}2$. c.a.o.	3
(ii)	Want $P(Z < X + Y)$ i.e. $P(Z - X - Y < 0)$ $Z - X - Y \sim N(-1,$ $17{\cdot}64)$ $\therefore P(\text{this} < 0)$ $= P\left(N(0,1) < \frac{0-(-1)}{4\cdot2} = 0\cdot2381\right) = 0\cdot594(1)$	M1 B1 B1 A1	Or $P(X + Y - Z > 0)$. Mean. Or "+1" for alternative method. Variance. Accept $\sigma = 4{\cdot}2$. N.B. Method and mean should be consistent with each other. c.a.o. Or $P\left(N(0,1) > \frac{0-1}{4\cdot2} = -0\cdot2381\right)$.	4
(iii)	Sample mean = 19·5, $s_{n-1} = 2{\cdot}065(36)$ CI is given by $19{\cdot}5 \pm$ $2{\cdot}365$ $\times \frac{2\cdot06536}{\sqrt{8}}$ $= 19{\cdot}5 \pm 1{\cdot}72(696) = (17{\cdot}7(73), 21{\cdot}2(27))$ This interval contains the former mean (20·4), <u>suggesting</u> that there has been no improvement.	B1 M1 B1 M1 A1 E1	Allow $s_n = 1{\cdot}931(97)$ only if used correctly in sequel. Must be c's $\bar{x}$ ± … From t_7. Allow c's s_{n-1}, but not 3·2. Allow $s_n/\sqrt{7}$ (see above). c.a.o. Must be written as an interval. Non-assertive comment.	6
(iv)	Reward <u>any reasonable</u> discussion probably to the effect that the <u>first</u> 8 are unlikely to be a random sample.	E2	(E2, E1, E0). Could include discussion in context about how the sample might have been chosen.	2
				15

Q3				
(i)	$H_0 : \mu = 540$ $H_1 : \mu < 540$	B1 B1	Do **not** allow any other symbol, including $\overline{X}$ or similar, unless it is clearly and explicitly stated to be a population mean. Allow statements in words (see below).	
	Where μ is the (population) mean efficiency measure for the fans.	B1	μ must be defined verbally. Must indicate "mean"; condone "average". Allow absence of "population" if correct notation μ is used, otherwise insist on "population".	
	$n = 12$, $\Sigma x = 6459{\cdot}0$, $\bar{x} = 538{\cdot}25$ ($\sigma = 14$ is given.)	B1		
	Test statistic is $\dfrac{538{\cdot}25-540}{\left(\frac{14}{\sqrt{12}}\right)}$	M1	Allow c's $\bar{x}$. Use of s_{n-1} or s_n gets M0. Allow alternative: 540 – (c's 1·645) × $\frac{14}{\sqrt{12}}$ (= 533·35) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ + (c's 1·645) × $\frac{14}{\sqrt{12}}$ (= 544·90) for comparison with 540.)	
	= –0·433(01)	A1	c.a.o. (but ft from here if this is wrong.) Use of 540 – $\bar{x}$ scores M1A0, but next 4 marks still available.	
	Refer to N(0, 1). Lower 5% point is –1·645. $\left(\Phi(-0\cdot4330) = 0\cdot3325, \text{for comparison with } 0\cdot05.\right)$	M1 A1	No ft from here if wrong. Must be minus 1·645 unless absolute values are being compared. No ft from here if wrong.	
	Not significant.	E1	ft only c's test statistic. Explicit comparison required.	
	Reasonable to suppose specification is being met.	E1	ft only c's test statistic. Should be in context with reference either to the mean or to the specification being met.	10
(ii)	If $\mu = 530$, $\overline{X} \sim N\left(530, \frac{14^2}{12}\right)$	M1	For the distribution of $\overline{X}$ with μ = 530, and c's standard error from above.	
	H_0 is accepted if $\overline{X} > 540 - 1.645 \times \frac{14}{\sqrt{12}} = 533\cdot35(18)$	M1	For the critical point for the test above. Allow c's –1·645.	
	So P(Type II error) = $P\left(N\left(530, \frac{14^2}{12}\right) > 533\cdot35\right)$	M1	M0 if RHS = 540 or 538·25.	
	= P(N(0, 1) > 0·8289)	m1	Standardising. Accept awrt 0·829. Depends on the first and third of the preceding M marks.	
	= 1 – (awrt 0·796 or 0·797) = awrt 0·203 or 0·204	A1	This mark is cao.	5
				15

Q4				
(i) (A)	Sample mean = $\frac{\sum fx}{\sum f} = \frac{200}{100} = 2$	B1	Beware printed answer.	1
(B)	x: 0, 1, 2, 3, 4, 5, 6, >6 o: 19, 25, 22, 18, 5, 9, 2, 0 p: 0·1353, 0·2707, 0·2707, 0·1804, 0·0902, 0·0361, 0·0121, 0·0045 * e: 13·53, 27·07, 27·07, 18·04, 9·02, 3·61, 1·21, 0·45 Combining last 3 cells: o = 11, e = 5·27 * From Poisson(2). These are from cumulative tables. Might differ slightly if calculated directly.			
		M1	For apparently correct method for e_i's. (>6 cell must be present and not empty, or equivalent if candidate obviously realises to group cells earlier.)	
		A1	If all correct <u>or</u> if Σe_i = 100. (But A0 if rounded to integers.)	
		M1	For grouping (cells where $e_i \le 5$).	
	X^2 = 2·21145 + 0·15829 + 0·94957 + 0·00009 + 1·79162 + 6·23015	M1	For evidence of correct method for X^2.	
	= 11·34(12)	A1	c.a.o. (but ft from here if this is wrong.) e_i to 1 d.p. gives X^2 = 11·27(12).	
	Refer to χ_ν^2, where ν = no of cells in candidate's table – 2 (ideally, ν = 4).	M1	Allow this mark if it agrees with candidate's table, and then ft as below. Accept anything that implies use of this distribution.	
	For ν = 4 upper 5% point is 9·488. (If ungrouped ν = 6 upper 5% point is 12·59.)	A1	Allow candidate's ν if preceding M1 awarded. No ft from here if not correct point from candidate's χ^2.	
	Significant.	E1	No f.t. of above M1 or A1 if wrong, except for <u>Special Case:</u> ν + 1 and its 5% point can get EITHER (but not both) of these 2 marks for the conclusion. (ν = 5, cv = 11·07)	
	Seems Poisson does not fit.	E1	"Model does not fit data" NOT "data do not fit model".	
	The main discrepancy is in the "top" cell, where there are substantially too many observations for the model to explain. Other discrepancies are comparatively small.	E1	Accept any reasonable descriptive comment e.g. about discrepancies.	10

(ii)	$\bar{x} = 24 \cdot 264 \quad s^2 = \frac{1216 \cdot 68}{99} = 12 \cdot 2897 = 3 \cdot 50566..^2$		Accept divisor 100: s^2 =12·1668 = 3·48809..^{2}.	
	CI is given by 24·264 ±	M1	Must be c's $\bar{x}$ ± …	
	1·96	B1	Must be from N(0, 1).	
	$\times \frac{\sqrt{12 \cdot 2897 \text{ or } 12 \cdot 1668}}{\sqrt{100}}$	M1	Allow c's s_{n-1} or s_n. Accept ../$\sqrt{99}$ if 12·1668 used $\left(\frac{s_n}{\sqrt{n}}\right)$.	
	= 24·264 ± 0·6871 = (23·577, 24·951) or 24·264 ± 0·6837 = (23·580, 24·948)	A1	c.a.o. Must be written as an interval.	4
				15

Mark Scheme 2616
June 2005

2616 Statistics 4

Q1	$X_1, \ldots, X_n \sim$ ind $N(\mu, \sigma^2)$ $\quad Y = \sum\left(X_i - \overline{X}\right)^2$ $E(Y) = (n-1)\sigma^2 \quad Var(Y) = 2(n-1)\sigma^4 \quad T = kY$			
(i)	$E(T) = k(n-1)\sigma^2$ $Var(T) = 2k^2(n-1)\sigma^4$	B1 B1		2
(ii)	Bias = $E(T) - \sigma^2$ $= k(n-1)\sigma^2 - \sigma^2$	M1 A1	Allow M1A0 if $\sigma^2 - E(T)$.	2
(iii)	MSE(T) = Variance + bias^2 $= 2k^2(n-1)\sigma^4 + \{k(n-1)\sigma^2 - \sigma^2\}^2$ $= 2k^2(n-1)\sigma^4 + \{k^2(n-1)^2 - 2k(n-1) + 1\}\sigma^4$ $= \sigma^4[2(n-1) + (n-1)^2]k^2 - 2\sigma^4(n-1)k + \sigma^4$	M1 A1 A2	If both terms present, even if wrong. If both correct. Divisible for algebra. BEWARE printed answer.	4
(iv)	Consider $\frac{d\,MSE(T)}{d\,k} = 0$ $\frac{d\,MSE(T)}{d\,k} = \sigma^4\left[2(n-1) + (n-1)^2\right]2k - 2\sigma^4(n-1)$ $= 0 \quad \to \quad k = \frac{n-1}{2(n-1)+(n-1)^2}$ $= \frac{1}{n+1}$ Check minimum by considering $\frac{d^2\,MSE(T)}{d\,k^2} = \sigma^4\left[2(n-1)+(n-1)^2\right]2$ $> 0 \quad \therefore$ min	M1 A1 A1 A1 M1 A1	To include "=0", possibly implied. Correct derivative. Isolate k. BEWARE printed answer. Or other methods. (Since $n > 1$).	6
(v)	With $k = \frac{1}{n+1}$, $MSE(T) = \sigma^4\left\{\frac{2(n-1)+(n-1)^2}{(n+1)^2} - \frac{2(n-1)}{n+1} + 1\right\}$ $= \frac{\sigma^4}{(n+1)^2}\left\{2n - 2 + n^2 - 2n + 1 - 2n^2 + 2 + n^2 + 2n + 1\right\}$ $= \frac{\sigma^4}{(n+1)^2}\{2n+2\} = \frac{2\sigma^4}{n+1}$	B2	Divisible for algebra. Answer not printed.	2
(vi)	From (ii), we want $k(n-1)\sigma^2 - \sigma^2 = 0$ $\Rightarrow k = \frac{1}{n-1}$ In this case, MSE(T) = Var(T) $= \frac{2\sigma^4}{n-1}$	M1 A1 M1 A1	For the converse argument, with no support of "only if", award SC B1. Or substitute in expression for MSE in (iii) – this is not difficult.	4
				20

Q2				
(i)	$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A \neq \mu_B$	B1	Both hypotheses. Do **not** allow any other symbols, including, e.g., $\overline{X}_A = \overline{X}_B$ or similar, unless they are clearly and explicitly stated to be population means. Allow statements in words (see below).	
	Where μ_A, μ_B are the population mean strengths for processes A and B.	B1	For adequate verbal definitions of μ_A, μ_B. Must indicate "mean"; condone "average". Allow absence of "population" if correct notation μ is used, otherwise insist on "population".	
	Normality of both populations. Same variance.	B1 B1		4
(ii)	$n_1 = 9,\ \bar{x} = 114\cdot6667,\ s_{n-1}{}^2 = 87\cdot25,\ (s_{n-1} = 9\cdot3408)$ $n_2 = 8,\ \bar{y} = 123\cdot75,\ s_{n-1}{}^2 = 109\cdot07,\ (s_{n-1} = 10\cdot4437)$	B1	If all means and variances correct. Accept s_n's ONLY if correctly used in sequel. $s_n{}^2 = 77\cdot\dot{5},\quad s_n = 8\cdot8066$ $s_n{}^2 = 95\cdot4375,\ s_n = 9\cdot7692$	
	Pooled $s^2 = \dfrac{698 + 763\cdot5}{15} = 97\cdot4\dot{3}$	M1 A1	For any reasonable attempt at pooling (and ft into test and CI). If correct.	
	Test statistic is $\dfrac{114\cdot6667 - 123\cdot75}{\sqrt{97\cdot4\dot{3}}\sqrt{\dfrac{1}{9}+\dfrac{1}{8}}} = \dfrac{-9\cdot0833}{\sqrt{23\cdot0051} = 4\cdot7964}$ $= -1\cdot89(38)$	M1 M1 A1	Overall structure. Allow c's pooled s. $\sqrt{\dfrac{1}{9}+\dfrac{1}{8}}$ ft c's pooled s^2.	
	Refer to t_{15}. Double tail 5% point is 2·131. Not significant. Seems mean strengths are the same for both processes.	M1 A1 E1 E1	No ft from here if wrong. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. Expect reference to means and context.	10
(iii)	CI is given by –9·0833 ± 2·947 × 4·7964 = –9·0833 ± 14·1349 = (–23·21(8), 5·05(2))	M1 B1 M1 A1	Must be c's $(\bar{x} - \bar{y}) \pm$ ... From t_{15}. Allow c's pooled s. c.a.o. Must be written as an interval.	4
(iv)	Wilcoxon Rank sum test	B1 B1	Or Mann-Whitney scores B2.	2
				20

Q3 (a)	$H_0 : \mu_D = 0$ or $\mu_E = \mu_S$ $H_1 : \mu_D \neq 0$ or $\mu_E \neq \mu_S$	B1	Both hypotheses. Do **not** allow any other symbols, including, e.g., $\overline{X}_E = \overline{X}_S$ or similar, unless they are clearly and explicitly stated to be population means. Allow statements in words (see below).	
	Where μ_D is "population mean for Experimental fertilizer – population mean for Standard fertilizer".	B1	For adequate verbal definition of μ. Must indicate "mean"; condone "average". Allow absence of "population" if correct notation μ is used, otherwise insist on "population".	
	Normality of differences is required.	B1	Must be explicit about the population.	
	MUST be PAIRED COMPARISON *t* test. Differences are 0·6 2·3 –0·8 0·6 0·9 –1·5 1·4 0·8 0·1 0·2	M1		
	$\bar{d} = 0\cdot 46,\ s_{n-1} = 1\cdot 0668(75),\ s_{n-1}^{\ 2} = 1\cdot 1382$	B1	Accept $s_n = 1\cdot 0121,\ s_n^{\ 2} = 1.0244$ ONLY if correctly used in sequel.	
	Test statistic is $\dfrac{0\cdot 46 - 0}{\left(\dfrac{1\cdot 0668(75)}{\sqrt{10}}\right)}$	M1	Allow c's $\bar{d}$ and/or s_{n-1}. Allow alternative: 0 (c's 2·262) × $\dfrac{1\cdot 0668(75)}{\sqrt{10}}$ (= ±0·7631) for subsequent comparison with $\bar{d}$. (Or $\bar{d}$ ± (c's 2·262) × $\dfrac{1\cdot 0668(75)}{\sqrt{10}}$ (= –0·303, 1·2231) for comparison with 0.)	
	= 1·36(35)	A1	c.a.o. (but ft from here if this is wrong.) Use of $\mu_D - \bar{d}$ scores M1A0, but next 4 marks still available.	
	Refer to t_9.	M1	No ft from here if wrong.	
	Double tail 5% point is 2·262.	A1	No ft from here if wrong.	
	Not significant.	E1	ft only c's test statistic.	
	Seems mean yield using experimental fertilizer is same as for standard.	E1	ft only c's test statistic. Expect reference to mean(s) and context.	11
(b)	Now need Normality for yields using experimental fertilizer.	B1		
	For these yields, $\bar{x} = 20\cdot 43,\ s_{n-1} = 4\cdot 0803,\ s_{n-1}^{\ 2} = 16\cdot 649$	B1	Accept $s_n = 3\cdot 8709,\ s_n^{\ 2} = 14.9841$ ONLY if correctly used in sequel.	
	One-sided CI (lower confidence bound) is given by 20·43	M1	Mean. Allow c's $\bar{x}$.	
	–	M1	Minus.	
	1·833	B1	From t_9.	
	$\times \dfrac{4\cdot 0803}{\sqrt{10}}$	M1	Allow c's s_{n-1}, or $s_n / \sqrt{9}$ (see above).	
	= 20·43 – 2·36(51) = 18·06(49)	A1	Depends on all 4 preceding marks.	
	In repeated sampling, lower confidence bounds obtained in this way would fall below the true mean on 95% of occasions.	E2	(E0, E1, E2). Comment should refer to lower bound rather than just the confidence interval.	9

				20

Q4				
(a)	Data 29 32 34 38 40 46 51 52 59 63 71 95 Median 60 Difference –31 –28 –26 –22 –20 –14 –9 –8 –1 3 11 35 Rank of |diff| 11 10 9 8 7 6 4 3 1 2 5 12			
		M1	For differences. ZERO in this section if differences not used.	
		M1	For ranks of |difference|.	
		A1	All correct. ft from here if ranks wrong.	
	$T = 2 + 5 + 12 = \underline{\underline{19}}$	B1	Or 1 + 3 + 4 + 6 + 7 + 8 + 9 + 10 + 11 = 59	
	Refer to tables of Wilcoxon single sample (/paired) statistic.	M1	No ft from here if wrong.	
	Lower (or upper if 59 used) 2½% tail is needed.	M1	No ft from here if wrong.	
	Value for n = 12 is $\underline{\underline{13}}$ (or 65 if 59 used).	A1	No ft from here if wrong.	
	Result is not significant.	E1	ft only c's test statistic.	
	No real evidence that median is not 60.	E1	ft only c's test statistic.	9
(b) (i)	$\mathrm{P}(80 < \mathrm{N}(62, \sigma = 27 \cdot 3) \le 100)$ $= \mathrm{P}(0 \cdot 6593(4) < \mathrm{N}(0, 1) \le 1 \cdot 3919(4))$ $= 0 \cdot 9180 - 0 \cdot 7452 = 0 \cdot 1728$	B1		
	$\therefore$ expected frequency = 200 × 0·1728 = 34·6	B1	BEWARE printed answer.	2
(ii)	Grouping the last two cells, X^2 = 5·6903 + 0·1946 + 18·3265 + 5·2024 + 8 9526	M1	Allow without grouping. This becomes … + 0·0769 +	
	+ 5·6195 = 43.98(59)	A1	21·7529. X^2 becomes 60·19(62). Then <u>must</u> have χ_4^2 below.	
	Refer to χ_3^2.	M1	<u>NEXT</u> mark not available if not χ_3^2.	
	Extremely highly significant – overwhelming evidence that Normal model does not fit data.	A1	ft only c's test statistic.	
	The fit is not particularly good in most of the intervals, but the main points are that the modal class is perhaps "half an interval lower" than expected, that there are many fewer low values than expected, and that there a lot of upper outliers.	E2	(E0, E1, E2)	6
(iii)	Part (a) has a small sample and it appears that the underlying distribution is not Normal – could be dangerous to use a t test.	E2	(E0, E1, E2)	
	There is also the point that, in the absence of Normality (or at least of symmetry), we could not use the t test for the mean as a proxy test for the median.	E1		3
				20

Mark Scheme 2617
June 2005

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

B marks are independent of all others. They are usually awarded for a single correct answer. Typically they are available for correct quotation of points such as 1.96 from tables.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.

- Please indicate incorrect working by ringing or underlining as appropriate.

- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.

- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.

- Probabilities should be given as fractions, decimals or percentages.

- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".

- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.

- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
ISW	Ignore subsequent working

Question 1

(i)	$X \sim$ Poisson (λ) Pgf of X is $\mathrm{G}_X(t) = \mathrm{E}\left[t^X\right]$ $= \sum_{x=o}^{\infty} t^x \dfrac{e^{-\lambda}\lambda^x}{x!}$ $= \mathrm{e}^{-\lambda}\sum_{x=o}^{\infty} \dfrac{(\lambda t)^x}{x!} = \mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t}$	**M1** **M1, A1**	3
(ii)	$\mu = \mathrm{G}'(1) = \mathrm{e}^{-\lambda}(\lambda e^{\lambda t})\Big\vert_{t=1} = \lambda$ $\sigma^2 = \mathrm{G}''(1) + \mu(1-\mu) = \lambda e^{-\lambda} \cdot \lambda e^{\lambda t}\Big\vert_{t=1} + \lambda - \lambda^2$ $= \lambda^2 + \lambda - \lambda^2 = \lambda$	**M1, A1** **M1** **A1**	
			5
(iii)	$Y(\sim \text{Poisson}(\mu))$ has pgf $\mathrm{e}^{-\mu}\ \mathrm{e}^{\mu t}$ $\therefore X + Y$ has pgf $(\mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t}) . (\mathrm{e}^{-\mu}\mathrm{e}^{\mu t})$ $= \mathrm{e}^{-(\lambda+\mu)}\mathrm{e}^{(\lambda+\mu)t}$ which is pgf of Poisson ($\lambda + \mu$) and as pgfs are unique it follows that $X + Y \sim$ Poisson ($\lambda + \mu$) [or, <u>much</u> longer, by convolution of probabilities]	**M1** **M1** product of pgfs **A1** **M1** **E1** **1**	6

(iv)	$\mathrm{P}(X = x \mid X + Y = n) = \dfrac{\mathrm{P}(X = x \cap X + Y = n)}{\mathrm{P}(X + Y = n)}$	**M1**	
	$= \dfrac{\mathrm{P}(X = x \cap Y = n - x)}{\mathrm{P}(X + Y = n)}$	**M1**	
	$= \dfrac{e^{-\lambda}\lambda^x}{x!} \cdot \dfrac{e^{-\mu}\mu^{n-x}}{(n-x)!} \cdot \dfrac{n!}{e^{-(\lambda+\mu)}(\lambda+\mu)^n}$ [for x=0,1,…,n]	o.e. **1**	
	$= \binom{n}{x}\left(\dfrac{\lambda}{\lambda+\mu)}\right)^x\left(\dfrac{\mu}{\lambda+\mu} = 1 - \dfrac{\lambda}{\lambda+\mu}\right)^{n-x}$	**1** for algebraic terms **1** for $\binom{n}{x}$	
	i.e. $\mathrm{B}(n, \dfrac{\lambda}{\lambda+\mu})$	**1**	6

Question 2

(i)	$\mathrm{M}_V(\theta) = \mathrm{E}(\mathrm{e}^{\theta V})$ $= \mathrm{E}(\mathrm{e}^{\theta(aU+b)})$ $= \mathrm{e}^{b\theta}\mathrm{E}(\mathrm{e}^{(a\theta)U})$ $\mathrm{M}_X(\theta) = \int_0^\infty \mathrm{e}^{\theta x}\mathrm{e}^{-x}\mathrm{d}x = \int_0^\infty \mathrm{e}^{-x(1-\theta)}\mathrm{d}x$ $= \mathrm{e}^{b\theta}\mathrm{M}_U(a\theta)$	**M1** **1** **1**	3
(ii)	$\mathrm{M}_X(\theta) = \int_0^\infty \mathrm{e}^{\theta x}\mathrm{e}^{-x}\mathrm{d}x = \int_0^\infty \mathrm{e}^{-x(1-\theta)}\mathrm{d}x$ $= \left[\frac{\mathrm{e}^{-x(1-\theta)}}{-(1-\theta)}\right]_0^\infty \quad = 0 + \frac{1}{1-\theta} = (1-\theta)^{-1}$ (OK for $\theta < 1$: candidates are not expected to discuss this) $\mathrm{E}(X) = \mathrm{M}_x{}'(0) = -1(1-\theta)^{-2}(-1)\Big\|_{\theta=0} = 1$ $\mathrm{E}(X^2) = \mathrm{M}_x{}''(0) = -2(1-\theta)^{-3}(-1)\Big\|_{\theta=0} = 2$ $\therefore$ Var (X) = 2 – 1 = 1 [or by series expansion]	**M1** **A1** beware printed answer **M1** Note – Answer is given **M1, A1** **1** Answer given	6
(iii)	$\mathrm{M}_Y(\theta) = \{\mathrm{M}_X(\theta)\}^n = (1-\theta)^{-n}$	**1**	1
(iv)	$\mathrm{M}_{\overline{X}}(\theta) = \mathrm{e}^{0\theta}\mathrm{M}_Y(\frac{1}{n}\theta) = (1-\frac{\theta}{n})^{-n}$ Mean 1 Variance $\frac{1}{n}$	**1** Answer given **1** **1**	3
(v)	$Z = \sqrt{n}\ \overline{X} - \sqrt{n}$ $\therefore \mathrm{M}_Z(\theta) = \mathrm{e}^{-\sqrt{n}\theta}\ \mathrm{M}_{\overline{X}}(\ \sqrt{n}\theta\ ) = \mathrm{e}^{-\sqrt{n}\theta}\ (\ 1 - \frac{\theta}{\sqrt{n}}\ )^{-n}$	**M1** **1** Answer given	2

(vi)	$\ln \mathrm{M}_Z(\theta) = -\sqrt{n}\theta - n\ln\left(1-\frac{\theta}{\sqrt{n}}\right)$ $= -\sqrt{n}\,\theta + n\left\{\frac{\theta}{\sqrt{n}} + \frac{\theta^2}{2n} + \frac{\theta^3}{3n^{3/2}} + \frac{\theta^4}{4n^2} + \cdots\right\}$ $= \frac{\theta^2}{2} + \frac{\theta^3}{3}n^{-1/2} + \frac{\theta^4}{4}n^{-1} + \cdots$ As $n \to \infty$, this $\to \frac{\theta^2}{2}$, so $M_z(\theta) \to e^{\theta^2/2}$ Which is mgf of N(0,1) and, as mgfs are unique this implies Z tends to N(0,1)	**M1** **1** Answer given **1** **E1** **1**	5

Question 3

(i)	$s_{n-1}{}^2 = 7.268 \quad \left[s_n^2 = 5.8144, \text{allow only if correctly used}\right]$ Test statistic is $\dfrac{(n-1)S^2}{\sigma_o{}^2}\left[n=5, \sigma_o{}^2 = 2\right] = 14.536$ Compare with χ_4^2 Upper 5% point is 9.488 Significant/reject H_O	**M1, A1** **1** No FT if wrong **1** No FT if wrong **1**	5
(ii)	$P(Y<y) = \int_0^y \dfrac{1}{4} te^{-t/2} dt$ $= \dfrac{1}{4}\left\{\left[-2te^{-t/2}\right]_0^y + 2\int_0^y e^{-t/2} dt\right\}$ $= \dfrac{1}{4}\left\{-2ye^{-y/2} + (-4)\left[e^{-t/2}\right]_0^y\right\}$ $= -\dfrac{1}{2}ye^{-y/2} - e^{-y/2} + 1 \quad = 1 - e^{-y/2}(1+\dfrac{y}{2})$	**M1** for attempt to integrate by parts **2**, divisible, for algebra BEWARE PRINTED ANSWER	3
(iii)	Want $P(Y>14.536) = 1 - \left\{1 - e^{-7.268}(8.268)\right\} = 0.0057\ (669)$	**M1, A1** BEWARE PRINTED ANSWER	2
(iv)	14.536 is between upper 0.01 point (13.28) and upper 0.005 point (14.86)	**E2** (E0,E1,E2)	2

(v)	H_0 is accepted if $\dfrac{(n-1)S^2}{\sigma_o^2}\left[i.e.\,\dfrac{4S^2}{2},\, i.e.\, 2S^2\right]$ is < 9.488 But we now have $\dfrac{(n-1)S^2}{\sigma^2}\left[i.e.\,\dfrac{4S^2}{\sigma^2}\right] \sim \chi_4^2$ So $2S^2 \sim \dfrac{\sigma^2}{2}\chi_4^2$, ie accept H_0 if $\dfrac{\sigma^2}{2}\chi_4^2 < 9.488$ ie if $\chi_4^2 < \dfrac{2}{\sigma^2}\times 9.488$	**M1** **1** **E1**	3
(vi)	$\sigma^2 = 3:$ Want $P(\chi_4^2 < \dfrac{2}{3}\times 9.488 = 6.325\dot{3})$ $= 1 - e^{-3.162\dot{6}}\ (4.162\dot{6})$ $= 0.823(866)$ $\sigma^2 = 6:$ Want $P(\chi_4^2 < \dfrac{2}{6}\times 9.488 = 3.162\dot{6})$ $= 1 - e^{-1.581\dot{3}}\ (2.581\dot{3})$ $= 0.469(018)$ Also given $\sigma^2 = 4\ :\ 0.685$ $\sigma^2 = 10\ :\ 0.245$ These are quite high probabilities of accepting H_O when it is false [or other sensible comments]	**M1** **; A1, A1** **E2** (E0,E1,E2)	5

Question 4

(a)	We have 54 out of 60 and 76 out of 100 90% CI for $p_1 - p_2$ is $$\frac{54}{60}-\frac{76}{100}\pm 1.645\sqrt{\frac{(\frac{54}{60}\cdot\frac{6}{60})}{60}+\frac{(\frac{76}{100}\cdot\frac{24}{100})}{100}}$$	**M1** for $\frac{54}{60}-\frac{76}{100}$ **B1** for 1.645 **M1** Two Terms **M1** Both Correct	
	$= 0.14 \pm 1.645\sqrt{0.0015 + 0.001824 (= 0.003324)}$	**A1**	
	$= 0.14 \pm 1.645 \times 0.0576\ (54)$ $= 0.14 \pm 0.094\ (84)$ $= (0.045\ (16)\ , 0.234(84))$	**A1 CAO**	
	The lower end of this interval is >0, which suggests that the new spray is better.	**E2** (E0,E1,E2)	8
(b)	$s_1^2 = 4.125 \quad (3.7125 \text{ with divisor } n)$ $s_2^2 = 1.887 \quad (1.651 \text{ with divisor } n)$	**B1**	
	Test statistic is $\frac{4.125}{1.887} = 2.186$	**1** [FT from candidates' values] Refer to $F_{9,7}$ **1** for *F* **1** for df No FT if wrong	

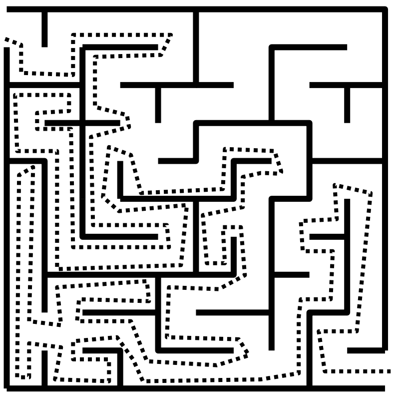
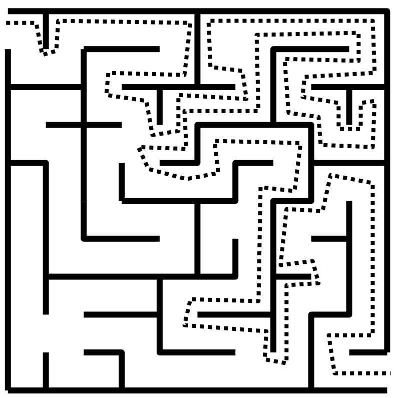
	$F_{9,7}$ is not in tables. Upper 2 ½ % point of $F_{8,7}$ is 4.90, of $F_{10,7}$ is 4.76. Accept any convincing explanation that result is not significant.	**E2** [Only **1** if based on 5% points : 3.73 and 3.64]	
	Seems underlying variances can be assumed equal	**1**	
	$$\frac{\left(\dfrac{S_1^2}{\sigma_1^2}\right)}{\left(\dfrac{S_2^2}{\sigma_2^2}\right)} \sim F_{n_1-1,\, n_2-1}$$	**1, 1** Accept $\left[\text{LHS as } \frac{S_1^2}{S_2^2}\right]$	
	Require Normality of both populations	**1**	
	This is also required for *t* test. *t* test needs same population variances – we have no evidence against this. So *t* test seems OK.	**E2** (E0,E1,E2)	12

Mark Scheme 2620
June 2005

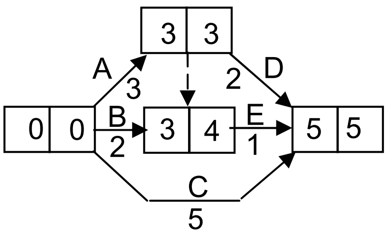
1.

(i)	Any connected tree.	M1 A1
	12 connections	B1
(ii)	14 connections	B1
(iii)	e.g. He might be able to save cable by using it. e.g. To avoid overloading.	B1

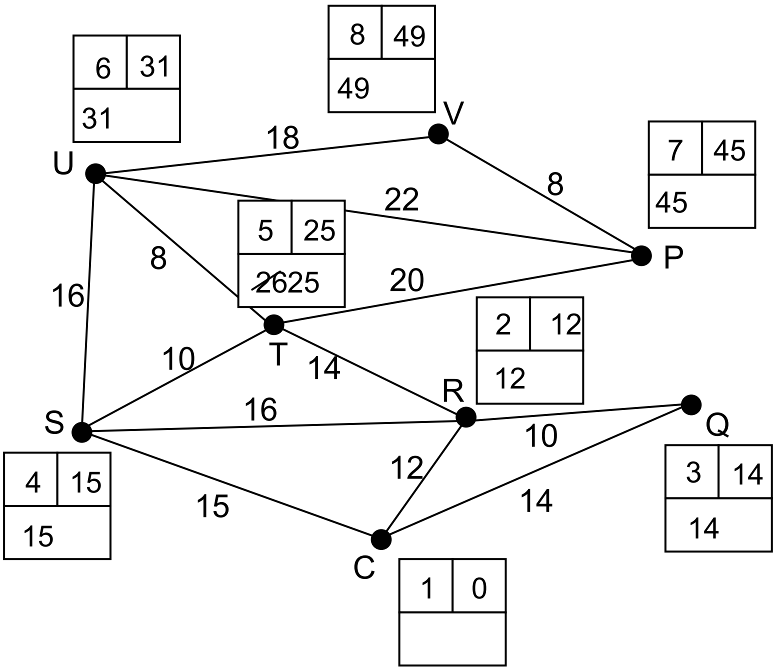
2.

(i)	Janet  John 	M1 A1 A1
(ii)	Yes Janet's route traces west and south walls plus "attachments". John's route traces north and east walls plus "attachments". – or equivalent (Any "islands" are irrelevant.)	M1 A1

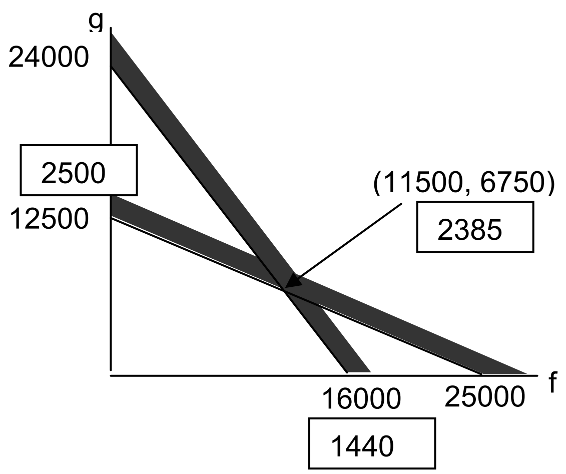
3.

(i)		M1 A1 B1 → B1 ←
(ii)	Critical – A, D and C	B1

4.

(i)	B1 starting at C
	M1 Dijkstra A1 labels A1 order of labelling A1 working values
P Q R S T U V 45 14 12 15 25 31 49	B1
P T S C	B1
(ii) PV ST CR RT UV Length = 80 TU QR	M1 A1 first 5 A1 last 2 B1 length
(iii) CP reduced to 26 CV reduced to 34	B1 (both and no more)
(iv) UV replaced by PQ New length = 74	B1
(v) Q Semi-Eulerian. (Order of P changed from 3 to 4, but order of Q changed from 2 to 3 − so still 2 odd vertices.) or Cross the bridge and proceed as before or A valid route	M1 A1

5.

(i) Let f be the number of litres of Flowerbase produced Let g be the number of litres of Growmuch produced	B1
Max $9f + 20g$ s.t. $0.75f + 0.5g \le 12000$ $f + 2g \le 25000$	M1 A1 M1 A1 A1
(ii) 	B1 labels + scales B1 B1 lines B1 shading
Max profit = £2500 by producing 12500 litres of Growmuch	M1 A1
(iii) (A) No effect	B1
(B) No effect The profit on Flowerbase will be reduced by more than that suffered by Growmuch, since it uses more fibre (the objective gradient will thus increase from −9/20), making it even less attractive to produce any Flowerbase.	B1 B1

6.

(i)	eg. 00–19 → 0 20–49 → 1 50–69 → 2 70–84 → 3 85–99 → 4	M1 sca at proportions A1
(ii)	1, 0, 2, 3, 1, 3, 4, 3, 0, 0	M1 A1
(iii)	eg. 00–15 → 0 16–39 → 1 40–63 → 2 64–95 → 3 96–99 → ignore	M1 missing some A1 times
(iv)	1, 0, 1, 0, 1, 1, 3, 3, 2, 2	B1 one ignored B1 rest
(v)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 0 0 0 0 2 4 Disptd 0 0 0 0 1 0 2 1 0 0 0	M1 A1 A1
(vi)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 1 0 0 1 3 5 Disptd 0 0 0 0 0 0 0 1 0 0 0 Yes – fewer disappointed	M1 using both ret dists A1 A1 B1

Mark Scheme 2621
June 2005

1.

(a)(i) If sidelights and headlights are on, and if the foglights are switched on. B1 B1

(ii) $\sim(\sim s \vee \sim h) \wedge f$ M1 A1

(iii)

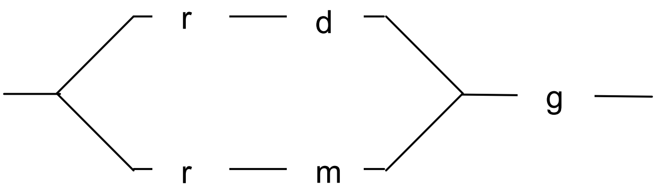
(s	$\wedge$	h)	$\wedge$	f	$\Leftrightarrow$	~	(~	s	$\vee$	~	h)	$\wedge$	f
0	0	0	0	0	1	0	1	0	1	1	0	0	0
0	0	0	0	1	1	0	1	0	1	1	0	0	1
0	0	1	0	0	1	0	1	0	1	0	1	0	0
0	0	1	0	1	1	0	1	0	1	0	1	0	1
1	0	0	0	0	1	0	0	1	1	1	0	0	0
1	0	0	0	1	1	0	0	1	1	1	0	0	1
1	1	1	0	0	1	1	0	1	0	0	1	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

M1 8 rows
A1 $s \wedge h \wedge f$
A1 $\sim(\sim s \vee \sim h) \wedge f$

Accept t/table showing $s \wedge h = \sim(\sim s \vee \sim h)$

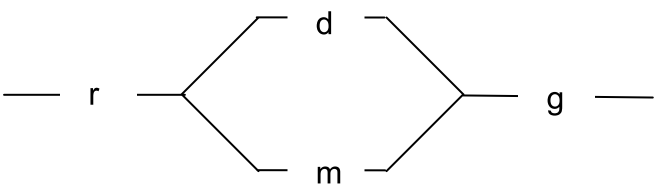
B1 comment re $\wedge f$
M1 4 lines
A1
alternativ

(b)(i)



M1
A1

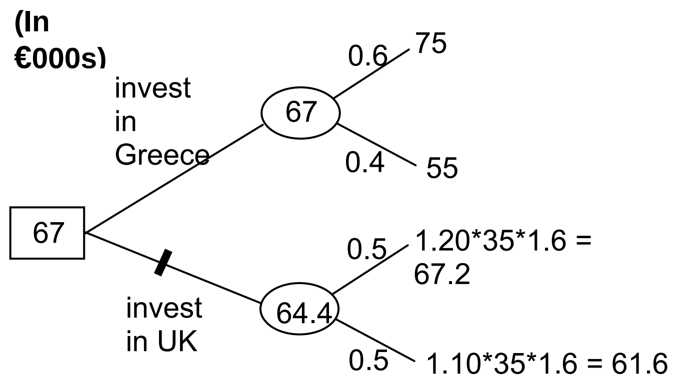
(ii)



B1

2.

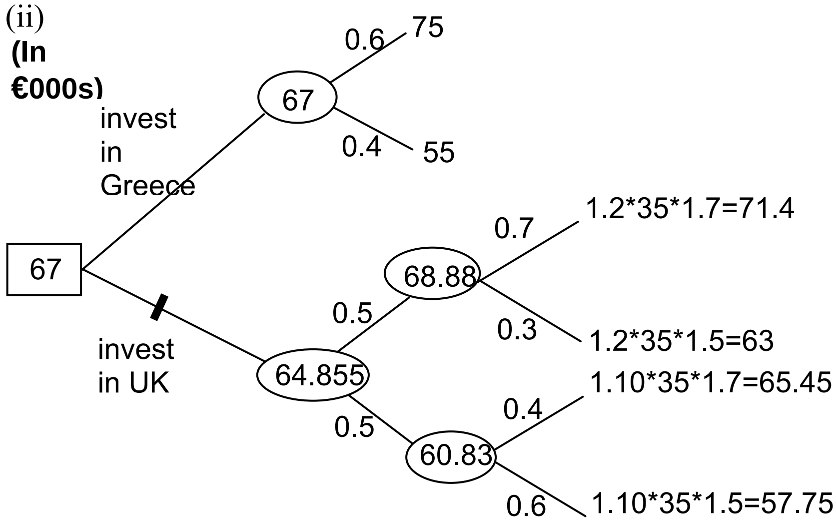
(i)



M1
A1 chance nodes
A1 choice node

B1 invest in Greece

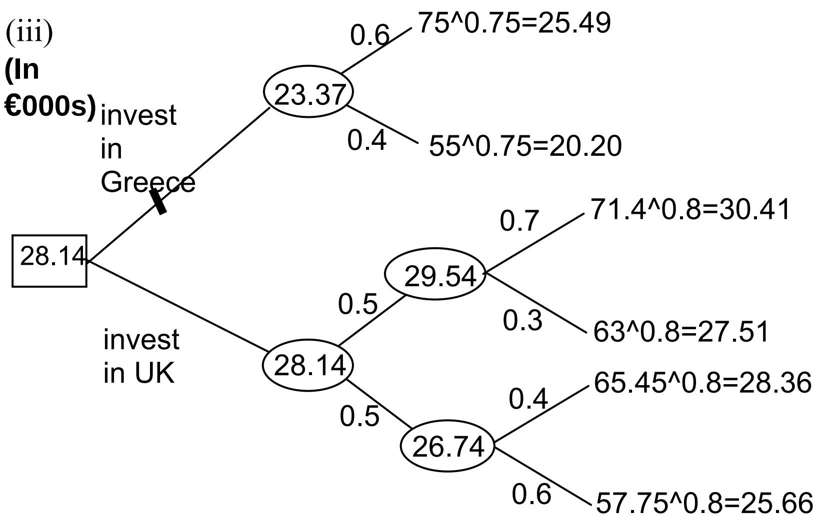
(ii)



M1 new chance nodes
A1 64.855 or .86 or .85

B1 invest in Greece

(iii)

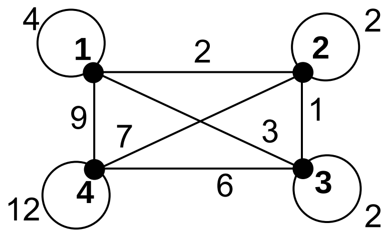


M1 utilities
A1 23.37 and 28.14

B1 invest in UK

3.

(i)



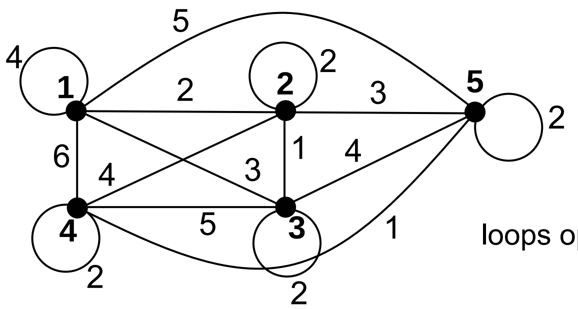
loops optional

M1
A1

(ii) First vertex en route is 3.
First vertex en route from 3 to 1 is 2.
First vertex en route from 2 to 1 is 1.

M1
A1

(iii)



loops optional

M1
A1 arcs
A1 weights

(iv)

	1	**2**	**3**	**4**	**5**
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

	1	**2**	**3**	**4**	**5**
1	2	2	2	2	2
2	1	3	3	5	5
3	2	2	2	2	2
4	5	5	5	5	5
5	2	2	2	4	4

B1 distance matrix

M1 route matrix
A1

(v) 1 2 3 5 4 1
14
1 2 3 2 5 4 5 2 1

M1
A1 cao
A1

(vi)

		1	2	4	3
	1	**2**	**3**	**4**	**5**
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

Lower bound is 5 + 2 + 3 = 10

M1 Prim on matrix
A1

B1 B1

(vii) e.g.
1 2 5 4 <u>3 2 3</u> 1
19

M1 A1 cao
B1

4.

(i) The objective is nonlinear. — B1

(ii)

P	x	y	S1	S2	S3	RHS
1	−1	1	0	0	0	0
0	1	0	1	0	0	10
0	0	1	0	1	0	6
0	1	−2	0	0	1	0
1	0	−1	0	0	1	0
0	0	2	1	0	−1	10
0	0	1	0	1	0	6
0	1	−2	0	0	1	0
1	0	0	1/2	0	1/2	5
0	0	1	1/2	0	−1/2	5
0	0	0	−1/2	1	1/2	1

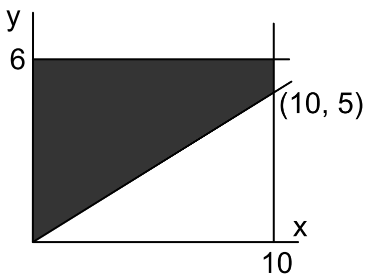
M1 tableau
A1

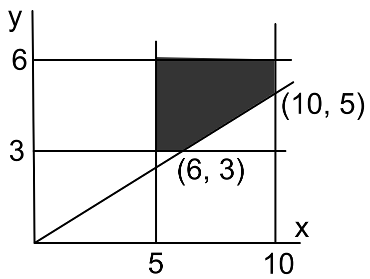
M1 pivot choice
A1 pivot

M1 pivot choice
A1 pivot

10 ml of oil and 5 ml of vinegar — B1

(iii)





B1 x ≤ 10 and y ≤ 6
B1 5 ≤ x and 3 ≤ y

B1 proportion line

B1 region 1
B1 region 2

(iv) Omitted constraints non-active — B1
(0, 0) not in feasible region. — B1

(v)

C	P	x	y	s1	s2	s3	s4	s5	a1	a2	RHS
1	0	1	1	0	–1	0	–1	0	0	0	8
0	1	1	–1	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	10
0	0	1	0	0	–1	0	0	0	1	0	5
0	0	0	1	0	0	1	0	0	0	0	6
0	0	0	1	0	0	0	–1	0	0	1	3

B1 > constraints

B1 artificial columns

B1 new objective

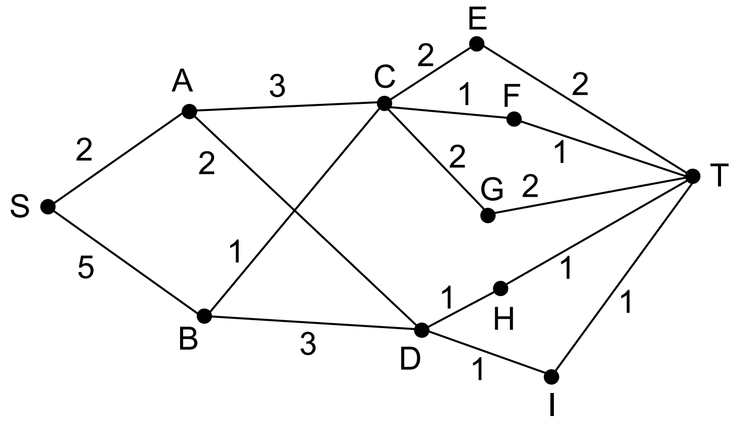
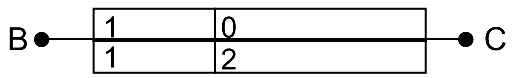
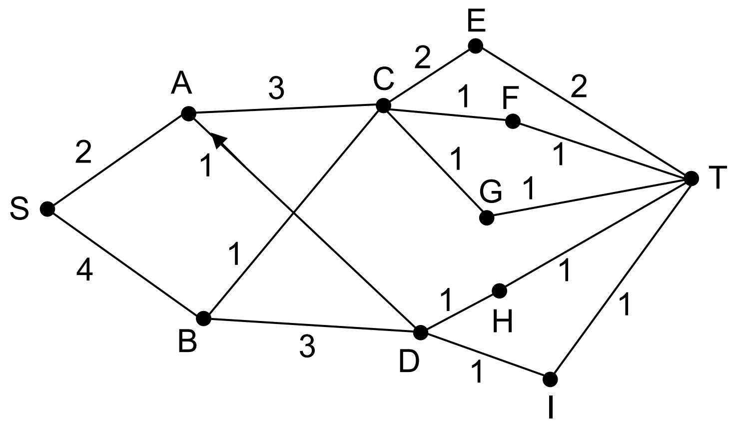
Minimise C, hopefully to zero. — B1
Thereafter delete C row and a1/a2 columns, and proceed as usual. — B1

Mark Scheme 2622
June 2005

Qu. 1

(i) 32*80 = 2560 calories	M1 A1
(ii) 3000/32 = 93.75 kg	M1 A1
(iii) Auxiliary equation is $(3x-1)(3x-2) = 0$	M1 A1 M1 particular A1 93.75 or 3rd eqn M1 gen homogeneous
Solution is $u_n = 13.75(1/3)^n - 27.5(2/3)^n + 93.75$	A1 correct form B1 case 1 ($u_0 = 80$) + case 2 ($u_1 = 80$) M1 simultaneous A1 13.75 and −27.5 B1 final answer
(iv) 90 90 85.83333 81.66667 78.65741 76.80556 75.78961 75.28807 75.06873 74.98871 74.96962 74.97276 74.98119 74.98876	M1 A1
(Oscillatory) convergence to 75 kg.	B1
(v) 90 90 82.77778 75.55556 70.33951 67.12963 65.36866 64.49931 64.11913 63.98043 63.94734 63.95278 63.9674 63.98052 63.98958	B1
Oscillatory convergence to 64 kg.	B1 B1

Qu. 2

(i) 	M1 A1
(ii) 	B1 1 and 1 B1 0 B1 2
(iii) S B D A C G	M1 A1
(iv) 	M1 A1
(v) S B | A C D E F G H I T Flow $\leq$ capacity of any cut Max flow = min cut Flow = capacity of a cut $\Rightarrow$ max flow and min cut.	M1 a cut, properly specified A1 correct cut B1 explanation

(vi)	
Max SA + SB + SD	B1
st SA+CA+DA-AD-AC = 0	
SB+CB+DB-BC-BD = 0	M1
AC+BC-CA-CB-CE-CF-CG = 0	A1
SD+BD+AD-DA-DB-DH-DI = 0	
SA < 2	
SB < 5	
SD < 1	
AD < 2	
DA < 2	B1
BC < 1	
CB < 1	
AC < 3	
BD < 3	
CE < 2	
CF < 1	
CG < 2	
DH < 1	
DI < 1	
end	
(vii)	
OBJECTIVE FUNCTION VALUE	
1) 6.00000	

VARIABLE	VALUE	REDUCED COST	
SA	2.000000	0.000000	
SB	3.000000	0.000000	
SD	1.000000	0.000000	M1
CA	0.000000	1.000000	
DA	1.000000	0.000000	A1
AD	0.000000	0.000000	
AC	3.000000	0.000000	
CB	0.000000	1.000000	
DB	0.000000	0.000000	
BC	1.000000	0.000000	
BD	2.000000	0.000000	
CE	2.000000	0.000000	
CF	0.000000	0.000000	
CG	2.000000	0.000000	
DH	1.000000	0.000000	B1
DI	1.000000	0.000000	

Flows are as listed in the "VALUE" column. B1

Not necessarily any improvement. In this solution Lynne's tree gets the maximum possible flow through CG, but there is another solution with CF = 1 and CG only = 1.

Qu. 3

(i)	Simulating service times (=lookup(rand(),cum.probs,times)) Accumulating (expectation is 207.5 seconds)	B1 B1 B1
(ii)	Repetitions Mean (not far off 207.5 seconds) sd (order of magnitude 5 seconds) $(2*1.96*s)^2$ = (about) 400 repetitions (assuming a 95% confidence interval half-width of 0.5s)	B1 B1 M1 A1 M1
(iii)	Rand()*120 fixed sorted	A1 B1 B1
(iv)	max(arrival time, gate available time) + service time queuing times finish time approx as in (i) mean Q time about 40s	B1 B1 B1 M1 A1
(v)	Test barrier free times to see which barrier passenger uses. Computation of barrier free times, eg: =if(bar=1, max(arrival t + service t, bar t + service t), bar t) finish time approx 130s mean Q time about 4s	M1 A1 B1 B1

Qu. 4

(i)

Sched.	City	Flight	City	Flight	City	Flight	City	Flight	City
S1	L	101	B	201	P	402	M	302	L
S2	L	101	B	201	P	403	L		
S3	L	101	B	202	M	302	L		
S4	L	101	B	202	M	301	P	403	L
S5	B	201	P	402	M	302	L	102	B
S6	B	201	P	402	M	303	B		
S7	B	201	P	403	L	102	B		
S8	B	202	M	301	P	403	L	102	B
S9	B	202	M	302	L	102	B		
S10	B	202	M	303	B				
S11	M	301	P	403	L	102	B	204	M
S12	M	302	L	102	B	204	M		
S13	M	303	B	204	M				
S14	P	401	B	201	P				
S15	P	401	B	202	M	301	P		
S16	P	401	B	203	P				
S17	P	401	B	202	M	303	B	203	P
S18	P	402	M	302	L	102	B	203	P
S19	P	402	M	303	B	203	P		
S20	P	403	L	102	B	203	P		

M1
A1 London
A1 Berlin
A1 Milan
A1 Paris

(ii) Number of schedules = number of pilots. — B1

(iii)

Min S1+S2+S3+S4+S5+S6+S7+S8+S9+S10+S11+ S12 +S13+S14+S15+S16+S17+S18+S19+S20 — M1 A1 objective

st S1+S2+S3+S4>1
S5+S7+S8+S9+S11+S12+S18+S20>1
S1+S2+S5+S6+S7+S14>1
S3+S4+S8+S9+S10+S15+S17>1
S16+S17+S18+S19+S20>1
S11+S12+S13>1
S4+S8+S11+S15>1
S1+S3+S5+S9+S12+S18>1
S6+S10+S13+S17+S19>1
S14+S15+S16+S17>1
S1+S5+S6+S18+S19>1
S2+S4+S7+S8+S11+S20>1

M1
A3 (−1 each error/omission)

(iv) OBJECTIVE FUNCTION VALUE 1) 3.000000 VARIABLE VALUE REDUCED COST S1 1.000000 0.000000 S2 0.000000 0.000000 S3 0.000000 1.000000 S4 0.000000 0.000000 S5 0.000000 0.000000 S6 0.000000 0.000000 S7 0.000000 0.000000 S8 0.000000 0.000000 S9 0.000000 1.000000 S10 0.000000 1.000000 S11 1.000000 0.000000 S12 0.000000 1.000000 S13 0.000000 1.000000 S14 0.000000 0.000000 S15 0.000000 0.000000 S16 0.000000 0.000000 S17 1.000000 0.000000 S18 0.000000 0.000000 S19 0.000000 0.000000 S20 0.000000 0.000000	M1 A1
The schedules used are those with Value = 1.	B1
3 pilots are used	B1
(v) Three more runs, with S1=0, S11=0 and S17=0 in turn. All require 4 pilots	M1 A1 (3 runs) A1 (4 pilots)
(vi) No account taken of pilot stress (workload/long day/short changeover)	B1

Mark Scheme 2623
June 2005

MEI Numerical Methods (2623)

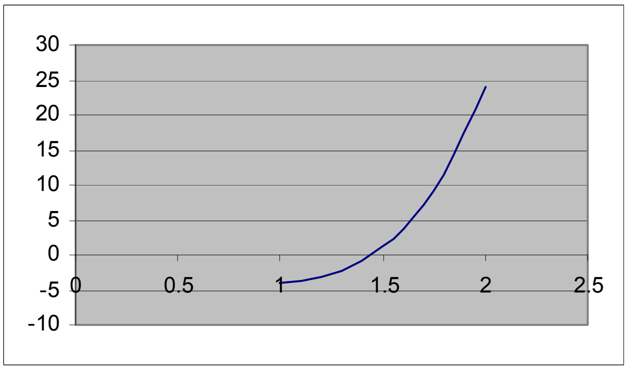
1 (a)	2/3 stored as 0.666 666 7	Absolute error 0.000 000 033...		**[A1A1]**
	mpe is 0.000 000 05			**[A1]**
	mpre is greatest when x is least			**[M1]**
	mpre is 0.000 000 05 / 0.1 =	5 * 10^ -7	5E-07	**[M1A1]**
				[subtotal 6]

(b)	Maximum possible, in theory, is 1p per call: £1000 per day	**[B1]**
	This would only arise if every call rounded downwards under tariff A	**[E1]**
	In practice, about half would round up and half would round down under tariff A	**[M1]**
	So likely benefit is £500 per day	**[A1]**
		[subtotal 4]

(c)	Computations of this type contain rounding errors	**[E1]**
	The rounding errors will be different when the two sums are computed	**[E1]**
	Adding from large to small loses precision (the small number is lost)	**[E1]**
	Adding from small to large allows each number to contribute to the sum	**[E1]**
	Hence the second sum is likely be more accurate	**[E1]**
		[subtotal 5]
		[TOTAL 15]

2 (i)	x	f(x)			
	1	-4			
	2	24	root in the interval (1, 2)		**[B1]**
					[subtotal 1]
(ii)	(A)	r	Xr	f(Xr)	
	secant	0	1	-4	
	method	1	2	24	**[M1]**
		2	1.142857	-3.47891	**[M1A1]**
		3	1.251374	-2.68556	**[M1A1]**
		4	1.618711	4.257268	**[A1]**
	(B)	r	Xr		
	fixed	0	1		
	point	1	1.37973		
		2	1.437547		
		3	1.44558		
		4	1.446682		**[M1A2]**
			root at 1.45 seems secure		**[A1]**
	Fixed point method (B) is much faster.				**[E1]**
					[subtotal 11]

(iii)



[G1]

The non-linearity of the function around the root [E1]

makes the secant method slow [E1]

[subtotal 3]

[TOTAL 15]

3	**(i)**	x	f(x)					
		2	0.832555		M1 =		2.096294	**[M1A1]**
		3	1.048147		T1 =		2.009965	**[A1]**
		4	1.177410		S1 =	(2*M1 + T1) / 3 =	2.067518	**[M1A1]**
								[subtotal 5]
		x	f(x)					
	(ii)							
		2.5	0.957231		T2 =	(M1 + T1) / 2 =	2.053129	**[A1]**
		3.5	1.119269		M2 =		2.076500	**[A1]**
		2.25	0.900517		S2 =	(2*M2 + T2) /3 =	2.068710	**[A1]**
		2.75	1.005784					
		3.25	1.085659		T4 =	(M2 + T2) / 2 =	2.064815	**[A1]**
		3.75	1.149676		M4 =		2.070818	**[A1]**
					S4 =	(2*M4 + T4) / 3 =	2.068817	**[A1]**
								[subtotal 6]
	(iii)	S1	2.067518	diffs	ratio			
		S2	2.068710	0.001192	of diffs			
		S4	2.068817	0.000107	0.089854	(= 1/16 approx)		**[M1A1]**

I = (16*S4 - S2) / 15 or equivalent extrapolation = 2.068824

2.0688 seems secure **[M1A1]**

[subtotal 4]

[TOTAL 15]

4 (i)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	
1	4				
		–3			
2	1		**6**		
		3		**a – 13**	
3	4		**a – 7**		
		a – 4		**87 – 3a**	**[A4]**
4	a		**80 – 2a**		**(-1 each error)**
		76 – a			
5	76				***[subtotal 4]***

(ii) 87 – 3a = a – 13 gives a = 25 **[M1]**

$f(x) = 4 - 3(x-1) + 6(x-1)(x-2)/2 + 12(x-1)(x-2)(x-3)/6$ **[M1A1A1A1A1]**

$= 4 - 3x + 3 + 3x^2 - 9x + 6 + 2x^3 - 12x^2 + 22x - 12$ **[A1]**

$= 2x^3 - 9x^2 + 10x + 1$ **[A1]**

[subtotal 8]

Algebra may appear in (iii) rather than (ii) for full credit

(iii) $f'(x) = 6x^2 - 18x + 10 = 0$ **[M1]**

$x = 2.26$ (2.26376) **[A1]**

$f(2.26...) = 0.718$ **[A1]**

[subtotal 3]

[TOTAL 15]

Mark Scheme 2624
June 2005

MEI Numerical Analysis (2624)

1 (i)	c = 0.7392	**[B1]**
		[subtotal 1]
(ii)	mpe is 0.5*pi /180 = 0.008727 radians	**[M1A1]** ***[subtotal 2]***
(iii)	$dc/d\alpha$ = - sqrt(3)/2 $\sin\alpha$ + 1/2 $\cos\alpha \cos\theta$	**[M1A1A1]**
	$dc/d\theta$ = - 1/2 $\sin\alpha \sin\theta$	**[M1A1]**
	Δc = (- sqrt(3)/2 $\sin\alpha$ + 1/2 $\cos\alpha \cos\theta$) $\Delta\alpha$ + (- 1/2 $\sin\alpha \sin\theta$) $\Delta\theta$	**[M1A1]**
	Δc = abs(-sqrt(3)/2 $\sin\alpha$ + 1/2 $\cos\alpha \cos\theta$)*max($\Delta\alpha$) + abs(-1/2 $\sin\alpha \sin\theta$)*max($\Delta\theta$)	**[A1]**
		[subtotal 8]
(iv)	mpe(c) = (abs(-sqrt(3)/2 sin60 + 1/2 cos60 cos45) + abs(-1/2 sin60 sin45)) 0.008727	**[A1]**
	= 0.007674 (0.0077)	**[A1]**
	(1/2) * 0.005 / 0.007674 = 0.326 (approx 1/3 degree)	**[M1A1]**
		[subtotal 4]
(v)	$dc/d\phi$ = - $\sin\phi$	**[M1A1]**
	mpe(c) = abs($\sin\phi$) mpe(ϕ) = sqrt(1 - c^2) mpe(ϕ) approx hence given result	**[M1A1A1]**
		[subtotal 5] **[TOTAL 20]**
2 (i)	$3x^2 + 3y^2 y' = 1 - y'$	**[M1A1A1]**
	when x = 0, y = 0 hence y' =1 (convincing)	**[M1]**
	$6x + 6y (y')^2 + 3y^2 y'' = - y''$	**[M1A1A1]**
	when x = 0, y = 0, y' =1 hence y" = 0	**[A1]**
	Taylor order 2: y = x	**[M1A1]**
	Obtained directly by noting that sufficiently near (0, 0) x^3 and y^3 are negligible.	**[E1]**
		[subtotal 11]
(ii)	y(1) = 0	**[M1]**
	y'(1) = -2	**[A1]**
	y"(1) = -6	**[A1]**
	Taylor order 2: $y(1+h) = -2h - 3h^2$	**[M1A1]**
	(or $y(x) = -2(x - 1) - 3 (x - 1)^2$)	***[subtotal 5]***
(iii)	y' = -2 - 6h = 0 at h = -1/3	**[M1A1]**
	Hence maximum estimated at (2/3, 1/3)	**[A1A1]**
		[subtotal 4] **[TOTAL 20]**

MEI Numerical Analysis (2624)

3 (i)	The summation would evidently take a long time to converge	**[E1]**
	There would be a large build-up of round-off error	**[E1]** ***[subtotal 2]***
(ii)	Terms from (n+1)th onwards can be approximated by the integral of	**[M1E1]**
	$x^{-5/4}$ from n+1/2 to infinity *limits*	**[A1A1]**
	This is $4/(n+1/2)^{1/4}$ (convincing algebra required)	**[M1A1]**

Hence:	N	10	20	40	80	
	Sn	2.373279	2.715327	3.009515	3.259716	
	Integral	2.222095	1.879843	1.585609	1.335399	**[M1A1A1]**
	Sum	4.595374	4.595170	4.595124	4.595115	**[A1A1]** ***[subtotal 11]***

(iii)	$f' = -5/4\ x^{-9/4}$	**[M1A1]**

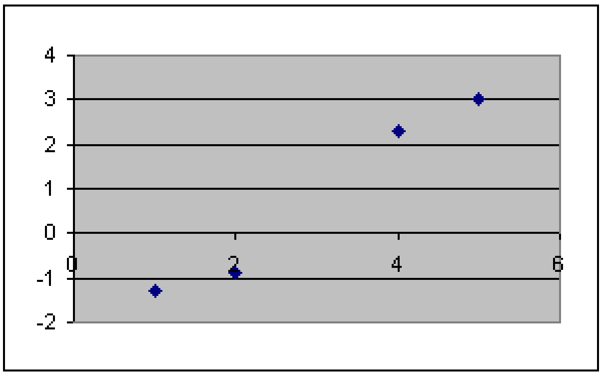
Hence:	N	10	20	40	80	
	Sum	4.595374	4.595170	4.595124	4.595115	
	Correction	0.000262	5.82E-05	1.26E-05	2.68E-06	**[M1A1A1]**
	Improved sum	4.595112	4.595112	4.595112	4.595112	**[A1]**

Complete agreement to 6 decimal places (7 significant figures) **[E1]** ***[subtotal 7]***

[TOTAL 20]

4 (i)	2	-0.9	1st DD			
	4	2.3	1.6	Line:	$y = -0.9 + 1.6(x-2)$	**[M1A1]**
					$y = 0$ gives $\alpha = 2.5625$ (2.6)	**[A1]** ***[subtotal 3]***
(ii)	2	-0.9	1st DD	2nd DD		
	4	2.3	1.6			
	1	-1.3	1.2	0.4		**[M1A1]**
				Quadratic:	$y = -0.9 + 1.6(x-2) + 0.4(x-2)(x-4)$	**[M1A1]**
					$y = 0.4x^2 - 0.8x - 0.9$	**[A1]**
					$y = 0$ gives $\alpha = 2.803$ (2.8)	**[M1A1]** ***[subtotal 7]***

(iii)



The parabolas through x = 1, 2, 4 and x = 2, 4, 5 have different curvature at the root. So one is likely to overestimate and the other will underestimate

[G2] **[E1]** **[E1]**

[subtotal 4]

(iv)

2	-0.9	1st DD	2nd DD	3rd DD	
4	2.3	1.6			
1	-1.3	1.2	0.4		
5	3.0	1.075	-0.125	-0.175	**[M1A1]**

Cubic = quadratic - 0.175 (x - 2)(x - 4)(x - 1) **[M1A1]**

Cubic(2.55) = -0.12268 < 0

Cubic(2.65) = 0.042378 > 0 hence result **[M1A1]**

[subtotal 6]

[TOTAL 20]

Mark Scheme 2625
June 2005

MEI Numerical Computation (2625) June 2005

1 (i) $-1 < g'(\alpha) < 1$ *and* x_0 sufficiently close to α **[B1B1]**

E.g. Multiply both sides of $x = g(x)$ by λ and add $(1 - \lambda)x$ to both sides. **[B1]**
Derivative of rhs set to zero: $\lambda g' + 1 - \lambda = 0$ **[M1A1]**
gives $\lambda = 1 / (1 - g')$ **[A1]**
Need an initial estimate of the root to estimate λ **[A1]**
[subtotal 7]

(ii)

x	1.8	2		
x - 3 lnx	0.03664	-0.07944	change of sign	**[M1A1]**

r	0	1	2	3	4	5	*6*
x_r	1.8	1.76336	1.701663	1.594818	1.40028	1.010016	0.029898
x_r	2	2.079442	2.196298	2.36032	2.576391	2.839169	3.130534

Diverging from each side of the root **[M1A1]**

$g' = 3 / x$
$1 / (1 - g')$ at root is about $1 / (1 - 3/2) = -2$
Hence $x_{r+1} = - 6 \ln x_r + 3 x_r$ **[M1A1]**

r	0	1	2	3	4	5	6
x_r	2	1.841117	1.861116	1.85629	1.857391	1.857136	1.857195

r	7	8	9	10	
x_r	1.857181	1.857184	1.857184	**1.857184**	**[M1A1]**

[subtotal 8]

(iii) Identify root e.g. in (4, 5)
E.g. starting value 4.5 **[M1A1]**

r	0	1	2	3	4	5	
lr	3	2.95224	2.952612	2.952612	2.952612	2.952612	
x_r	4.5	4.536697	4.536404	4.536404	4.536404	**4.536404**	**[M1A1A1]**

[subtotal 5]

[TOTAL 20]

2 (i)

$$T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \ldots$$
$$T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \ldots$$ **[M1A1]**
$$4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \ldots$$ **[M1]**
$$4T_{2n} - T_n - 3\,I = b_4h^4 + b_6h^6 + \ldots$$
$$(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \ldots$$ **[A1]**

($T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given)

$T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6 **[B1]**

[subtotal 5]

(ii)

x	f(x)	M	T	T*	T**	T***		
0	1							
1	0.707107		0.853553					
0.5	0.979796	0.979796	0.916675	0.937715			*x, f(x)*	**[M1A1]**
0.25	1.052267							
0.75	0.84664	0.949454	0.933064	0.938527	0.938582		*M*	**[A1]**
0.125	1.044342							
0.375	1.028036						*T*	**[A1]**
0.625	0.916678							
0.875	0.775536	0.941148	0.937106	0.938454	**0.938449**	0.938446	*T**	**[M1A1]**
0.0625	1.026766							
0.1875	1.052715						*T***	**[M1A1]**
0.3125	1.043718							
0.4375	1.006338						*answer*	**[A1]**
0.5625	0.949555							
0.6875	0.882105							
0.8125	0.810942							
0.9375	0.740825	0.939121	0.938113	0.938449	**0.938449**	0.938449		

[subtotal 9]

(iii)

x	f(x)	M	T	T*	T**	T***	k	
0	1						1.466	
1	0.785175		0.892588					
0.5	1.053148	1.053148	0.972868	0.999628				
0.25	1.100211							
0.75	0.927338	1.013774	0.993321	1.000139	1.000173		*modification*	**[M1A1]**
0.125	1.071038							
0.375	1.09141						J(1.5) = 1.004 (3dp)	**[A1]**
0.625	0.995442							
0.875	0.855718	1.003402	0.998361	1.000042	1.000035	1.000033		
0.0625	1.040743						*evidence*	
0.1875	1.090757						*of trial*	
0.3125	1.100097						*and error*	**[M1A1]**
0.4375	1.075336							
0.5625	1.026117						*answer*	
0.6875	0.962204						k = 1.466	**[A1]**
0.8125	0.89163							
0.9375	0.820105	1.000874	0.999618	1.000036	1.000036	1.000036		

[subtotal 6]

[TOTAL 20]

3 (i) This method will integrate quadratics exactly / find solutions up to cubics. **[E1]**
It will not be exact integrating cubics / finding quartic solutions. **[E1]**

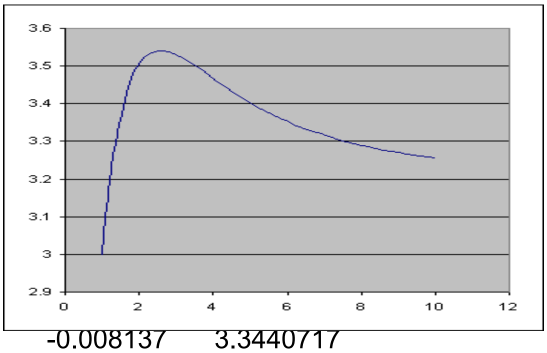
h	x	y	k_1	k_2	k_3	new y	
0.25	0	0	0	0.001736	0.006944	0.005208	**[M1A1]**
0.25	0.25	0.005208	0.015625	0.027778	0.043403	0.041667	**[A1]**
0.25	0.5	0.041667	0.0625	0.085069	0.111111	0.140625	
0.25	0.75	0.140625	0.140625	0.173611	0.210069	0.333333	
0.25	1	**0.333333**	**= 1/3 as required**				**[A1]**

h	x	y	k_1	k_2	k_3	new y	
0.25	0	0	0	0.000145	0.001157	0.000868	**[A1]**
0.25	0.25	0.000868	0.003906	0.009259	0.018084	0.015408	
0.25	0.5	0.015408	0.03125	0.049624	0.074074	0.078776	
0.25	0.75	0.078776	0.105469	0.144676	0.192564	0.249566	
0.25	1	**0.249566**	**not = 1/4 as required**				**[A1]**

[subtotal 8]

(ii)

h	x	y	k_1	k_2	k_3	new y		
0.2	1	3	0.228224	0.2005942	0.1780432	3.1905884		
0.2	1.2	3.1905884	0.1568714	0.1376721	0.1219375	3.3212594		
0.2	1.4	3.3212594	0.1071168	0.0936214	0.0824901	3.4099061	*setup*	**[M1A1]**
0.2	1.6	3.4099061	0.0719789	0.0623677	0.0543935	3.468696		
0.2	1.8	3.468696	0.0468509	0.0399328	0.0341693	3.5060357	*sol'n*	**[M1A1]**
0.2	2	3.5060357	0.0287142	0.0237031	0.0195204	3.5278545		
0.2	2.2	3.5278545	0.0155634	0.0119295	0.0088976	3.5384186	*graph*	**[M1A1]**
0.2	2.4	3.5384186	0.0060348	0.0034116	0.0012295	3.5408494		
0.2	2.6	3.5408494	-0.000824	-0.002696	-0.004245	3.53746		
0.2	2.8	3.53746	-0.005693	-0.007004	-0.008077	3.5299786		
0.2	3	3.5299786	-0.009072	-0.009962	-0.010678	3.5197018		
0.2	3.2	3.5197018	-0.011333	-0.011906	-0.012355	3.5076022		
0.2	3.4	3.5076022	-0.012755	-0.013091	-0.013342	3.494407	*max*	
0.2	3.6	3.494407	-0.013552	-0.013714	-0.013818	3.4806555	x=2.6	
0.2	3.8	3.4806555	-0.013889	-0.013923	-0.01392	3.4667434	y=3.541	**[A1A1]**
0.2	4	3.4667434		-0.013832	-0.013753	3.4529561		
			-0.01389					
0.2	4.2	3.4529561	-0.013652	-0.013529				
0.2	4.4	3.4394946	-0.013249	-0.013082				
0.2	4.6	3.4264957	-0.012735	-0.01254				
0.2	4.8	3.4140475	-0.012153	-0.011943				
0.2	5	3.4022017	-0.011534	-0.011317				
0.2	5.2	3.3909832	-0.010901	-0.010683				
0.2	5.4	3.3803972	-0.010271	-0.010057				
0.2	5.6	3.3704346	-0.009656	-0.009449				
0.2	5.8	3.3610769	-0.009062	-0.008864				
0.2	6	3.352299	-0.008497	-0.008309	-0.008137	3.3440717		
0.2	6.2	3.3440717	-0.007962	-0.007785	-0.007623	3.3363637		
0.2	6.4	3.3363637	-0.007458	-0.007293	-0.007142	3.3291428		
0.2	6.6	3.3291428	-0.006987	-0.006833	-0.006692	3.3223769		
0.2	6.8	3.3223769	-0.006548	-0.006404	-0.006274	3.3160345		
0.2	7	3.3160345	-0.00614	-0.006007	-0.005885	3.3100854		
0.2	7.2	3.3100854	-0.005762	-0.005638	-0.005525	3.304501		
0.2	7.4	3.304501	-0.005411	-0.005296	-0.005192	3.2992543		
0.2	7.6	3.2992543	-0.005086	-0.00498	-0.004884	3.2943199		
0.2	7.8	3.2943199	-0.004786	-0.004688	-0.004599	3.2896743		
0.2	8	3.2896743	-0.004508	-0.004418	-0.004335	3.2852957		
0.2	8.2	3.2852957	-0.004252	-0.004168	-0.004092	3.281164		
0.2	8.4	3.281164	-0.004014	-0.003937	-0.003866	3.2772607		
0.2	8.6	3.2772607	-0.003795	-0.003723	-0.003658	3.2735687		
0.2	8.8	3.2735687	-0.003591	-0.003525	-0.003465	3.2700724		
0.2	9	3.2700724	-0.003403	-0.003341	-0.003286	3.2667574		
0.2	9.2	3.2667574	-0.003229	-0.003171	-0.003119	3.2636107		
0.2	9.4	3.2636107	-0.003067	-0.003013	-0.002965	3.2606202		
0.2	9.6	3.2606202	-0.002916	-0.002867	-0.002822	3.2577748		
0.2	9.8	3.2577748	-0.002776	-0.00273	-0.002688	3.2550645		
0.2	10	3.2550645	-0.002646	-0.002603	-0.002564	3.2524801		



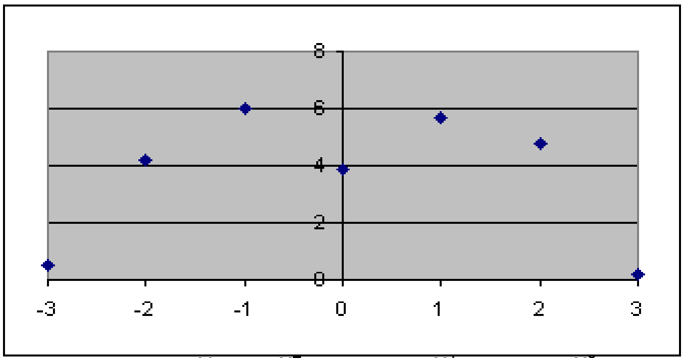
reduce h:	0.005	(or less, in which case FT)	**[M1]**
	2.57	to 2 dp	**[A1]**
	2.5407	4 dp is justified	**[A1A1]**
			[subtotal 12]
			[TOTAL 20]

4 (i) $Q = \Sigma (y - a - bx^2 - cx^4)^2$

dQ/da = 0 gives	$\Sigma y =$	$na + b \Sigma x^2 + c \Sigma x^4$	as given	**[M1A1]**
other equations:	$\Sigma x^2 y =$	$a \Sigma x^2 + b \Sigma x^4 + c \Sigma x^6$		**[B1]**
	$\Sigma x^4 y =$	$a \Sigma x^4 + b \Sigma x^6 + c \Sigma x^8$		**[B1]**

[subtotal 4]

(ii)



[G2]

rough symmetry **[B1]**

3 turning points **[B1]**

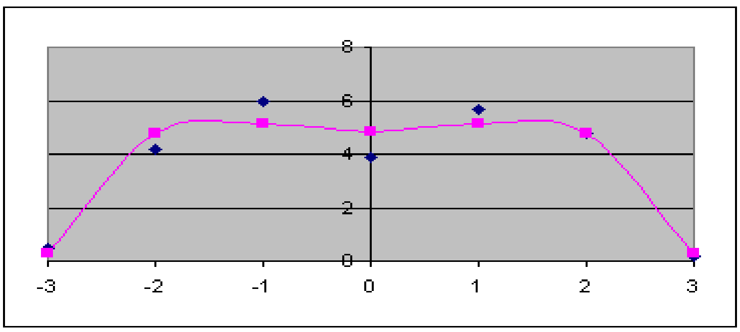
[subtotal 4]

(iii)

x	y	x^2	x^4	x^6	x^8	x^2y	x^4y	
-3	0.5	9	81	729	6561	4.5	40.5	
-2	4.2	4	16	64	256	16.8	67.2	
-1	6.0	1	1	1	1	6	6	
0	3.9	0	0	0	0	0	0	
1	5.7	1	1	1	1	5.7	5.7	
2	4.8	4	16	64	256	19.2	76.8	
3	0.2	9	81	729	6561	1.8	16.2	
0	**25.3**	**28**	**196**	**1588**	**13636**	**54**	**212.4**	**[M1A1]**

Equations:	7	28	196	25.3		*eqns*	**[M1A1]**
	28	196	1588	54			
any method	**196**	1588	13636	212.4	a=	4.85671	**[A1]**
of solution		-28.7143	-291	17.71429			
e.g. Gauss		**-30.8571**	-360	23.65714	b=	0.373485	**[A1]**
elimination			44	-4.3	c=	-0.09773	**[A1]**

x	y	yfit	res	res^2		
-3	0.5	0.302165	0.197835	0.039139		
-2	4.2	4.787013	-0.58701	0.344584		
-1	6.0	5.132468	0.867532	0.752613		
0	3.9	4.85671	-0.95671	0.915294		
1	5.7	5.132468	0.567532	0.322093		
2	4.8	4.787013	0.012987	0.000169		
3	0.2	0.302165	-0.10216	0.010438	*yfit*	**[A1]**
0	**25.3**		**-4.2E-15**	**2.384329**	*rss*	**[M1A1]**



graph **[M1A1]**

[subtotal

12]

[TOTAL 20]

Mark Scheme 4751
June 2005

Section A

1	40	2	M1 subst of 3 for x or attempt at long divn with $x^3 - 3x^2$ seen in working; 0 for attempt at factors by inspection	2
2	$[x =]\dfrac{6y}{3+m}$ as final answer	3	M1 for $3x + mx = y + 5y$ o.e. and M1 for $x(3 + m)$ or ft sign error	3
3	$n + 1$ and $n + 2$ both seen $3n + 3$ $=3(n + 1)$ o.e.	1 M1 A1	 condone e.g. a instead of n for last 2 marks or starting again with full method for middle number = y etc or 3 a factor of both terms so divisible by 3	3
4	–0.6 o.e. (4, 0) (0, 12/5) o.e.	2 1 1	M1 for 0.6 or $-0.6x$ o.e. or rearrangement to 'y =' form [need not be correct] condone values of x and y given	4
5	$8 - 12x + 6x^2 - x^3$ isw	4	B3 for 3 terms correct or all correct except for signs; B2 for two terms correct including at least one of $-12x$ and $6x^2$; B1 for 1 3 3 1 soi or for 8 and $-x^3$	4
6	(i) 1 (ii) a^8 cao (iii) $\dfrac{1}{3a^3b}$ or $\frac{1}{3}a^{-3}b^{-1}$ isw	1 1 3	 M2 for two 'terms' correct or M1 for $3a^3b$ or $\dfrac{1}{(9a^6b^2)^{\frac{1}{2}}}$ or $\dfrac{1}{\sqrt{9a^6b^2}}$; ignore ±	5
7	(i) $3\sqrt{6}$ or $\sqrt{54}$ isw (ii) $10 + 2\sqrt{7}$	2 3	M1 for $\sqrt{(4\times6)}$ or $2\sqrt{6}$ or $3\sqrt{2}\sqrt{3}$seen M1 for attempt to multiply num. and denom. by $5 + \sqrt{7}$ and M1 for 18 or $25 - 7$ seen	5
8	$x(30 - 2x) = 112$ $x(15 - x) = 56$ or $30x - 2x^2 = 112$ $(x - 7)(x - 8)$ $x = 7$ or 8 7 by 16 or 8 by 14	M1 A1 1 1 1	allow M1 for length = $30 - 2x$ soi NB answer given 0 for formula or completing sq etc must be explicit; both values required allow for 16 and 14 found following 7 and 8; both required	5
9	$[y=]\ 3x + 2 = 3x^2 - 7x + 1$ $[0 =]\ 3x^2 - 10x - 1$ or $-3x^2 + 10x + 1$ $x = \dfrac{10 \pm \sqrt{100+12}}{6}$ $= \dfrac{10\pm\sqrt{112}}{6}$ or $\dfrac{5\pm\sqrt{28}}{3}$ o.e. isw	M1 M1 M1 A2	or rearrangement of linear and subst for x in quadratic attempted condone one error; dep on first M1 attempt at formula [dep. on first M1 and quadratic = 0]; M2 for whole method for completing square or M1 to stage before taking roots A1 for two of three 'terms' correct [with correct fraction line] or for one root	5

Section B

10	**i**	$(x-4)^2+9$	3	B1 for 4, B2 for 9 or M1 for 25 – 16	3
	ii	(4, 9) or ft	1+1		
		parabola right way up 25 at intersection on y-axis (mark intent)	G1 G1	condone stopping at y axis ignore posn of min: can ft theirs	4
	iii	$x > 7$ or $x < 1$	3	M1 for $x^2 - 8x + 7$ [> 0] and M1 for $(x-7)(x-1)$ [>0] <u>or</u> M1 for $(x-4)^2$ [>] 9 and M1 for $x - 4 > 3$ and $x - 4 < -3$ or B2 for 1 and 7	3
	iv	[y =] $x^2 - 8x + 5$	1	or [y =] $(x-4)^2 - 11$	1
11	**i**	$(6-0)^2 + (10-2)^2$ AC = 10 AB = √98 and BC = √2 clear correct use of Pythagoras's theorem	M1 A1 1 1	or 1 for grad AB = 1 and grad BC = –1 and 1 for comment/ showing $m_1m_2 = -1$ o.e.	4
	ii	[angle in a semicircle so]AC diameter [so radius = 5] midpt of AC = (6/2, [10+2]/2) $(x-3)^2 + (y-6)^2 = 5^2$ o.e. isw	1 1 2	d or diameter needed; NB ans given method must be shown; NB ans givn B1 for one side correct	4
	iii	[grad AC =] 8/6 or 4/3 grad tgt = –3/4 $y - 10 = [-3/4](x-6)$ o.e. [e.g. $3x + 4y = 58$] or ft (58/3, 0) and (0, 58/4) o.e. isw	1 M1 M1 A2	for grad tgt = –1/their grad AC or M1 for y = their $m\,x + c$ then subst (6, 10) to find c 1 each cao; condone not as coords	5
12	**i**	$(x+1)(x-2)(x-5)$ $(x+1)(x^2-7x+10)$ correct step shown towards completion [answer given]	M1 A1 A1	o.e. with two other factors; condone missing brackets if expanded correctly; A2 for $x^3 - 5x^2 - 2x^2 + x^2 + 10x - 5x - 2x + 10$	3
	ii	cubic the right way up –1, 2 and 5 indicated on x axis 10 indicated at intn on y axis	G1 G1 G1	must extend beyond $x = -1$ and 5 at intersections of curve and axis	3
	iii	f(4) attempted = 64 – 96+ 12 + 10 attempt at long division of $x^3 - 6x^2 + 3x + 20$ by $x - 4$ as far as $x^3 - 4x^2$ in working $x^2 - 2x - 5 = 0$	M1 A1 M2 A2	or f(4) + 10; or '4 a root implies (x − 4) a factor' or vv or 5 × 2 × –1 etc or correct long division if first M1 earned or M2 for $(x-4)(x^2 + .. - 5)$ or $(x-4)(x^2 - 2x + k)$ seen; M1 for realising long divn by $x - 4$ needed but not doing it A1 for $x^2 - 2x - 5$ SC2 for finding f(x) ÷ (x − 4) = $x^2 - 2x - 5$ rem − 10 without further explanation	6

Mark Scheme 4752
June 2005

Section A

1	$1+\frac{3}{2}x^{\frac{1}{2}}$	1+3	B2 for $kx^{\frac{1}{2}}$, or M1 for $x^{\frac{3}{2}}$ seen before differentiation or B1 ft their $x^{\frac{3}{2}}$ correctly differentiated	4
2	1170	4	B1 for a = 11 and B1 for d = 5 or 20th term = 106 and M1 for 20/2[their (a) + their(106)] or 20/2[2their (a)+ (20-1)×their(d)] <u>OR</u> M1 for 6×20 and M2 for $5\left(\frac{20}{2}[20+1]\right)$ o.e.	4
3	$\pm\sqrt{13}/4$	3	B2 for (-) $\sqrt{13}/4$ or $\pm\sqrt{\frac{13}{16}}$ or M1 for $\sqrt{13}$ or $\sin^2\theta + \cos^2\theta = 1$ used	3
4	$x + x^{-1}$ soi $y' = 1 - 1/x^2$ subs x = 1 to get y' = 0 $y''=2x^{-3}$ attempted Stating y'' > 0 so min cao	B1 B1 B1 M1ft A1	$1 - x^{-2}$ is acceptable Or solving $1 - x^{-2} = 0$ to obtain x = 1 or checking y' before and after x = 1 Valid conclusion First quadrant sketch scores B2	5
5	(i) 1 (ii) –2 (iii) 6log x	1 2 2	 M1 for $1/9=3^{-2}$ or $\log(1) - \log(3^2)$ base not requd; M1 for 5 log x or $\log(x^6)$	5
6	Correct curve thro' y axis (0, 1) indicated on sketch or table 5.64	G1 G1 3	y, y' & y'' all positive independent B2 for other versions of 5.64(3....) or B1 for other ans 5.6 to 5.7 or M1 for x log 2 = log 50 and M1 for x = log 50 ÷ log 2	5
7	$y = 7 - 3/x^2$ oe	5	B3 for (y =) $-3/x^2 + c$ [B1 for each of k/x^2, $k = -6/2$ and $+c$] and M1 for substituting (1, 4) in their attempted integration with + c, the constant of integration	5
8	(i) 66° or 66.4 or 66.5…. 293.58 …. to 3 or more sf cao (ii) stretch (one way) parallel to the x-axis sf 0.5	B1 B1 1 1 1	Allow 1.16 or 73.8 Lost for extras in range. Ignore extras outside the range Horizontal, from y axis, in x axis, oe	5
				36

Section B

9	**i**	$3x^2 - 20x + 12$	2	B1 if one error "+c" is an error	2
	ii	$y - 64 = -16(x - 2)$ o.e. eg $y = -16x + 96$	4	M1 for subst $x = 2$ in their y' A1 for $y' = -16$ and B1 for $y = 64$	4
	iii	Factorising $f(x)$ $\equiv (x+2)(x-6)^2$ OR Expanding $(x+2)(x-6)^2$	B3 M2 E1	or B1 for $f(-2) = -8-40-24+72 = 0$ and B1 for $f'(6) = 0$ and B1dep for $f(6)=0$	3
	iv	$\frac{x^4}{4} - \frac{10x^3}{3} + 6x^2 + 72x$ value at ($x = 6$) ~ value at ($x = -2$) 341(.3..) cao	B2 M1 A1	-1 for each error Must have integrated $f(x)$	4
10	**i**	AB = 7.8(0), 7.798 to 7.799 seen area = 52.2 to 52.3	2 2	M1 for correct use of sine rule For long methods M1A1 for art 7.8 M1 for [2×][0.5 ×] their AB × 11.4 × sin 36°	4
	ii	tan 0.91 = ST/12.6 ST = 12.6 × tan 0.91 and completion (16.208...) area OSTR = [2×][0.5 ×]12.6 × their(16.2) nb 204. …. area of sector = $0.5 \times 12.6^2 \times 1.82$ =144.47… Logo = 59.6 to 60.0 arc = 12.6 × 1.82 [=22.9...] perimeter = 55.3 to 55.4	M1 E1 M1 M1 A1 A1 M1 A1	 Accept 16.2 if ST is explicit but for long methods with pa check that their explicit expression = 16.2 oe using degrees soi by correct ans Accept 144, 144.5 oe using degrees	8
11	**i**	81	1		1
	ii	$(1\text{x})3^{n-1}$	1		1
	iii	(GP with) $a = 1$ and $r = 3$ clear correct use GP sum formula	M1 M1	or M1 for $= 1+3+9+ \ldots +3^{n-1}$	2
	iv	(A) 6 www (B) 243	2 1	M1 for $364 = (3^n - 1)/2$	3
	v	their (ii) > 900 $(y - 1)\log 3 > \log 900$ $y - 1 > \log 900 \div \log 3$ $y = 8$ cao	M1ft M1ft M1 B1	-1 once for = or < seen: condone wrong letter / missing brackets / no base	4

Mark Scheme 4753
June 2005

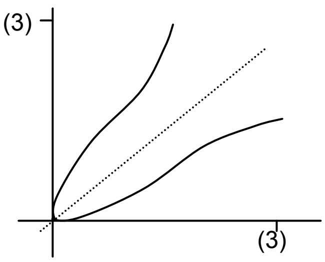
Section A

1 $3x + 2 = 1 \Rightarrow x = -1/3$ $3x + 2 = -1$ $\Rightarrow x = -1$	B1 M1 A1	$x = -1/3$ from a correct method – must be exact
or $(3x + 2)^2 = 1$ $\Rightarrow 9x^2 + 12x + 3 = 0$ $\Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x + 1)(x + 1) = 0$ $\Rightarrow x = -1/3$ or $x = -1$	M1 B1 A1 [3]	Squaring and expanding correctly $x = -1/3$ $x = -1$
2 $x = ½$ $\cos\theta = ½$ $\Rightarrow \theta = \pi/3$	B1 M1 A1 [3]	 M1A0 for 1.04… or 60°
3 $\text{fg}(x) = \ln(x^3)$ $= 3\ln x$ Stretch s.f. 3 in y direction	M1 A1 B1 [3]	$\ln(x^3)$ $= 3\ln x$
4 $T = 30 + 20e^0 = 50$ $dT/dt = -0.05 \times 20e^{-0.05t} = -e^{-0.05t}$ When $t = 0$, $dT/dt = -1$ When $T = 40$, $40 = 30 + 20e^{-0.05t}$ $\Rightarrow e^{-0.05t} = ½$ $\Rightarrow -0.05t = \ln ½$ $\Rightarrow t = -20 \ln ½ = 13.86..$ (mins)	B1 M1 A1cao M1 M1 A1cao [6]	50 correct derivative -1 (or 1) substituting $T = 40$ taking lns correctly or trial and improvement – one value above and one below or 13.9 or 13 mins 52 secs or better www condone secs

5 $\int_0^1 \frac{x}{2x+1}\,dx$ let $u = 2x + 1$		
$\Rightarrow du = 2dx,\ x = \frac{u-1}{2}$	M1	Substituting $\frac{x}{2x+1} = \frac{u-1}{2u}$ o.e.
When $x = 0$, $u = 1$, when $x = 1$, $u = 3$	A1	$\frac{1}{4}\int \frac{u-1}{u}\,du$ o.e. [condone no du]
$= \int_1^3 \frac{\frac{1}{2}(u-1)}{u}\frac{1}{2}\,du = \frac{1}{4}\int_1^3 \frac{u-1}{u}\,du$	B1	converting limits
$= \frac{1}{4}\int_1^3 (1-\frac{1}{u})\,du$		
$= \frac{1}{4}\left[u - \ln u\right]_1^3$	M1	dividing through by u
$= \frac{1}{4}[3 - \ln 3 - 1 + \ln 1]$	A1	$\frac{1}{4}\left[u - \ln u\right]$ o.e. – ft their ¼ (only)
$= ¼\,(2 - \ln 3)$	E1 [6]	must be some evidence of substitution
6 $y = \frac{x}{2+3\ln x}$		
$\Rightarrow \frac{dy}{dx} = \frac{(2+3\ln x).1 - x.\frac{3}{x}}{(2+3\ln x)^2}$	M1 B1 A1	Quotient rule consistent with their derivatives or product rule + chain rule on $(2+3x)^{-1}$; $\frac{d}{dx}(\ln x) = \frac{1}{x}$ soi; correct expression
$= \frac{2+3\ln x - 3}{(2+3\ln x)^2}$		
$= \frac{3\ln x - 1}{(2+3\ln x)^2}$		
When $\frac{dy}{dx} = 0$, $3\ln x - 1 = 0$	M1	their numerator = 0 (or equivalent step from product rule formulation) M0 if denominator = 0 is pursued
$\Rightarrow \ln x = 1/3$		
$\Rightarrow x = e^{1/3}$	A1cao	$x = e^{1/3}$
$\Rightarrow y = \frac{e^{1/3}}{2+1} = \frac{1}{3}e^{1/3}$	M1 A1cao [7]	substituting for their x (correctly); Must be exact: – 0.46… is M1A0
7 $y^2 + y = x^3 + 2x$		
$x = 2 \Rightarrow y^2 + y = 12$	M1	Substituting $x = 2$
$\Rightarrow y^2 + y - 12 = 0$		
$\Rightarrow (y-3)(y+4) = 0$	A1	$y = 3$
$\Rightarrow y = 3$ or -4.	A1	$y = -4$
$2y\frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$	M1	Implicit differentiation – LHS must be correct
$\Rightarrow \frac{dy}{dx}(2y+1) = 3x^2 + 2$		
$\Rightarrow \frac{dy}{dx} = \frac{3x^2+2}{2y+1}$	A1cao	
At (2, 3), $\frac{dy}{dx} = \frac{12+2}{6+1} = 2$	M1	substituting $x = 2$, $y = 3$ into their dy/dx, but must require both x and one of their y to be substituted
	A1 cao	2
At (2, –4), $\frac{dy}{dx} = \frac{12+2}{-8+1} = -2$	A1 cao [8]	–2

Section B

8 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6}\sin\frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow$ Q($\pi/6$, $\pi/6$) lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x\sin 3x$ $\Rightarrow \frac{dy}{dx} = x.3\cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6}.3\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ = gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	d/dx (sin $3x$) = 3cos $3x$ Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative = 1 ft dep 1st M1 = gradient of $y = x$ (www)
(iv) Area under curve = $\int_0^{\frac{\pi}{6}} x\sin 3x dx$ Integrating by parts, $u = x$, dv/dx = sin3x $\Rightarrow v = -\frac{1}{3}\cos 3x$ $\int_0^{\frac{\pi}{6}} x\sin 3x dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3}\cos 3x dx$ $= -\frac{1}{3}.\frac{\pi}{6}\cos\frac{\pi}{2} + \frac{1}{3}.0.\cos 0 + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{9}$ Area under line = $\frac{1}{2}\times\frac{\pi}{6}\times\frac{\pi}{6} = \frac{\pi^2}{72}$ So area required = $\frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$ *	 M1 A1cao A1ft M1 A1 B1 E1 [7]	 Parts with $u = x$ dv/dx = sin $3x$ $\Rightarrow$ $v = -\frac{1}{3}\cos 3x$ [condone no negative] $\ldots + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

9 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 +- x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ $= \frac{1}{u}.2x = \frac{2x}{1+x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule $1/u$ soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for g(x) $= 0 \le x \le \ln 10$ $y = \ln(1 + x^2)$ $x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow$ $e^x = 1 + y^2$ $\Rightarrow$ $e^x - 1 = y^2$ $\Rightarrow$ $y = \sqrt{(e^x - 1)}$ so $g(x) = \sqrt{(e^x - 1)}$* *or* $g\ f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	g(x) is f(x) reflected in $y = x$ Reasonable shape and domain, i.e. no –ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$* www forming g f(x) or f g(x) $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = ½ (e^x - 1)^{-1/2} . e^x$ $\Rightarrow g'(\ln 5) = ½ (e^{\ln 5} - 1)^{-1/2} . e^{\ln 5}$ $= ½ (5 - 1)^{-1/2} . 5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	$½ u^{-1/2}$ soi $\times e^x$ substituting ln 5 into g′ - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.

Mark Scheme 4754
June 2005

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 − 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓
   - For error in follow through work, use ✓

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not been earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR − 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

c.a.o.	: correct answer only
b.o.d.	: benefit of doubt (where full work is not shown)
X ... X	: work of no mark value between crosses
s.o.i.	: seen or implied
s.c.	: special case (as defined in the mark scheme)
w.w.w	: without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting prior to the batch 1 date, send a further sample of about 40 scripts, from complete centres. You should record the marks for these scripts on your marksheets. They will not be returned to you, but you will receive feedback on them. If all is well, you will then be given clearance to send your batch 1 scripts and marksheets to Cambridge.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

SECTION A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25,\ R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.927$ $f(\theta) = 7 + 5\cos(\theta - 0.927)$ $\Rightarrow$ Range is 2 to 12 Greatest value of $\dfrac{1}{7+3\cos\theta+4\sin\theta}$ is ½ .	**B1** **M1** **A1** **M1** **A1** **B1ft** **[6]**	*R*=5 tan α=4/3 oe ft their *R* 0.93 or 53.1° or better their cos (θ - 0. 927) = 1 or -1 used (*condone use of graphical calculator)* 2 and 12 seen cao simplified
2 $\sqrt{4+2x} = 2(1+\frac{1}{2}x)^{\frac{1}{2}}$ $= 2\{1+\frac{1}{2}.(\frac{1}{2}x)+\frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(\frac{1}{2}x)^2+\frac{\frac{1}{2}.(-\frac{1}{2}).(-\frac{3}{2})}{3!}(\frac{1}{2}x)^3+...\}$ $= k\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3+...\right)$ $= \left(2+\frac{1}{2}x-\frac{1}{16}x^2+\frac{1}{64}x^3+...\right)$ Valid for $-2 < x < 2$.	**M1** **M1** **A2,1,0** **A1cao** **B1cao** **[6]**	Taking out 4 oe correct binomial coefficients $\frac{1}{4}x, -\frac{1}{32}x^2, +\frac{1}{128}x^3$
3 $\sec^2\theta = 4$ $\Rightarrow \dfrac{1}{\cos^2\theta} = 4$ $\Rightarrow \cos^2\theta = ¼$ $\Rightarrow \cos\theta = ½$ or $-½$ $\Rightarrow \theta = \pi/3, 2\pi/3$ **OR** $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow \tan^2\theta = 3$ $\Rightarrow \tan\theta = \sqrt{3}$ or $-\sqrt{3}$ $\Rightarrow \theta = \pi/3, 2\pi/3$	**M1** **M1** **A1 A1** **M1** **M1** **A1 A1** **[4]**	sec θ = 1/cos θ used ± ½ allow unsupported answers ± √3 allow unsupported answers

4 $V = \int \pi y^2 dx$ $= \int_0^1 \pi(1+e^{-2x})dx$ $= \pi\left[x - \frac{1}{2}e^{-2x}\right]_0^1$ $= \pi(1 - ½ e^{-2} + ½)$ $= \pi(1½ - ½ e^{-2})$	**M1** **M1** **B1** **M1** **A1** **[5]**	Correct formula $k\int_0^1 (1+e^{-2x})dx$ $\left[x - \frac{1}{2}e^{-2x}\right]$ substituting limits. Must see *0* used. Condone omission of π o.e. but must be exact
5 $2\cos 2x = 2(2\cos^2 x - 1) = 4\cos^2 x - 2$ $\Rightarrow 4\cos^2 x - 2 = 1 + \cos x$ $\Rightarrow 4\cos^2 x - \cos x - 3 = 0$ $\Rightarrow (4\cos x + 3)(\cos x - 1) = 0$ $\Rightarrow \cos x = -3/4$ or 1 $\Rightarrow x = 138.6°$ or $221.4°$ or 0	**M1** **M1** **M1dep** **A1** **B1 B1** **B1** **[7]**	Any double angle formula used *getting a quadratic in cos x* attempt to solve for -3/4 and 1 139,221 or better www -1 extra solutions in range
6 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$ $= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$ $= 4$	**M1** **E1** **[2]**	Substituting for *x* and *y* in terms of *t* oe
(ii) EITHER $dx/dt = 1 + 1/t^2$, $dy/dt = 1 - 1/t^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1-1/t^2}{1+1/t^2}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ * **OR** $2y\frac{dy}{dx} - 2x = 0$ $\Rightarrow \frac{dy}{dx} = \frac{x}{y} = \frac{t-1/t}{t+1/t}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ **OR** $y=\sqrt{(4+x^2)}$, $\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{4+x^2}}$ $= \frac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$ $= \frac{t-1/t}{\sqrt{(t+1/t^2)}} = \frac{t-1/t}{(t+1/t)}$ $= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ $\Rightarrow dy/dx = 0$ when $t = 1$ or -1 $t = 1, \Rightarrow (0, 2)$ $t = -1 \Rightarrow (0, -2)$	**B1** **M1** **E1** **B1** **M1** **E1** **B1** **M1** **E1** **M1** **A1 A1** **[6]**	For both results

SECTION B

7 (i) $\int \frac{t}{1+t^2}dt = ½ \ln(1+t^2) + c$ **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, $du = 2t dt$ $= \int \frac{1/2}{u} du$ $= ½ \ln u + c$ $= ½ \ln(1 + t^2) + c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k \ln(1 + t^2)$ $½ \ln(1 + t^2)$ $[+c]$ substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow$ $1 = A(1+t^2) + (Bt + C)t$ $t = 0 \Rightarrow 1 = A$ coeff[t] of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff[t] of t $\Rightarrow 0 = C$ $\Rightarrow$ $\frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	**M1** **M1** **A1** **A1** **A1** **[5]**	Equating numerators substituting or equating coeff[t]s dep 1[st] M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow$ $\int \frac{1}{M} dM = \int \frac{1}{t(1+t^2)} dt = \int [\frac{1}{t} - \frac{t}{1+t^2}] dt$ $\Rightarrow$ $\ln M = \ln t - ½ \ln(1 + t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow$ $M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	**M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½ \ln(1 + t^2) + c$ combining $\ln t$ and $½ \ln(1 + t^2)$ $K = e^c$ o.e.
(iv) $t = 1, M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow$ $K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.
8 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (−15, 20, 30) $\Rightarrow$ $\overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow$ $\overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$ $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$ $\Rightarrow$ $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane	**M1** **E1**	Scalar product with 1 vector in the plane OR vector x product oe
$\Rightarrow$ equation of plane is $2x + 3y + 2z = c$	**M1**	$2x + 3y + 2z = c$ **or** an appropriate vector form
At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow$ $c = 2\times0 + 3\times10 + 2\times30 = 90$	**M1dep**	substituting to find c **or** completely eliminating parameters
$\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**A1 cao** **[5]**	
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$	**B1**	
$\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$	**M1**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S
$=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$	**A1ft**	**Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
So T is $(-5, 16\frac{2}{3}, 25)$ *	**E1** **[4]**	
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$	**B1,B1**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\dots \dots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$
At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$	**M1** **A1**	Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ
1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**E1** **[5]**	oe www

COMPREHENSION

1. The masses are measured in units. The ratio is dimensionless	**B1** **B1** **[2]**	
2. Converting from base 5, $3.03232 = 3 + \frac{0}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5}$ $= 3.14144$	**M1** **A1** **[2]**	
3. (table below)	**B1**	Condone variations in last digits
4. $\frac{\phi}{1} = \frac{1}{\phi - 1}$ $\Rightarrow \phi^2 - \phi = 1 \quad \Rightarrow \phi^2 - \phi - 1 = 0$ Using the quadratic formula gives $\phi = \frac{1 \pm \sqrt{5}}{2}$	**M1** **E1**	Or complete verification B2
5. $\frac{1}{\phi} = \frac{1}{\frac{1+\sqrt{5}}{2}} = \frac{2}{1+\sqrt{5}}$ $= \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$ $= \frac{2(\sqrt{5}-1)}{(\sqrt{5})^2 - 1} = \frac{2(\sqrt{5}-1)}{4} = \frac{\sqrt{5}-1}{2}$ **OR** $\frac{1}{\phi} = \phi - 1$ $= \frac{\sqrt{5}+1}{2} - 1 = \frac{\sqrt{5}-1}{2}$	**M1** **M1** **E1** **M1** **M1** **E1** **[3]**	Must discount ± Must discount ± Substituting for ϕ and simplifying

Table for question 3:

n	x_n
0	0.5
1	0.8
2	0.512
3	**0.7995392**
4	**0.5128840565**
5	**0.7994688035**

6. Let $r = \dfrac{a_{n+1}}{a_n}$ and $r = \dfrac{a_n}{a_{n-1}}$	**M1**	For either ratio used
$a_{n+1} = 2a_n + 3a_{n-1}$		
dividing through by $a_{n\cdot}$ $\Rightarrow r = 2 + \dfrac{3}{r}$	**M1**	$r = 2 + \dfrac{3}{r}$
$\Rightarrow r^2 - 2r - 3 = 0$ $\Rightarrow (r-3)(r+1) = 0$	**A1**	
$\Rightarrow$ $r = 3$ (discounting -1)	**E1** **[4]**	SC B2 if dividing terms as far as $a_9/a_8 = a_{10}/a_9$ =3.00
7. The length of the next interval = l, where $\dfrac{0.0952...}{l} = 4.669...$	**M1**	
$\Rightarrow$ $l = 0.0203$	**A1**	
So next bifurcation at 3.5437… + 0.0203…≈ 3.564	**DM1** **A1** **[4]**	

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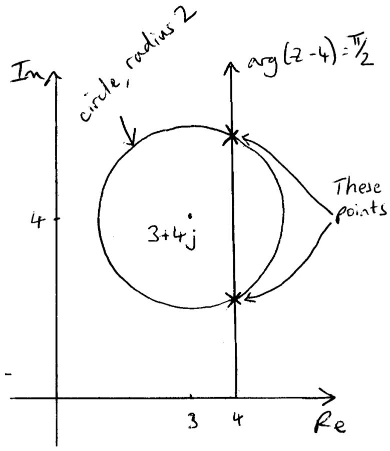
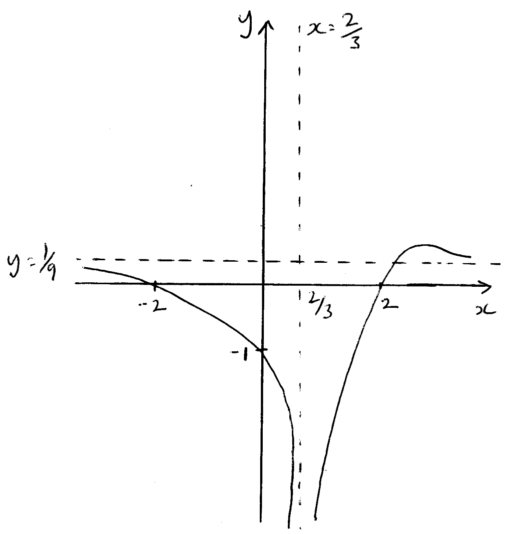
Section A			
1(i)	$\mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$	M1 A1	Dividing by determinant
1(ii)	$\frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 22 \\ -21 \end{pmatrix}$ $\Rightarrow x = \frac{22}{5},\ y = \frac{-21}{5}$	M1 A1(ft), A1(ft) **[5]**	Pre-multiplying by their inverse Follow through use of their inverse No marks for solving without using inverse matrix
2	$4 - \mathrm{j},\ 4 + \mathrm{j}$ $\sqrt{17}\left(\cos 0.245 + \mathrm{j}\sin 0.245\right)$ $\sqrt{17}\left(\cos 0.245 - \mathrm{j}\sin 0.245\right)$	M1 A1 **[2]** M1 F1, F1 **[3]**	Use of quadratic formula Both roots correct Attempt to find modulus and argument One mark for each root Accept $(r,\ \theta)$ form Allow any correct arguments in radians or degrees, including negatives: 6.04, 14.0°, 346°. Accuracy at least 2s.f. S.C. F1 for consistent use of their incorrect modulus or argument (not both, F0)
3	$\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x = 3x - y,\ y = 2x$ $\Rightarrow y = 2x$	M1 A1 A1 **[3]**	M1 for $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ (allow if implied) $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} k \\ mk \end{pmatrix} = \begin{pmatrix} K \\ mK \end{pmatrix}$ can lead to full marks if correctly used. Lose second A1 if answer includes two lines
4(i) **4(ii)** **4(iii)**	$\alpha + \beta = 2,\ \alpha\beta = 4$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 8 = -4$ Sum of roots = $2\alpha + 2\beta = 2(\alpha + \beta) = 4$ Product of roots = $2\alpha \times 2\beta = 4\alpha\beta = 16$ $x^2 - 4x + 16 = 0$	B1 M1A1 (ft) M1 A1(ft) **[5]**	Both Accept method involving calculation of roots Or substitution method, or method involving calculation of roots The = 0, or equivalent, is necessary for final A1

5(i)	Sketch of Argand diagram with: Point $3+4j$. Circle, radius 2.	 B1 B1 **[2]**	Circle must not touch either axis. B1 max if no labelling or scales. Award even if centre incorrect.
5(ii)	Half-line: Starting from (4, 0) Vertically upwards	 B1 B1 **[2]**	
5(iii)	Points where line crosses circle clearly indicated. 	 B1 **[1]**	Identifying 2 points where their line cuts the circle

Qu	Answer	Mark	Comment
Section A (continued)			
6	For $k=1,\ 1^3=1$ and $\frac{1}{4}1^2(1+1)^2=1$, so true for $k=1$ Assume true for $n=k$ Next term is $(k+1)^3$ Add to both sides RHS = $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ $=\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ $=\frac{1}{4}(k+1)^2(k+2)^2$ $=\frac{1}{4}(k+1)^2((k+1)+1)^2$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k=1$ it is true for *k* = 1, 2, 3, … .	B1 B1 B1 M1 M1 A1 E1 **[7]**	Assuming true for *k*, (*k* +1)th term - for alternative statement, give this mark if whole argument logically correct Add to both sides Factor of $(k+1)^2$ Allow alternative correct methods For fully convincing algebra leading to true for *k* $\Rightarrow$ true for *k* + 1 Accept 'Therefore true by induction' only if previous A1 awarded S.C. Give E1 if convincing explanation of induction following acknowledgement of earlier error
7	$3\sum r^2-3\sum r$ $=3\times\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)[(2n+1)-3]$ $=\frac{1}{2}n(n+1)(2n-2)$ $=n(n+1)(n-1)$	M1,A1 M1,A1 M1 A1 c.a.o. **[6]**	Separate sums Use of formulae Attempt to factorise, only if earlier M marks awarded Must be fully factorised
			Section A Total: 36

Qu	Answer		Comment
Section B			
8(i)	$x = \frac{2}{3}$ and $y = \frac{1}{9}$	B1, B1 **[2]**	-1 if any others given. Accept min of 2s.f. accuracy
8(ii)	Large positive *x*, $y \to \frac{1}{9}^{+}$ (e.g. consider $x = 100$) Large negative *x*, $y \to \frac{1}{9}^{-}$ (e.g. consider $x = -100$)	M1 A1 E1 **[3]**	Approaches horizontal asymptote, not inconsistent with their (i) Correct approaches Reasonable attempt to justify approaches
8(iii)	Curve $x = \frac{2}{3}$ shown with correct approaches $y = \frac{1}{9}$ shown with correct approaches (from below on left, above on right). (2, 0), (-2, 0) and (0, -1) shown 	B1(ft) B1(ft) B1(ft) B1 B1 **[5]**	1 for each branch, consistent with horizontal asymptote in (i) or (ii) Both *x* intercepts *y* intercept (give these marks if coordinates shown in workings, even if not shown on graph)
8(iv)	$-1 = \frac{x^2 - 4}{(3x-2)^2} \Rightarrow -9x^2 + 12x - 4 = x^2 - 4$ $\Rightarrow 10x^2 - 12x = 0$ $\Rightarrow 2x(5x - 6) = 0$ $\Rightarrow x = 0$ or $x = \frac{6}{5}$ From sketch, $y \geq -1$ for $x \leq 0$ and $x \geq \frac{6}{5}$	M1 A1 B1 F1 **[4]**	Reasonable attempt at solving inequality Both values – give for seeing 0 and $\frac{6}{5}$, even if inequalities are wrong For $x \leq 0$ Lose only one mark if any strict inequalities given

9(i)	2 - j 2j	B1 B1 **[2]**	
9(iii)	$(x-2-\mathrm{j})(x-2+\mathrm{j})(x+2\mathrm{j})(x-2\mathrm{j})$ $=(x^2-4x+5)(x^2+4)$ $=x^4-4x^3+9x^2-16x+20$ So A = -4, B = 9, C = -16 and D = 20	M1, M1 A1,A1 A4 **[8]**	M1 for each attempted factor pair A1 for each quadratic - follow through sign errors Minus 1 each error – follow through sign errors only
OR	$-\mathrm{A}=\sum\alpha=4\Rightarrow \mathrm{A}=-4$ $\mathrm{B}=\sum\alpha\beta=9\Rightarrow \mathrm{B}=9$ $-\mathrm{C}=\sum\alpha\beta\gamma=16\Rightarrow \mathrm{C}=-16$ $\mathrm{D}=\sum\alpha\beta\gamma\delta=20\Rightarrow \mathrm{D}=20$	M1, A1 M1, A1 M1, A1 M1, A1 **[8]**	M1s for reasonable attempt to find sums S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values
OR	Attempt to substitute two correct roots into $x^4+Ax^3+Bx^2+Cx+D=0$ Produce 2 correct equations in two unknowns A = -4, B = 9, C = -16, D = 20	M1 M1 A2 A4	One for each root One for each equation One mark for each correct. S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values

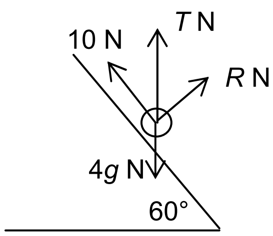
10(i)	$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \sum_{r=1}^{n}\left[\frac{1}{r} - \frac{2}{(r+1)} + \frac{1}{(r+2)}\right]$	M1	Give if implied by later working
	$= \left(\frac{1}{1} - \frac{2}{2} + \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right) +$	M1	Writing out terms in full, at least three terms
	$........ + \left(\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right)$	A2	All terms correct. A1 for at least two correct
	$= \frac{1}{1} - \frac{2}{2} + \frac{1}{2} + \frac{1}{n+1} - \frac{2}{n+1} + \frac{1}{n+2}$	M1 A3	Attempt at cancelling terms Correct terms retained (minus 1 each error)
	$= \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$		
	$= \frac{1}{2} - \frac{1}{(n+1)(n+2)}$	M1	Attempt at single fraction leading to given answer.
		[9]	
10(ii)	$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + ...$		
	$= \frac{1}{2}\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$	M1 M1	M1 relating to previous sum, M1 for recognising that $\frac{1}{(n+1)(n+2)} \to 0$ as $n \to \infty$ (could be implied)
	$\Rightarrow \frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + ... = \frac{1}{4}$	A1	
		[3]	

Mark Scheme 4761
June 2005

Q 1		mark		Sub
(i)	Acceleration is 8 m s $^{-2}$ speed is $0+0.5\times4\times8$ = 16 m s $^{-1}$	B1 B1		2
(ii)	a = 2t	B1		1
(iii)	t = 7 a > 0 for t < 7 and a < 0 for t > 7	B1 E1	 Full reason required	2
(iv)	Area under graph $0.5\times2\times8-0.5\times1\times4=6$ so 6 m s $^{-1}$ Increase	M1 B1 E1	Both areas under graph attempted. Accept both positive areas. If 2×3 seen accept ONLY IF reference to average accn has been made. Award for $v=-2t^2+28t+c$ seen or 24 and 30 seen Award if 6 seen. Accept '24 to 30'. This must be clear. Mark dept. on award of M1	3
	total	8		

Q 2		mark		Sub
(i)	$a=24-12t$	M1 A1	Differentiate cao	2
(ii)	Need $24t-6t^2=0$ t = 0, 4	M1 A1	Equate v = 0 and attempt to factorise (or solve). Award for one root found. Both. cao.	2
(iii)	$s=\int_0^4\left(24t-6t^2\right)\,\mathrm{d}t$ = $\left[12t^2-2t^3\right]_0^4$ $\left(12\times16-2\times64\right)-0$ = 64 m	M1 A1 M1 A1	Attempt to integrate. No limits required. Either term correct. No limits required Sub t = 4 in integral. Accept no bottom limit substituted or arb const assumed 0. Accept reversed limits. FT **their** limits. cao. Award if seen. [If trapezium rule used. M1 At least 4 strips: M1 enough strips for 3 s. f. A1 (dep on 2nd M1) One strip area correct: A1 cao]	4
	total	8		

Q 3		mark		Sub
(i)	$\mathbf{R} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	M1	Sum to zero	
	$\mathbf{R} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$	A1	Award if seen here or in (ii) or used in (ii).	
			[SC1for $\begin{pmatrix} 18 \\ -3 \end{pmatrix}$]	2
(ii)	$\lvert \mathbf{R} \rvert = \sqrt{18^2 + 3^2}$	M1	Use of Pythagoras	
	= 18.248… so 18.2 N (3 s. f.)	A1	Any reasonable accuracy. FT **R** (with 2 non-zero cpts)	
	angle is $180 - \arctan\left(\frac{3}{18}\right)$ = 170.53…°	M1	Allow arctan $\left(\frac{\pm 3}{\pm 18}\right)$ or arctan $\left(\frac{\pm 18}{\pm 3}\right)$	
	so 171° (3 s. f.)	A1	Any reasonable accuracy. FT **R** provided their angle is obtuse but not 180°	4
	total	6		

Q 4		mark		Sub
(i)		B1	All forces present. No extras. Accept *mg*, *w* etc. All labelled with arrows. Accept resolved parts only if clearly additional. Accept no angles	1
(ii)	Resolve parallel to the plane 10 + *T* cos 30 = 4*g* cos 30	M1	All terms present. Must be resolution in at least 1 term. Accept $\sin \leftrightarrow \cos$. If resolution in another direction there must be an equation only in *T* with no forces omitted. No extra forces.	
		A1	All correct	
	T = 27.65299… so 27.7 N (3 s. f.)	A1	Any reasonable accuracy	3
(iii)	Resolve perpendicular to the plane *R* + 0.5 *T* = 2*g*	M1	At least one resolution correct . Accept resolution horiz or vert if at least 1 resolution correct. All forces present. No extra forces.	
		A1	Correct. FT *T* if evaluated.	
	R = 5.7735… so 5.77 N (3 s. f.)	A1	Any reasonable accuracy. cao.	3
	total	7		

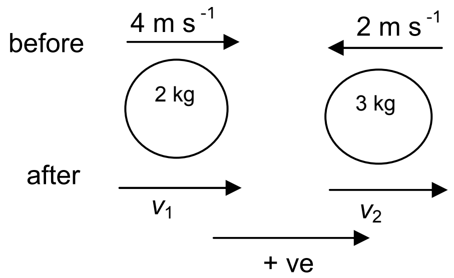
Q 5		mark		Sub
(i)	$x = 2 \Rightarrow t = 4$ $t = 4 \Rightarrow y = 16 - 1 = 15$	B1 F1	cao FT **their** *t* and *y*. Accept 15 **j**	2
(ii)	$x = \frac{1}{2}t$ and $y = t^2 - 1$ Eliminating *t* gives $y = \left((2x)^2 - 1\right) = 4x^2 - 1$	M1 E1	Attempt at elimination of expressions for *x* and *y* in terms of *t* Accept seeing $(2x)^2 \text{-} 1 = 4x^2 - 1$	2
(iii)	**either** We require $\frac{dy}{dx}$ = 1 so 8*x* = 1 $x = \frac{1}{8}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$ **or** Differentiate to find **v** equate **i** and **j** cpts so $t = \frac{1}{4}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	 M1 B1 A1 M1 M1 A1	 This may be implied Differentiating correctly to obtain 8*x* Equating the **i** and **j** cpts of **their v**	 3
	total	7		

Q 6		mark		Sub
(i)	$2000 = 1000a$ so $a = 2$ so 2 m s^{-2}	B1		
		M1	Use of appropriate *uvast* for *t*	
	$12.5 = 5 + 2t$ so $t = 3.75$ so 3.75 s	A1	cao	3
(ii)	$2000 - R = 1000 \times 1.4$	M1	N2L. Accept $F = mga$. Accept sign errors. Both forces present. Must use $a = 1.4$	
	$R = 600$ so 600 N (AG)	E1		2
(iii)	$2000 - 600 - S = 1800 \times 0.7$	M1	N2L overall or 2 paired equations. $F = ma$ and use 0.7. Mass must be correct. Allow sign errors and 600 omitted.	
		A1	All correct	
	$S = 140$ so 140 N (AG)	E1	Clearly shown	3
(iv)	$T - 140 = 800 \times 0.7$	M1	N2L on trailer (or car). $F = 800a$ (or $1000a$). Condone missing resistance otherwise all forces present. Condone sign errors.	
		B1	Use of 140 (or 2000 – 600) and 0.7	
	$T = 700$ so 700 N	A1		3
(v)	N2L in direction of motion car and trailer			
	$-600 - 140 - 610 = 1800a$	M1	Use of $F = 1800a$ to find new accn. Condone 2000 included but not *T*. Allow missing forces.	
		A1	All forces present; no extra ones Allow sign errors.	
	$a = -0.75$	A1	Accept $\pm$. cao.	
	For trailer $T - 140 = -0.75 \times 800$	M1	N2Lwith their a ($\neq 0.7$) on trailer or car. Must have correct mass and forces. Accept sign errors	
	so $T = -460$ so 460	A1	cao. Accept ± 460	
	thrust	F1	Dep on M1. Take tension as +ve unless clear other convention	6
	total	17		

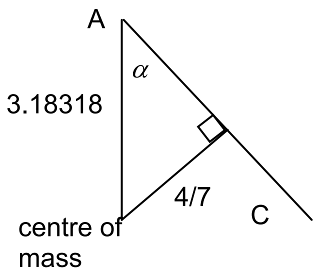
Q 7		mark		Sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$	B1	Accept any accuracy 2 s. f. or better	
	$\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3s.f.)	M1	Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** 15.62cosθ = 10 or **their** 15.62sinθ = 12)	
		A1	[FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u$cosθ = 10 and $u$sinθ = 12 seen]	3
(ii)	vert $12t - 0.5 \times 10t^2 + 9$	M1	Use of $s = ut + 0.5at^2$, $a = \pm 9.8$ or ± 10 and u = 12 or 15.62.. Condone $-9 = 12t - 0.5 \times 10t^2$, condone $y = 9 + 12t - 0.5 \times 10t^2$. Condone g.	
		A1	All correct with origin of u = 12 clear; accept 9 omitted	
	= $12t - 5t^2 + 9$ (AG)	E1	Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	
	horiz $10t$	B1		4
(iii)	$0 = 12^2 - 20s$	M1	Use of $v^2 = u^2 + 2as$ or equiv with u = 12, v = 0. Condone $u \leftrightarrow v$	
	s = 7.2 so 7.2 m	A1	From CWO. Accept 16.2.	2
(iv)	We require 0 = $12t - 5t^2 + 9$ Solve for t the + ve root is 3 range is 30 m	M1 M1 A1 F1	Use of y equated to 0 Attempt to solve a 3 term quadratic Accept no reference to other root. cao. FT root and **their** x. [If range split up M1 all parts considered; M1 valid method for each part; A1 final phase correct; A1]	4
(v)	Horiz displacement of B: $20\cos 60t$ = $10t$	B1	Condone unsimplified expression. Award for 20cos60 = 10	
	Comparison with Horiz displacement of A	E1	Comparison clear, must show $10t$ for each or explain.	2
(vi)	vertical height is $20\sin 60t - 0.5 \times 10t^2$ = $10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and 20sin60 = $10\sqrt{3}$ not explained.	1
(vii)	Need $10\sqrt{3}t - 5t^2$ = $12t - 5t^2 + 9$	M1	Equating the **given** expressions	
	$\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$	A1	Expression for t obtained in any form	
	t = 1.6915… so 1.7 s (2 s. f.) (AG)	E1	Clearly shown. Accept 3 s. f. or better as evidence. Award M1 A1 E0 for 1.7 sub in each ht	3

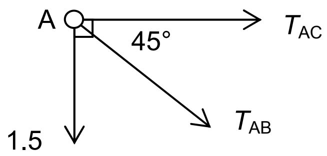
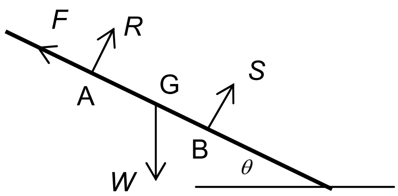
	total	19			

Mark Scheme 4762
June 2005

Q 1		mark		Sub
(a) (i)	240 **i** N s $\rightarrow$	B1		1
(ii) (A)	240 **i** = 70 **i** +50 **v** so **v** = 3.4 **i** m s^{-1}	M1 A1	Equating to **their** 240 **i** in this part FT 240 **i**	
(B)	240 **i** = 70u **i** – 50u **i**	M1	Must have u in both RHS terms and opposite signs	
	u = 12 so **v** = $-$12 **i** m s^{-1}	A1	FT 240 **i**	
(C)	240 **i** = 280(**i** + **j**) +50$\mathbf{v}_B$	M1	FT 240 **i** Must have all terms present	
	so $\mathbf{v}_B$ = (-0.8 **i** – 5.6 **j**) m s^{-1}	A1	cao	6
(b) (i)				
	NEL $\dfrac{v_2 - v_1}{-2-4} = -0.5$	M1	NEL	
	so $v_2 - v_1 = 3$	A1	Any form	
	PCLM $8-6 = 2v_1 + 3v_2$	M1 A1	PCLM Any form	
	Solving v_2 = 1.6 so 1.6 m s^{-1} $\rightarrow$ v_1 = $-$1.4 so 1.4 m s^{-1} $\leftarrow$	A1 A1	Direction must be clear (accept diagram) Direction must be clear (accept diagram). [Award A1 A0 if v_1 & v_2 correct but directions not clear]	6
(ii)	1.6 m s^{-1} at 60° to the wall (glancing angles both 60°)	B1 B1	FT **their** 1.6	
	No change in the velocity component parallel to the wall as no impulse	E1	Must give reason	
	No change in the velocity component perpendicular to the wall as perfectly elastic	E1	Must give reason	4
	total	17		

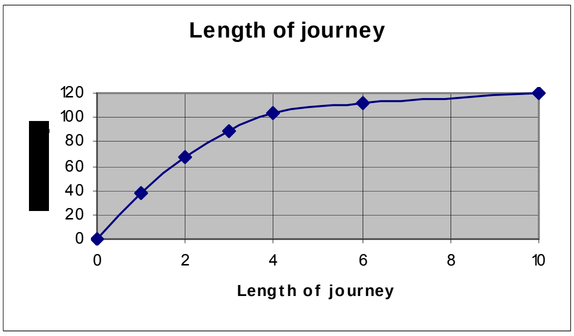
Q 2		**mark**		**Sub**
(i)	We need $\frac{mgh}{t} = \frac{850 \times 9.8 \times 60}{20}$ = 24 990	M1	Use of $\frac{mgh}{t}$	
	so approx 25 kW	E1	Shown	2
(ii)	Driving force – resistance = 0	B1	May be implied	
	$25000 = 800v$	M1	Use of $P = Fv$	
	so v = 31.25 and speed is 31.25 m s $^{-1}$	A1		3
(iii)	Force is $\frac{25000}{10} = 2500$ N	B1		
	N2L in direction of motion			
	$2500 - 800 = 850a$	M1	Use of N2L with all terms	
	a = 2 so 2 m s $^{-2}$	A1		3
(iv)	$0.5 \times 850 \times 20^2 = 0.5 \times 850 \times 15^2$	M1	W-E equation with KE and power term	
		B1	One KE term correct	
	$+25000 \times 6.90$	B1	Use of *Pt* .Accept wrong sign	
	$-800x$	B1	WD against resistance. Accept wrong sign	
		A1	All correct	
	x = 122.6562… so 123 m (3 s. f.)	A1	cao	6
(v)	**either**			
	$0.5 \times 850 \times v^2 = 0.5 \times 850 \times 20^2$	M1	W-E equation inc KE, GPE and WD	
	$-850 \times 9.8 \times \frac{105}{20}$	M1	GPE term with attempt at resolution	
		A1	Correct. Accept expression. Condone wrong sign.	
	-800×105	B1	WD term. Neglect sign.	
	$v^2 = 99.452...$ so 9.97 m s $^{-1}$	A1	cao	
	or			
	N2L + ve up plane			
	$-(800 + 850g \times 0.05) = 850a$	M1	N2L. All terms present. Allow sign errors.	
	a = -1.43117…	A1	Accept $\pm$	
	$v^2 = 20^2 + 2 \times (-1.43117...) \times 105$	M1	Appropriate *uvast*. Neglect signs.	
		A1	All correct including consistent signs. Need not follow sign of a above.	
	$v^2 = 99.452...$ so 9.97 m s $^{-1}$	A1	cao	5
		19		

Q 3		mark		Sub
(i)	$28\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=16\begin{pmatrix}2\\2\end{pmatrix}+2\begin{pmatrix}5\\0\end{pmatrix}+2\begin{pmatrix}6\\1\end{pmatrix}+2\begin{pmatrix}5\\2\end{pmatrix}$ $+2\begin{pmatrix}0\\5\end{pmatrix}+2\begin{pmatrix}1\\6\end{pmatrix}+2\begin{pmatrix}2\\5\end{pmatrix}$	M1 B1 B1	Complete method Total mass correct 3 c. m. correct (or 4 *x*- or *y*-values correct)	
	$\bar{x}=2.5$	A1		
	$\bar{y}=2.5$	A1		
			[Allow A0 A1 if only error is in total mass] [If $\bar{x}=\bar{y}$ claimed by symmetry and only one component worked replace final A1, A1 by B1 explicit claim of symmetry A1 for the 2.5]	5
(ii)	$\bar{x}=\bar{y}$	B1	Or by direct calculation	
		M1	Dealing with 'folded' parts for $\bar{x}$ or for $\bar{z}$	
	$28\bar{x}=16\times2+6\times4+2\times0+2\times1+2\times2$	A1	At least 3 terms correct for $\bar{x}$	
	$\bar{x}=\frac{31}{14}$ (2.21428…)	A1		
	$\bar{z}=\frac{8\times(-1)+4\times(-2)}{28}=-\frac{4}{7}$ (- 0.57142…)	A1	All terms correct allowing sign errors	
		A1		
	Distance is $\sqrt{\left(\frac{31}{14}\right)^2+\left(\frac{31}{14}\right)^2+\left(\frac{4}{7}\right)^2}$	M1	Use of Pythagoras in 3D on **their** c.m.	
	= 3.18318.. so 3.18 m (3 s. f.)	F1		8
(iii)		M1	c.m. clearly directly below A	
		B1	Diagram showing α and known lengths (or equivalent). FT their values. Award if final answer follows **their** values.	
	$\sin\alpha=\frac{4}{7}/3.18318..$	M1	Appropriate expression for α. FT **their** values.	
	so α = 10.3415… so 10.3° (3 s. f.)	A1	cao	4
	total	17		

Q 4		mark		Sub
(a) (i)	Moments c.w. about A $2R = 5L$ so $R = 2.5L$ Resolve $\rightarrow$ $U = 0$ Resolve $\uparrow$ $V + R = L$ so $V = -1.5L$	 E1 E1 M1 E1	 Resolve vertically or take moments about B (or C)	 4
(ii)	 For equilibrium at A $\uparrow$ $T_{AB}\cos 45 + 1.5L = 0$ so $T_{AB} = -\dfrac{3\sqrt{2}L}{2}$ so $\dfrac{3\sqrt{2}L}{2}$ N (C) in AB $\rightarrow$ $T_{AC} + T_{AB}\cos 45 = 0$ so $T_{AC} = \dfrac{3L}{2}$ so $\dfrac{3L}{2}$ N (T) in AC At C $\downarrow$ $L + T_{BC}\cos\theta = 0$ $\tan\theta = 3/2 \Rightarrow \cos\theta = 2/\sqrt{13}$ so $T_{BC} = -\dfrac{\sqrt{13}L}{2}$ so $\dfrac{\sqrt{13}L}{2}$ N (C) in BC	M1 M1 A1 F1 M1 B1 A1 F1	Equilibrium at a pin-joint Attempt at equilibrium at A or C including resolution with correct angle (2.12L (3 s. f.)) (1.5L) Must include attempt at angle (1.80 L (3 s. f.)) Award for T/C correct from their internal forces. Do not award without calcs	 8
(b) (i)		B1	All forces present with arrows and labels. Angles and distances not required.	 1
(ii)	c.w.moments about B $R \times 3 - W \times 1\cos\theta = 0$ so $R = \dfrac{1}{3}W\cos\theta$	 M1 A1 A1	 If moments about other than B, then need to resolve perp to plank as well Correct	 3
(iii)	Resolve parallel to plank $F = W\sin\theta$ $\mu = \dfrac{F}{R} = \dfrac{W\sin\theta}{\frac{1}{3}W\cos\theta} = 3\tan\theta$	 B1 M1 A1	 Use of $F = \mu R$ and their F and R Accept any form.	 3
	total	19		

Mark Scheme 4766
June 2005

Statistics 1 (4766)

Qn	Answer	Mk	Comment
1 (i)	Mean = 657/20 = 32.85	B1 cao	
	Variance = $\frac{1}{19}\left(22839 - \frac{657^2}{20}\right) = 66.13$	M1 A1 cao	
(ii)	Standard deviation = 8.13		
	32.85 + 2(8.13) = 49.11	M1 ft	Calculation of 49.11
	none of the 3 values exceed this so no outliers	A1 ft	
2 (i)		G1 G1 G1	For calculating 38,68,89,103,112,120 Plotting end points Heights inc (0,0)
(ii)	Median = 1.7 miles	B1	
	Lower quartile = 0.8 miles	M1	
	Upper quartile = 3 miles	M1	
	Interquartile range = 2.2 miles	A1 ft	
(iii)	The graph exhibits positive skewness	E1	

Statistics 1 (4766) June 2005 **Final Mark Scheme**

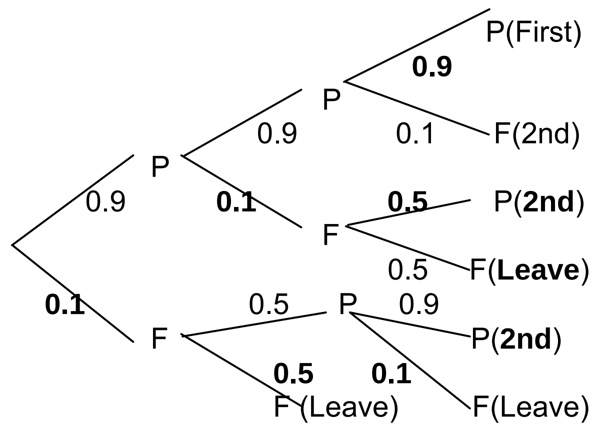
3 (i)	P(X = 4) = $\frac{1}{40}$(4)(5) = $\frac{1}{2}$ (Answer given)	B1	Calculation must be seen
(ii)	E(X) = (2+12+36+80)$\frac{1}{40}$ So E(X) = 3.25	M1 A1 cao	Sum of rp
	Var (X) = (2+24+108+320)$\frac{1}{40}$ - 3.25² = 11.35 – 10.5625 = 0.7875	M1 M1 dep A1 cao	Sum of r²p -3.25²
(iii)	Expected number of weeks = $\frac{6}{40}$x45 = 6.75 weeks	M1 A1	Use of np
4 (i)	Number of choices = $\binom{6}{3}$ = 20	M1 A1	For $\binom{6}{3}$
(ii)	Number of ways = $\binom{6}{3}\times\binom{7}{4}\times\binom{8}{5}$ = $20\times35\times56$ = 39200	M1 M1 A1 cao	Correct 3 terms Multiplied
(iii)	Number of ways of choosing 12 questions = $\binom{21}{12}$ = 293930 Probability of choosing correct number from each section = 39200/293930 = 0.133	M1 M1 ft A1 cao	For $\binom{21}{12}$

5 (i)

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	2	6	4	10	6
3	3	6	3	12	15	6
4	4	4	12	**4**	**20**	12
5	5	10	15	**20**	**5**	**30**
6	6	6	6	12	**30**	**6**

Part	Answer	Mark	Comment
5 (i)	Table above	B1	All correct
(ii)	(A) P(LCM > 6) = 1/3	B1	
	(B) P(LCM = 5n) = 11/36	B1	
	(C) P(LCM > 6 ∩ LCM = 5n) = 2/9	M1 A1 cao	Use of diagram
(iii)	$\frac{1}{3} \times \frac{11}{36} \neq \frac{2}{9}$	M1	Use of definition
	Hence events are not independent	E1	

Statistics 1 (4766)

6 (i)		G1 G1	Probabilities Outcomes
(ii) (A)	P(First team) = 0.9^3 = 0.729	M1 A1	
(B)	P(Second team) = $0.9\times0.9\times0.1+0.9\times0.1\times0.5+0.1\times0.9\times0.5$ = 0.081 + 0.045 + 0.045 = 0.171	M1 M1 A1	1 correct triple 3 correct triples added
(iii)	P(asked to leave) = 1 -0.729 – 0.171 = 0.1	B1	
(iv)	P(Leave after two games given leaves) = $\frac{0.1\times0.5}{0.1}$ = $\frac{1}{2}$	M1 ft A1 cao	Denominator
(v)	P(at least one is asked to leave) $=1-0.9^3$ = 0.271	M1 ft M1 A1 cao	Calc'n of 0.9 1 – ( )³
(vi)	P(Pass a total of 7 games) =P(First, Second, Second) + P(First, First, Leave after three games) = $3\times0.729\times0.171^2$ + $3\times0.729^2\times0.05$ = 0.064 + 0.080 = 0.144	M1 M1 ft M1 ft M1 A1 cao	Attempts both 0.729(0.171)² 0.05(0.729)² multiply by 3

7 (i)	$X \sim B\left(15, \frac{1}{6}\right)$ $P(X=0) = \left(\frac{5}{6}\right)^{15} = 0.065$	M1 A1 cao	$\left(\frac{5}{6}\right)^{15}$
(ii)	$P(X=4) = \binom{15}{4} \times \left(\frac{1}{6}\right)^4 \times \left(\frac{5}{6}\right)^{11}$ $= 0.142$ (or 0.9102-0.7685)	M1 M1 A1 cao	$\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{11}$ multiply by $\binom{15}{4}$
(iii)	$P(X > 3) = 1 - P(X \le 3)$ $= 1 - 0.7685 = 0.232$	M1 A1	
(iv) (A)	Let p = probability of a six on any throw $H_0 : p = \frac{1}{6}$ $H_1 : p < \frac{1}{6}$ $X \sim B\left(15, \frac{1}{6}\right)$ $P(X=0) = 0.065$ 0.065 < 0.1 and so reject H_0 Conclude that there is sufficient evidence at the 10% level that the dice are biased against sixes.	B1 B1 M1 M1 dep E1 dep	Definition of p Both hypotheses 0.065 Comparison
(B)	Let p = probability of a six on any throw $H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$ $X \sim B\left(15, \frac{1}{6}\right)$ $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.910 = 0.09$ 0.09 < 0.1 and so reject H_0 Conclude that there is sufficient evidence at the 10% level that the dice are biased in favour of sixes.	B1 M1 M1 dep E1 dep	Both hypotheses 0.09 Comparison
(v)	Conclusions contradictory. Even if null hypothesis is true, it will be rejected 10% of the time purely by chance. Or other sensible comments.	E1 E1	Contradictory By chance

Mark Scheme 4767
June 2005

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

B marks are independent of all others. They are usually awarded for a single correct answer. Typically they are available for correct quotation of points such as 1.96 from tables.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.

- Please indicate incorrect working by ringing or underlining as appropriate.

- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.

- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.

- Probabilities should be given as fractions, decimals or percentages.

- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".

- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.

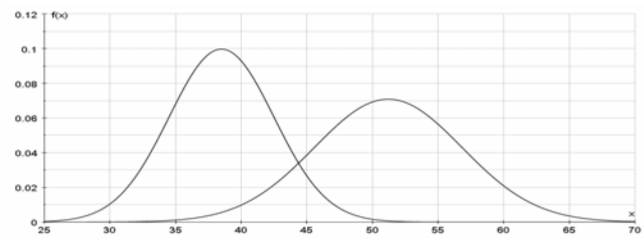
- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
ISW	Ignore subsequent working

Question 1

(i)	Uniform average rate of occurrence; Successive arrivals are independent. Suitable arguments for/against each assumption: Eg Rate of occurrence could vary depending on the weather (any reasonable suggestion)	E1,E1 for suitable assumptions E1, E1 must be in context	**4**
(ii)	Mean = $\frac{\Sigma xf}{n}$ = $\frac{39+40+36+32+15}{100}$ = $\frac{162}{100}$ = 1.62 Variance = $\frac{1}{n-1}\left(\Sigma fx^2 - n\bar{x}^2\right)$ = $\frac{1}{99}\left(430-100\times1.62^2\right)$ = 1.69 (to 2 d.p.)	B1 for mean ***NB answer given*** M1 for calculation A1	**3**
(iii)	Yes, since mean is close to variance	B1FT	**1**
(iv)	P(X = 2) = $e^{-1.62}\frac{1.62^2}{2!}$ = 0.260 (3 s.f.) *Either:* Thus the expected number of 2's is 26 which is reasonably close to the observed value of 20. *Or*: This probability compares reasonably well with the relative frequency 0.2	M1 for probability calc. M0 for tables unless interpolated A1 B1 for expectation of 26 or r.f. of 0.2 E1	**4**
(v)	λ = 5×1.62 = 8.1 Using tables: P($X \geq$ 10) = 1 – P($X \leq$ 9) = 1 – 0.7041 = 0.2959	B1FT for mean (SOI) M1 for probability from using tables to find 1 – P($X \leq$ 9) A1 FT	**3**
(vi)	Mean no. of items in 1 hour = 360 × 1.62 = 583.2 Using Normal approx. to the Poisson, X ~ N(583.2, 583.2): P($X \leq$ 550.5) = P$\left(Z \leq \frac{550.5-583.2}{\sqrt{583.2}}\right)$ = P($Z \leq$ -1.354) = 1 - Φ(1.354) = 1 – 0.9121 = 0.0879 (3 s.f.)	B1 for Normal approx. with correct parameters (SOI) B1 for continuity corr. M1 for probability using correct tail A1 CAO, **(**but FT wrong or omitted CC)	**4**
			19

Question 2

(i)	$X \sim N(38.5,16)$ $P(X > 45) = P\left(Z > \frac{45-38.5}{4}\right)$ $= P(Z > 1.625)$ $= 1 - \Phi(1.625) = 1 - 0.9479$ $= 0.0521$ (3 s.f.) *or* 0.052 (to 2 s.f.)	M1 for standardizing A1 for 1.625 M1 for prob. with tables and correct tail A1 CAO (min 2 s.f.)	**4**
(ii)	From tables $\Phi^{-1}(0.90) = 1.282$ $\frac{x-38.5}{4} = -1.282$ $x = 38.5 - 1.282 \times 4 = 33.37$ So 33.4 should be quoted	B1 for 1.282 seen M1 for equation in x and negative z-value A1 CAO	**3**
(iii)	$Y \sim N(51.2, \sigma^2)$ From tables $\Phi^{-1}(0.75) = 0.6745$ $\frac{55-51.2}{\sigma} = 0.6745$ $3.8 = 0.6745\,\sigma$ $\sigma = 5.63$	B1 for 0.6745 seen M1 for equation in σ with z-value A1 ***NB answer given***	**3**
(iv)		G1 for shape G1 for means, shown explicitly or by scale G1 for lower max height in diesel G1 for higher variance in diesel	**4**
(v)	$P(\text{Diesel} > 45) = P\left(Z > \frac{45-51.2}{5.63}\right)$ $= P(Z > -1.101) = \Phi(1.101) = 0.8646$ P(At least one over 45) = 1 – P(Both less than 45) $= 1 - (1 - 0.0521) \times (1 - 0.8646)$ $= 1 - 0.9479 \times 0.1354 = 0.8717$ NB allow correct alternatives based on: P(D over, P under)+P(D under, P over)+ P(both over) *or* P(D over) + P(P over) – P(both over)	M1 for prob. calc. for diesel M1 for correct structure M1*dep* for correct probabilities A1 CAO (2 s.f. min)	**4**
			18

Question 3

(i)	$\bar{x} = 4.5$, $\bar{y} = 26.85$ $b = \dfrac{Sxy}{Sxx} = \dfrac{983.6 - 36 \times 214.8/8}{204 - 36^2/8} = \dfrac{17}{42} = 0.405$ OR $b = \dfrac{983.6/8 - 4.5 \times 26.85}{204/8 - 4.5^2} = \dfrac{2.125}{5.25} = 0.405$ hence least squares regression line is: $y - \bar{y} = b(x - \bar{x})$ $\Rightarrow \quad y - 26.85 = 0.405(x - 4.5)$ $\Rightarrow \quad y = 0.405x + 25.03$	B1 for $\bar{x}$ and $\bar{y}$ used (SOI) M1 for attempt at gradient (b) A1 for 0.405 **cao** M1 *indep* for equation of line A1FT for complete equation	**5**
(ii)	$x = 4 \Rightarrow$ predicted $y = 0.405 \times 4 + 25.03 = 26.65$ Residual = 27.5 – 26.65 = 0.85	M1 for prediction A1FT for ± 0.85 B1FT for sign (+)	**3**
(iii)	The new equation would be preferable, since the equation in part (i) is influenced by the unrepresentative point (4,27.5)	B1 E1	**2**
(iv)	H_0: $\rho = 0$; H_1: $\rho > 0$ where ρ represents the population correlation coefficient Critical value at 5% level is 0.3783 Since 0.209 < 0.3783, there is not sufficient evidence to reject H_0, i.e. there is not sufficient evidence to conclude that there is any correlation between cycling and swimming times.	B1 for H_0 and H_1 B1 for defining ρ B1 for 0.3783 M1 for comparison leading to conclusion A1*dep on cv* for conclusion in words in context	**5**
(v)	Underlying distribution must be bivariate normal. The distribution of points on the scatter diagram should be approximately elliptical.	B1 E1	**2**
			17

Question 4

(a) (i)	H_0: μ = 166500; H_1: μ > 166500 Where μ denotes the mean selling price in pounds of the population of houses on the large estate	B1 for both correct B1 for definition of μ	**2**
(ii)	n = 6, Σx = 1018500, $\bar{x}$ = £169750 Test statistic = $\frac{169750-166500}{14200/\sqrt{6}} = \frac{3250}{5797}$ = 0.5606 5% level 1 tailed critical value of z = 1.645 0.5606 < 1.645 so not significant. There is insufficient evidence to reject H_0 It is reasonable to conclude that houses on this estate are not more expensive than in the rest of the suburbs.	B1CAO M1 must include √6 A1FT B1 for 1.645 M1 for comparison leading to a conclusion A1 for conclusion in words in context	**6**
(b)	H_0: no association between customer and drink types; H_1: some association between customer and drink types;	B1	

Observed		Type of drink: Alcoholic	Type of drink: Soft drinks	Row totals
Type of customer	Business	54	63	117
	Tourist	95	41	136
	Local	71	76	147
Column totals		220	180	400

Expected		Type of drink: Alcoholic	Type of drink: Soft drinks	Row totals
Type of customer	Business	64.35	52.65	117
	Tourist	74.80	61.20	136
	Local	80.85	66.15	147
Column totals		220	180	400

Chi squared contribution		Type of drink: Alcoholic	Type of drink: Soft drinks	Row totals
Type of customer	Business	1.665	2.035	3.699
	Tourist	5.455	6.667	12.122
	Local	1.200	1.467	2.667

	X^2 = 18.49	M1 A1 for expected values (to 2dp) M1 for valid attempt at $(O-E)^2/E$ M1dep for summation A1CAO for X^2	**6**
	Refer to Ξ_2^2 Critical value at 5% level = 5.991 Result is significant There is some association between customer type and type of drink. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not	B1 for 2 deg of f B1 CAO for cv B1*dep on cv* E1	**4**
			18

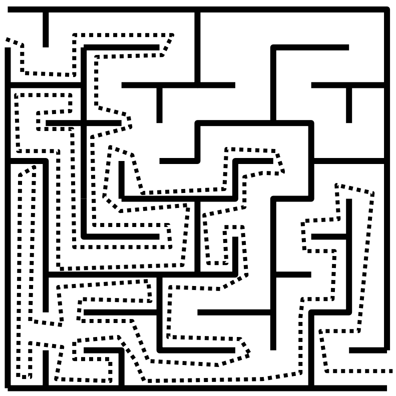
Mark Scheme 4771
June 2005

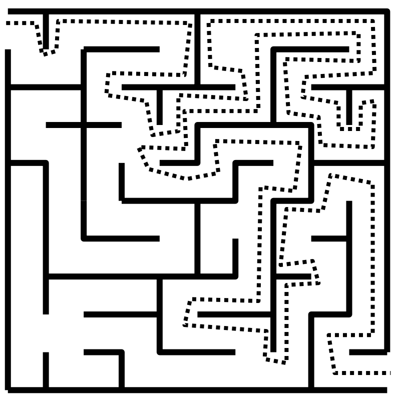
1.

(i)	Any connected tree.	M1 A1
	12 connections	B1
(ii)	14 connections	B1
(iii)	e.g. He might be able to save cable by using it. e.g. To avoid overloading.	B1
(iv)	Yes. A minimum connector is a tree. This gives the min number of arcs (n–1). This gives the minimum no of connections (2(n–1)).	B1 B1 B1

2.

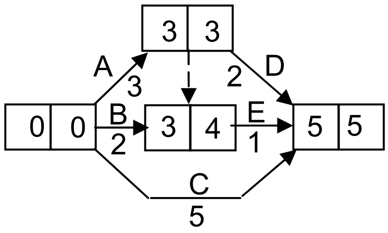
(i) Janet John



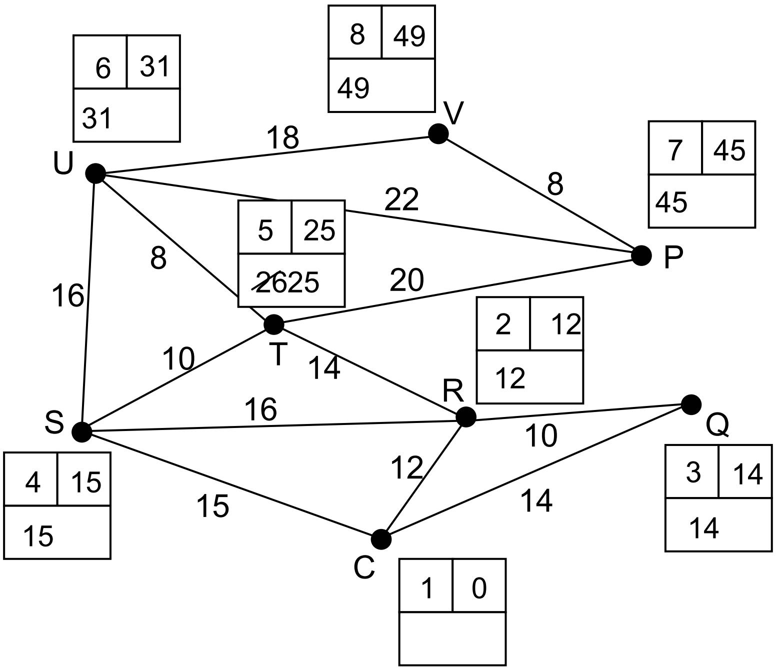


(i)	(maze diagrams above)	M1 A1 A1
(ii)	Yes Janet's route traces west and south walls plus "attachments". John's route traces north and east walls plus "attachments". – or equivalent (Any "islands" are irrelevant.)	M1 A1 B1
(iii)	Yes	B1
(iv)	Yes All avenues covered by forward and backward pass (i.e. by John's original route + Janet's route).	B1

3.

(i) 	M1 A1 B1 B1
(ii) Critical − A, D and C	B1
(iii) Total float for B = 2 Independent float for B = 1 Total float for E = 1 Independent float for E = 0	B1 both total floats A1 B's independent A1 E's independent

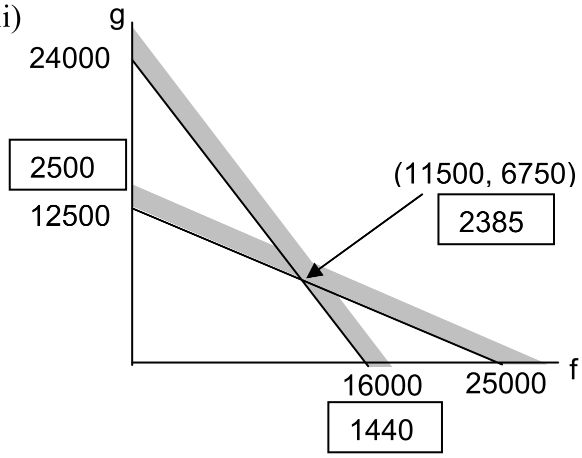
4.

(i)	B1 starting at C
	M1 Dijkstra A1 labels A1 order of labelling A1 working values
P Q R S T U V 45 14 12 15 25 31 49	B1
P T S C	B1
V U S C	B1
(ii) PV ST CR RT UV Length = 80 TU QR	M1 A1 first 5 A1 last 2 B1 length
(iii) CP reduced to 26 CV reduced to 34	B1 (both and no more)
(iv) UV replaced by PQ New length = 74	B1
(v) Q	M1
Semi-Eulerian. (Order of P changed from 3 to 4, but order of Q changed from 2 to 3 − so still 2 odd vertices.)	A1
or Cross the bridge and proceed as before or A valid route	

5.

(i)	eg. 00–19 → 0 20–49 → 1 50–69 → 2 70–84 → 3 85–99 → 4	M1 sca at proportions A1
(ii)	1, 0, 2, 3, 1, 3, 4, 3, 0, 0	M1 A1
(iii)	eg. 00–15 → 0 16–39 → 1 40–63 → 2 64–95 → 3 96–99 → ignore	M1 missing some A1 times
(iv)	1, 0, 1, 0, 1, 1, 3, 3, 2, 2	B1 one ignored B1 rest
(v)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 0 0 0 0 2 4 Disptd 0 0 0 0 1 0 2 1 0 0 0	M1 A1 A1
(vi)	Day 0 1 2 3 4 5 6 7 8 9 10 Stock 3 3 3 2 0 1 0 0 1 3 5 Disptd 0 0 0 0 0 0 0 1 0 0 0	M1 using both ret dists A1 A1
	Only 1 disappointed under new policy against 4 under old policy.	B1
	Not definitely, but pretty convincingly.	B1

6.

(i) Let f be the number of litres of Flowerbase produced Let g be the number of litres of Growmuch produced	B1
Max $9f + 20g$ s.t. $0.75f + 0.5g \leq 12000$ $f + 2g \leq 25000$	M1 A1 M1 A1 A1
(ii) 	B1 labels + scales B1 B1 lines B1 shading
Max profit = £2500 by producing 12500 litres of Growmuch	M1 A1
(iii) No effect	B1
(iv) No effect The profit on Flowerbase will be reduced by more than that suffered by Growmuch, since it uses more fibre. The objective gradient will thus increase from −9/20, making it even less attractive to produce any Flowerbase.	M1 A1
(v) £3000	B1

Mark Scheme 4772
June 2005

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

M marks can be implied by correct answers.

A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

B marks are for specific results or statements, and are independent of method.

$\surd$ marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of –1, –2 or –3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

SC special case

1.

(a)

(i) If sidelights and headlights are on, and if the foglights are switched on. — B1 B1

(ii) $\sim(\sim s \vee \sim h) \wedge f$ — M1 A1

(iii)

(s	$\wedge$	h)	$\wedge$	f	$\Leftrightarrow$	~	(~	s	$\vee$	~	h)	$\wedge$	f
0	0	0	0	0	1	0	1	0	1	1	0	0	0
0	0	0	0	1	1	0	1	0	1	1	0	0	1
0	0	1	0	0	1	0	1	0	1	0	1	0	0
0	0	1	0	1	1	0	1	0	1	0	1	0	1
1	0	0	0	0	1	0	0	1	1	1	0	0	0
1	0	0	0	1	1	0	0	1	1	1	0	0	1
1	1	1	0	0	1	1	0	1	0	0	1	0	0

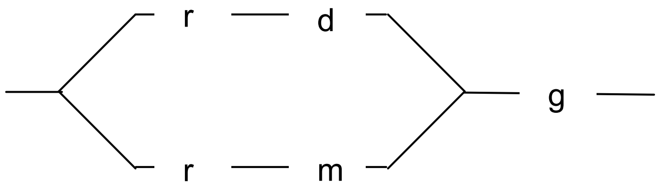
M1 8 rows
A1 $s \wedge h \wedge f$
A1 $\sim(\sim s \vee \sim h) \wedge f$

Accept t/table showing $s \wedge h = \sim(\sim s \vee \sim h)$

B1 comment re $\wedge f$
M1 4 lines
A1
alternativ

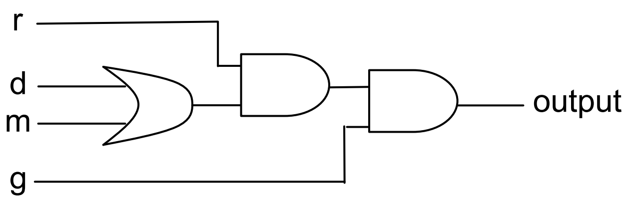
(b)

(i)



M1
A1

(ii)



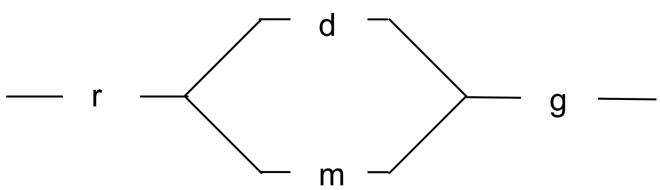
M1
A1 "or"
A1 first "and"
A1 second "and"

(iii) $r \wedge (d \vee m) \wedge g = (r \wedge (d \vee m)) \wedge g$ by associativity

$= ((r \wedge d) \vee (r \wedge m)) \wedge g$ by distributivity

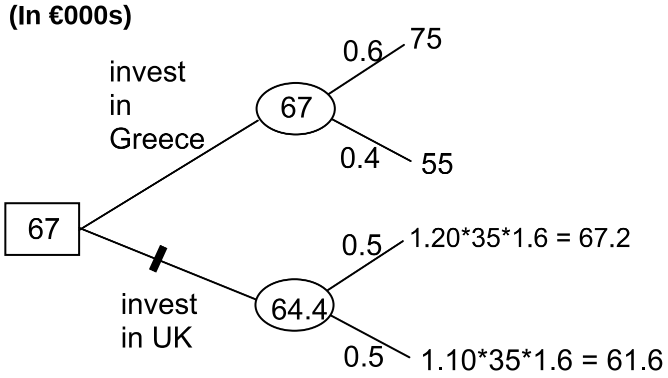
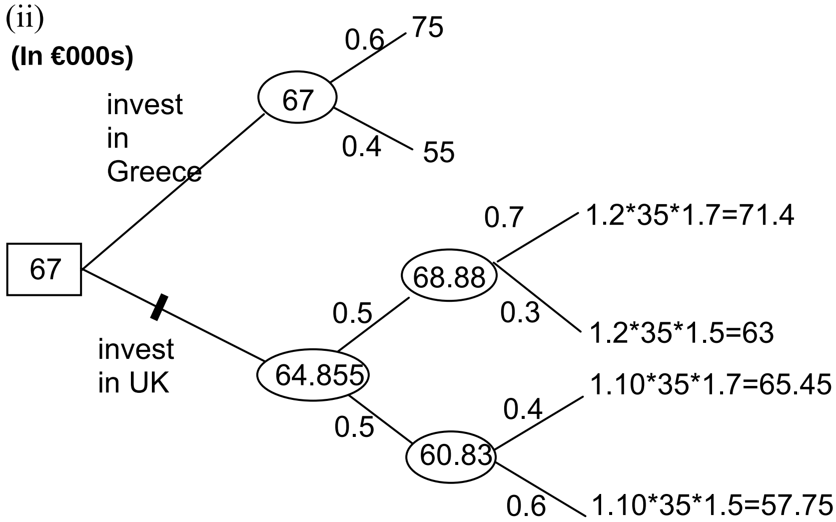
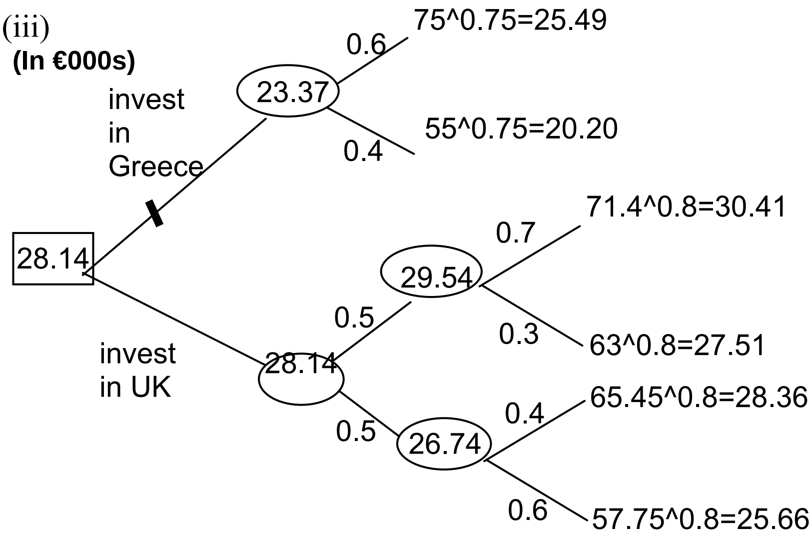
M1 distributive law
A1 handling brackets
(law names not needed)

(iv)



B1

2.

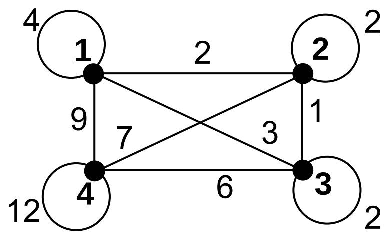
(i) **(In €000s)**  	M1 A1 chance nodes A1 choice node B1√ invest in Greece
(ii) **(In €000s)**  	M1 new chance nodes A1 64.855 or .86 or .85 B1 invest in Greece
(iii) **(In €000s)**  	M1 utilities A1 23.37 and 28.14 B1 invest in UK

2 (cont)

(iv) Require $\frac{1.2+1.1}{2}\times 35\times x = 67$, giving x = 1.665	M1 A1 cao
(v) Require $\frac{(1.2\times 35\times y)^{0.8}+(1.1\times 35\times y)^{0.8}}{2} = 23.37$. Trying y = 1.277: $(1.2\times 35\times 1.277)^{0.8} = 24.185$ $(1.1\times 35\times 1.277)^{0.8} = 22.559$ (24.185+22.559)/2 = 23.37	M1 cash M1 house A1 one bracket evaluated correctly A1

3.

(i)



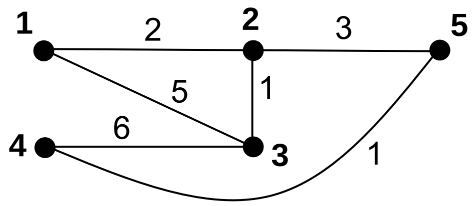
loops optional

M1
A1

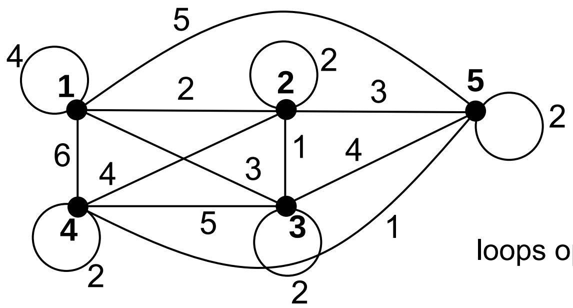
(ii) First vertex en route is 3.
First vertex en route from 3 to 1 is 2.
First vertex en route from 2 to 1 is 1.

M1
A1

(iii)



B1



loops optional

M1
A1

(iv)

	1	2	3	4	5
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

	1	2	3	4	5
1	2	2	2	2	2
2	1	3	3	5	5
3	2	2	2	2	2
4	5	5	5	5	5
5	2	2	2	4	4

B1 distance matrix

M1 route matrix
A1 cao

(v) 1 2 3 5 4 1
14
1 2 3 2 5 4 5 2 1

M1
A1
A1

(vi)

		1	2	4	3
	1	2	3	4	5
1	4	2	3	6	5
2	2	2	1	4	3
3	3	1	2	5	4
4	6	4	5	2	1
5	5	3	4	1	2

M1 Prim on matrix
A1

Lower bound is 5 + 2 + 3 = 10

B1 B1

(vii) e.g. 1 2 5 4 <u>3 2 3</u> 1 19	 M1 A1 cao B1

4.

(i) The objective is nonlinear. — B1

(ii)

P	x	y	S1	S2	S3	RHS
1	−1	1	0	0	0	0
0	1	0	1	0	0	10
0	0	1	0	1	0	6
0	1	−2	0	0	1	0
1	0	−1	0	0	1	0
0	0	2	1	0	−1	10
0	0	1	0	1	0	6
0	1	−2	0	0	1	0
1	0	0	1/2	0	1/2	5
0	0	1	1/2	0	−1/2	5
0	0	0	−1/2	1	1/2	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

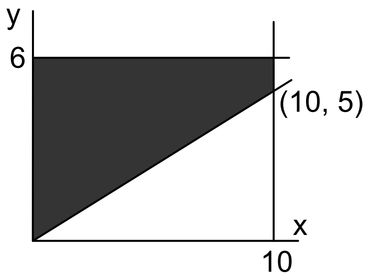
M1 tableau
A1

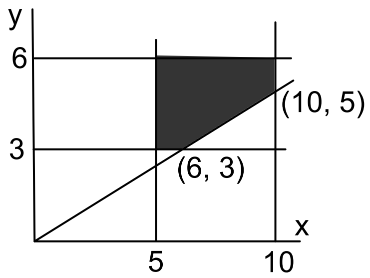
M1 pivot choice
A1 pivot

M1 pivot choice
A1 pivot

10 ml of oil and 5 ml of vinegar — B1

(iii)





B1 $x \le 10$ and $y \le 6$
B1 $5 \le x$ and $3 \le y$

B1 proportion line

B1 region 1
B1 region 2

(iv) Omitted constraints non-active — B1
(0, 0) not in feasible region. — B1

(v)

C	P	x	y	s1	s2	s3	s4	s5	a1	a2	RHS
1	0	1	1	0	–1	0	–1	0	0	0	8
0	1	1	–1	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0	10
0	0	1	0	0	–1	0	0	0	1	0	5
0	0	0	1	0	0	1	0	0	0	0	6
0	0	0	1	0	0	0	–1	0	0	1	3
0	0	1	–2	0	0	0	0	1	0	0	0

B1 > constraints

B1 artificial columns

B1 new objective

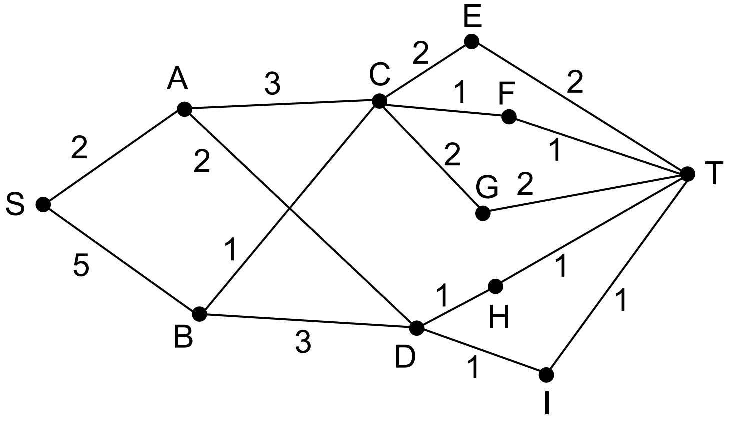
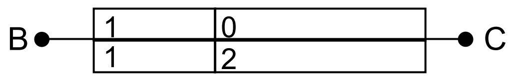
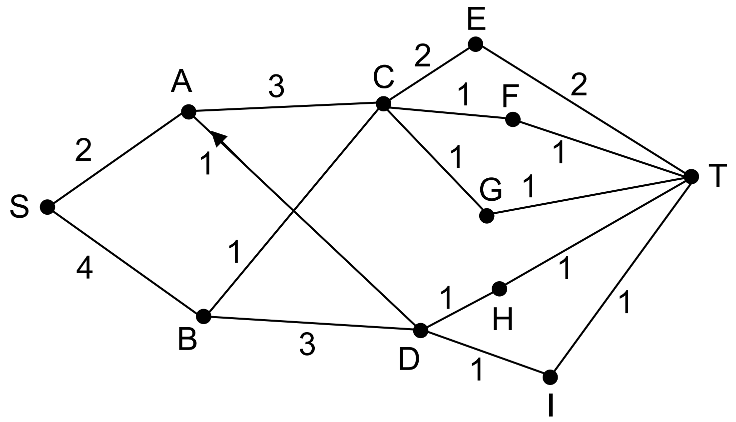
Minimise C, hopefully to zero. — B1
Thereafter delete C row and a1/a2 columns, and proceed as usual. — B1

Mark Scheme 4773
June 2005

Qu. 1

(i) 32*80 = 2560 calories	M1 A1
(ii) 3000/32 = 93.75 kg	M1 A1
(iii) Auxiliary equation is (3x−1)(3x−2) = 0 Solution is $u_n = 13.75(1/3)^n - 27.5(2/3)^n + 93.75$	M1 A1 M1 particular A1 93.75 or 3rd eqn M1 gen homogeneous A1 correct form B1 case 1 (u_0 = 80) + case 2 (u_1 = 80) M1 simultaneous A1 13.75 and −27.5 B1 final answer
(iv) 90 90 85.83333 81.66667 78.65741 76.80556 75.78961 75.28807 75.06873 74.98871 74.96962 74.97276 74.98119 74.98876 (Oscillatory) convergence to 75 kg.	M1 A1
(v) 90 90 82.77778 75.55556 70.33951 67.12963 65.36866 64.49931 64.11913 63.98043 63.94734 63.95278 63.9674 63.98052 63.98958	B1 B1

Qu. 2

(i) 	M1 A1
(ii) 	B1 1 and 1 B1 0 B1 2
(iii) S B D A C G (T)	M1 A1
(iv) 	M1 A1
(v) S B | A C D E F G H I T or S A B D | C E F G H I T	M1 a cut, properly specified A1 correct cut

(vi)	
Max SA + SB + SD	B1
st SA+CA+DA-AD-AC = 0	
SB+CB+DB-BC-BD = 0	M1
AC+BC-CA-CB-CE-CF-CG = 0	A1
SD+BD+AD-DA-DB-DH-DI = 0	
SA < 2	
SB < 5	
SD < 1	
AD < 2	
DA < 2	B1
BC < 1	
CB < 1	
AC < 3	
BD < 3	
CA < 3	
CE < 2	
CF < 1	
CG < 2	
DB < 3	
DH < 1	
DI < 1	
end	
(vii)	
OBJECTIVE FUNCTION VALUE	
1) 6.00000	
VARIABLE VALUE REDUCED COST	
SA 2.000000 0.000000	M1
SB 3.000000 0.000000	A1
SD 1.000000 0.000000	
CA 0.000000 1.000000	
DA 1.000000 0.000000	
AD 0.000000 0.000000	
AC 3.000000 0.000000	
CB 0.000000 1.000000	
DB 0.000000 0.000000	
BC 1.000000 0.000000	
BD 2.000000 0.000000	
CE 2.000000 0.000000	
CF 0.000000 0.000000	
CG 2.000000 0.000000	
DH 1.000000 0.000000	B1
DI 1.000000 0.000000	
Flows are as listed in the "VALUE" column.	

Qu. 3

(i)	Simulating service times	B1
	(=lookup(rand(),cum.probs,times))	B1
	Accumulating (expectation is 207.5 seconds)	
		B1
(ii)	Repetitions	B1
	Mean (not far off 207.5 seconds)	B1
	sd (order of magnitude 5 seconds)	M1
	$(2*1.96*s)^2$ = (about) 400 repetitions	A1
	(assuming a 95% confidence interval half-width of 0.5s)	
		M1
(iii)	Rand()*120	A1
	fixed	B1
	sorted	
		B1
(iv)	max(arrival time, gate available time)	B1
	+ service time	B1
	finish time approx as in (i)	
		M1 A1
(v)	Test barrier free times to see which barrier passenger uses.	M1
	Computation of barrier free times, eg:	A1
	=if(bar=1, max(arrival t + service t, bar t + service t), bar t)	B1
	finish time approx 130s	

Qu. 4

(i)

Sched.	City	Flight	City	Flight	City	Flight	City	Flight	City
S1	L	101	B	201	P	402	M	302	L
S2	L	101	B	201	P	403	L		
S3	L	101	B	202	M	302	L		
S4	L	101	B	202	M	301	P	403	L
S5	B	201	P	402	M	302	L	102	B
S6	B	201	P	402	M	303	B		
S7	B	201	P	403	L	102	B		
S8	B	202	M	301	P	403	L	102	B
S9	B	202	M	302	L	102	B		
S10	B	202	M	303	B				
S11	M	301	P	403	L	102	B	204	M
S12	M	302	L	102	B	204	M		
S13	M	303	B	204	M				
S14	P	401	B	201	P				
S15	P	401	B	202	M	301	P		
S16	P	401	B	203	P				
S17	P	401	B	202	M	303	B	203	P
S18	P	402	M	302	L	102	B	203	P
S19	P	402	M	303	B	203	P		
S20	P	403	L	102	B	203	P		

M1
A1 London
A1 Berlin
A1 Milan
A1 Paris

(ii)

Min
S1+S2+S3+S4+S5+S6+S7+S8+S9+S10+S11+S12
+S13+S14+S15+S16+S17+S18+S19+S20

M1 A1 objective

st
S1+S2+S3+S4>1
S5+S7+S8+S9+S11+S12+S18+S20>1
S1+S2+S5+S6+S7+S14>1
S3+S4+S8+S9+S10+S15+S17>1
S16+S17+S18+S19+S20>1
S11+S12+S13>1
S4+S8+S11+S15>1
S1+S3+S5+S9+S12+S18>1
S6+S10+S13+S17+S19>1
S14+S15+S16+S17>1
S1+S5+S6+S18+S19>1
S2+S4+S7+S8+S11+S20>1

M1
A3 (−1 each error/omission)

(iii)	OBJECTIVE FUNCTION VALUE 1) 3.000000 VARIABLE VALUE REDUCED COST S1 1.000000 0.000000 S2 0.000000 0.000000 S3 0.000000 1.000000 S4 0.000000 0.000000 S5 0.000000 0.000000 S6 0.000000 0.000000 S7 0.000000 0.000000 S8 0.000000 0.000000 S9 0.000000 1.000000 S10 0.000000 1.000000 S11 1.000000 0.000000 S12 0.000000 1.000000 S13 0.000000 1.000000 S14 0.000000 0.000000 S15 0.000000 0.000000 S16 0.000000 0.000000 S17 1.000000 0.000000 S18 0.000000 0.000000 S19 0.000000 0.000000 S20 0.000000 0.000000	M1 A1
	3 pilots are used	B1
(iv)	Three more runs, with S1=0, S11=0 and S17=0 in turn. All require 4 pilots	M1 A1 (3 runs) A1 (4 pilots)
(v)	No account taken of pilot stress (workload/long day/short changeover)	B1

Mark Scheme 4776
June 2005

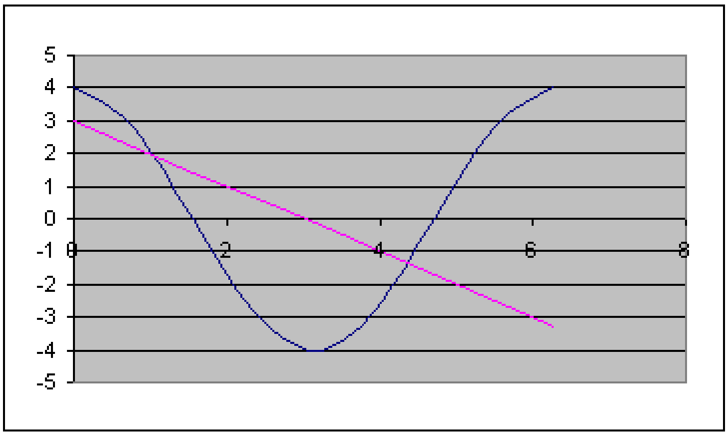
1(i)	h	0.4	0.2	0.1				
	est f '(2)	3.195	2.974	2.871			**M1A1A1A1**	**[4]**
(ii)	differences		-0.221	-0.103			**M1A1**	
	differences approximately halving so extrapolate to 2.871 - 0.103 = 2.768.							
	Last figure unreliable so 2.77.			Accept argument to 2.8.			**M1A1**	**[4]**
2(i)	E.g. 2/3 rounded to 0.666 666 7, chopped to 0.666 666 6						**E1**	**[1]**
(ii)	2/3 stored as 0.666 666 7			Absolute error 0.000 000 033...			**A1A1**	
	mpe is 0.000 000 05						**A1**	
	mpre is greatest when x is least						**M1**	
	mpre is 0.000 000 05 / 0.1 =			5 * 10^ -7	5E-07		**M1A1**	**[6]**
3	x	f(x)						
	1.4	-0.82176						
	1.5	1.09375	root in the interval (1.4, 1.5)				**B1**	
		r	Xr	f(Xr)				
		0	1.4	-0.82176				
		1	1.5	1.09375			**M1**	
		2	1.4429	-0.07436			**M1A1**	
		3	1.446535	-0.00609			**A1**	
		4	1.446859	3.88E-05			**M1A1**	
	Root at 1.447 seems secure.						**B1**	**[8]**
4	x	f(x)						
	2	1.553774		M =		3.305783	**M1A1**	
	3	1.652892		T =		3.285825	**A1**	
	4	1.732051		S =	(2*M + T) / 3 =	3.299130	**M1A1**	
	S(h=2)	3.299130	diffs					
	S(h=1)	3.299231	0.0001					
	S(h=0.5)	3.299238	7 E -06					
	Differences reducing very rapidly. 3.29924 seems secure.						**M1A1A1**	**[8]**
5	Computations of this type contain rounding errors						**E1**	
	The rounding errors will be different when the two sums are computed						**E1**	
	Adding from large to small loses precision (the small number is lost)						**E1**	
	Adding from small to large allows each number to contribute to the sum						**E1**	
	Hence the second sum is likely to be more accurate						**E1**	

6(i)

x	f(x)	Δf(x)	Δ²f(x)	Δ³f(x)		
1	4					
		–3				
2	1		**6**			
		3		**a – 13**		
3	4		**a – 7**			
		a – 4		**87 – 3a**		
4	a		**80 – 2a**		**A4**	
		76 – a			**(-1 each**	
5	76				**error)**	**[4]**

(ii)	87 – 3a = a – 13 gives a = 25		**M1**	
	f(x) = 4 - 3(x-1) + 6(x-1)(x-2)/2 + 12(x-1)(x-2)(x-3)/6		**M1A1A1A1A1**	
	= 4 - 3x + 3 + $3x^2$ - 9x + 6 + $2x^3$ - $12x^2$ + 22x -12		**A1**	
	= $2x^3$ - $9x^2$ + 10x + 1		**A1**	**[8]**
(iii)	f '(x) = $6x^2$ - 18x + 10 = 0	*Algebra may appear in (iii) rather than (ii) for full credit*	**M1**	
	x = 2.26 (2.26376)		**A1**	
	f(2.26...) = 0.718		**A1**	**[3]**
(iv)	f(x) = 4 (x - 2)(x - 3)(x - 5)/(1 - 2)(1 - 3)(1 - 5) + three similar terms		**M1A1A1**	**[3]** **Total 18**

7(i)



		G1	
		G1	
	shows two roots	**E1**	**[3]**

(ii) E.g.:

r	0	1	2	3	4	5	6	7
Xr	1	1.04720	1.06077	1.06465	1.06576	1.06608	1.06617	1.06620

M1A1A1

alpha = 1.066 correct to 3 decimal places **A1** **[4]**

(iii)

0	1	2	3	4	5	6	
1	1.04720	1.06077	1.06465	1.06576	1.06608	1.06617	
diffs	0.04720	0.01357	0.00388	0.00111	0.00032	0.00009	
ratio of diffs		0.28756	0.28615	0.28575	0.28564	0.28561	**M1A1**

ratios (approx) constant so first order convergence. **E1** **[3]**

(iv) Obtain N-R iteration (beware printed answer) **M1A1**

E.g.:

r	0	1	2	3	4	
Xr	5	4.35177	4.36435	4.36432	4.36432	**M1A1**

beta = 4.3643 correct to 4 decimal places **A1** **[5]**

(v)

0	1	2	3	4	
5	4.35177	4.36435	4.36432	4.36432	
diffs	-0.64823	0.01258	-0.00002	0.00000	
ratio of diffs		-0.01940	-0.00184	0.00000	**M1A1**

ratios getting (much) smaller so faster than first order **E1** **[3]**

Total 18

Report on the Units
June 2005

2601 - Pure Mathematics 1

General Comments

There were more candidates than expected for this legacy paper. There were a few centres who use the A level specification in a linear way and therefore entered whole groups of candidates. There were also many entries of a small number of retake candidates per centre, with candidates from across the ability spectrum using this paper in an attempt to improve their uniform score on the P1 unit.

Candidates, in general, had sufficient time to complete the paper and presented their work well.

Comments on Individual Questions

Section A

1) The differentiation was very good.

2) The conversion of radians to degrees was usually correct. A small number of candidates made errors such as inverting the conversion factor or cancelling incorrectly.

3) There was a full range of responses, from the concise ${}^8C_3 \times 2^5 = 1792$ to attempts at full algebraic expansions. The common errors were to forget the 2 or to write $2\left(1+\frac{x}{2}\right)^8$.

4) Most candidates found the equation of the line correctly. There were the usual errors such as inverting the gradient or not handling the negative correctly.

5) Candidates who handled the trapezium rule correctly were able to find the answer rather more easily than those who worked with separate trapezia or rectangles and triangles. More candidates than usual substituted *x*-values instead of y-values into the trapezium rule formula, often quoting 'first + last + twice the others' rather than using the given formula.

6) Most candidates were able to solve the associated quadratic equation correctly, although many resorted to the formula rather than factorising. However, the handling of the inequality was then poor, with many concluding that $(5x + 1)(x - 2) > 0$ implies $5x + 1 > 0$ or $x - 2 > 0$. The few who produced a sketch graph were usually successful.

7) The vast majority attempted to use their calculators here to find the angle or a decimal form of the fraction. Those who used the common correct methods of a right-angled triangle with $\sqrt{11}$ on the hypotenuse, or used $\sin^2\theta + \cos^2\theta = 1$, often made errors in squaring.

8) Many did not use the correct formula for finding the volume of revolution, and many of those who did made errors in squaring $2x^2$ and received only partial credit.

9) Some candidates were not able to start this question. Those who did realise that the other two numbers were $n - 1$ and $n + 1$ did not always understand 'the sum of the squares'. Many did get going and most of those were able to obtain $3n^2 + 2$ correctly, even if they then had difficulty in explaining how this result showed that the sum was not divisible by 3.

Section B

10) (i) Most candidates used differentiation to find the coordinates of C, then in the next part realised that they needed the coordinates of A. A few found A and B first by solving the quadratic equation, then used symmetry to find C.

(ii) Most candidates successfully demonstrated that the length of AC is $\sqrt{20}$. Most knew the correct form for the equation of a circle, but there were frequent errors in applying it, with some using the wrong centre and some having $\sqrt{20}$ instead of 20 for r^2, for instance. Some candidates elegantly solved these first two parts in a few lines; some took a couple of pages.

(iii) Showing angle ACB is 0.93 radians was not found easy, with many working in degrees and then converting. There were the usual errors in working with sectors and triangles; with relatively few realising they could use the simple $\frac{1}{2} \times 4 \times 4 = 8$ for the area of the triangle instead of using the angle.

(iv) This required a standard integration, with the usual errors, but many were able to complete it successfully.

11) (i) Many produced the factors immediately and usually expanded them well. A few worked backwards and factorised the given cubic.

(ii) This was generally well done, although a few candidates ignored the hint of giving answers to 2 decimal places and attempted factorisation.

(iii) The derivation of the equation of the normal was very well done, with the given answer giving candidates confidence to proceed from correctly finding $y' = -7$.

(iv) Except for the weakest candidates, many were able to equate the normal and the cubic and tidy up the resulting equation to obtain the given result. However, few realised that they needed to use the known factor of $(x + 1)$ to find the required quadratic factor.

2602 - Pure Mathematics 2

General Comments

This paper gave opportunities for candidates of all standards and attracted the full range of marks. Although there were fewer candidates scoring the very highest marks, 55 or more out of the maximum 60, than in previous years, and a significant number scoring less than 10 marks, it appeared that the majority of candidates were familiar with the methods and techniques required by the questions.

There were some very good performances with well presented scripts, and little evidence of candidates being short of time.

There were several points where success depended on careful reading of the question to identify what was required. The request for exact answers in Question 3 was often overlooked or misunderstood and in all questions marks were frequently lost through slips in algebra.

Comments on Individual Questions

1) Question 1 gave opportunities for most candidates, who tended to score well here. The quotient rule and the chain rule were well applied.

In part (a) some weaker candidates misidentified *u* and *v*, and errors in simplifying the answer were quite common. Only a small minority used the product rule.

In part (b) it was good to see that only a very few candidates used the mistaken form $f'\,g'(x)$, in this case $\frac{1}{2}(3x^2)^{-\frac{1}{2}}$, instead of $f'\,g(x).g'(x)$.

Part (c) saw slips and elementary errors of algebra and the final verification in (iii) was often not achieved.

A surprising number went from $y-1=x^{\frac{1}{3}}$

to $y^3-1=x$

and $x=(y-1)^{\frac{1}{3}}$ was often seen.

Another error was to put $(x^{-\frac{2}{3}})^{-1}$ equal to $x^{\frac{3}{2}}$

A few candidates began (iii) by swapping *x* and y, as if looking for an inverse function, which led to confusion.

Part (d) was usually done well but many candidates misread the integrand as $1+x^{\frac{1}{3}}$
The Mark Scheme was generous towards this. Others used unhelpful substitutions.

2) Question 2 was the least well answered question. Whilst there were many excellent solutions, weaker candidates made algebraic slips or had trouble remembering the formulae.

A common error for the sum formula in part (ii) was $\frac{1}{2}n\left[a+(n-1)d\right]$.

Setting up an equation often defeated them, and many ended up with, for example

$$3 \times 10^{th} \text{ term} = 10^{th} \text{ term}$$

In part (i), errors such as 3(-8 + 9*d*) = -24 + 9*d* were seen.

In part (ii) $3(ar^9) = 3a \times 3r^9$ was common.

Many of those who established $3ar^9 = ar^{19}$ could not solve this equation.

In part (iv) able candidates quickly reduced the initial equation to $r^{20} - 1 = 3(r^{10} - 1)$ in one step, to their advantage.

A small number of the stronger candidates misread the question and answered

$$3 \times 20^{th} \text{ term} = 10^{th} \text{ term}$$

and 3 x sum of first 20 terms = sum of first 10 terms throughout.

3) Question 3 was generally well answered, although the candidates' algebra was often not up to scoring full marks in part (i). Most candidates knew that the function was odd, but some attempted to verify this by substituting particular values rather than (-*x*).

The differentiation in part (ii) was done well by many.

Some candidates confused $\text{f}(x)$ with $\text{e}^{-\frac{1}{2}x^2}$ and so thought that the result of differentiating $\text{e}^{-\frac{1}{2}x^2}$ should be $(1-x^2)\text{e}^{-\frac{1}{2}x^2}$.

In part $\sqrt{1}$ (iii) appeared in many solutions unresolved, and many gave decimal approximations for the *y*- coordinates, rather than the exact forms.

In part (iv), the technique of integration by substitution was widely understood, but many candidates, having obtained the correct expression, were unable to integrate correctly.

$\int \text{e}^{-u} du$ was often given as $\frac{\text{e}^{-u}}{-u}$ or $\frac{\text{e}^{-u+1}}{-u+1}$ or $\frac{\text{e}^{-2u}}{2}$

4) Perhaps it was because this was the last question, and required some careful thinking, that marks were lost by many candidates. Nevertheless, some weaker candidates, who were familiar with the topic, were able to make up ground here and the question was well answered by those confident about the logarithmic notation and laws. There were frequent slips and errors from others.

In part (i) ln*y* = ln *a* x *x* ln *b* was common and the correct form, written as ln *y* = *x* ln *b* + ln *c* led many to offer *x* as the gradient.

Part (ii) was done well.

In part (iii) mistakes arose from misreading scales, reading from the wrong line, problems with signs and generally whether to use x, y, ln x, ln y, c or ln c.

In part (iv) some wasted time trying to solve rather than estimating from the graph; part (v) was generally done well.

2603 - Pure Mathematics 3

General Comments

There was a wide range of marks for this paper, fewer candidates than in past years scoring in the range 65 plus and more than previously receiving marks of 15 or less.
The majority of candidates scored marks between these two extremes. Marks were pulled down by poorer performances on the comprehension paper on which some 50 per cent of candidates scored 6 marks or fewer. Having said that, there were some very good performances and quite a pleasing number of candidates scored well on both papers. These candidates presented their work well, in contrast to weaker candidates whose work was often very difficult to follow and, occasionally, difficult even to read.

Marks were often lost by candidates who missed, or misread, small parts of questions. For example an appreciable number of candidates failed to find the area of the triangle in question 4(i), and quite a large number found f(0.01) instead of f′(0.01) in question 2(iv). There was little evidence of candidates being short of time unless time had been wasted with overlong solutions or work crossed out and repeated with little improvement.

Comments on Individual Questions

1) (a) This question was generally well answered although the value of α was frequently given in degrees. Other errors that occurred were $\tan\alpha = 5/6$ or $-6/5$ or $\alpha = 0.88\pi$.

(b) Almost all candidates realized that integration by parts was required and chose the correct functions, $u = x$ and dv/dx = sin 2x. Integration of the latter function however, was not so sure; $v = ½ \cos 2x$ or $v = -2 \cos 2x$, or similar, were often seen. These errors were often repeated or introduced at the next stage when integrating *v.* Most candidates arriving at a result of integration were able to substitute the limits correctly, although 1/2x= 45° at the upper limit was seen occasionally.

(c) Most candidates were able to show that the point (π/2, 1) lies on the given curve. Solutions to the main part of the question involving implicit differentiation were often spoiled by the omission of brackets,

$$1+\frac{1}{y}\frac{dy}{dx}=\cos x,$$

by attempts to fudge the result after an error,

$$y+\frac{1}{y}\frac{dy}{dx}=\cos x \Rightarrow \frac{dy}{dx}=\frac{\cos x}{y+\frac{1}{y}}=\frac{\cos x}{\frac{y+1}{y}}=\frac{y\cos x}{y+1},$$

or by poor notation; the misuse of the symbol $\frac{dy}{dx}$ was often seen.

In the final part of this question it was surprising to find candidates interpreting $\frac{0}{2}$ as undefined or infinity.

2) **Partial fractions, differential equations, the binomial theorem**

(i) The partial fractions were obtained correctly by almost all candidates, just a few errors in the coefficients. For example, 5A=5 ⇒A=5 was seen on more than one occasion.

(ii) The differential equation was not so well done, the separation of variables often being incorrect or completely absent. Candidates realized that the partial fractions had to be used and so the most common errors were,

$$\int 5y\,dy = \int \left(\frac{1}{2+x} + \frac{2}{1-2x}\right)dx\,;$$

or, simply, $$y = \int \left(\frac{1}{2+x} + \frac{2}{1-2x}\right)dx\,;$$

although d*y* and d*x* were often omitted. This meant that most candidates had the opportunity to score two marks at least for integrating their partial fractions. For those candidates who separated the variables correctly and integrated, the next common errors were the omission of an arbitrary constant or incorrect log work in the process of finding the value of the constant,

$$\ln y = \ln(2+x) - \ln(1-2x) + C$$

$$\Rightarrow \quad y = \frac{2+x}{1-2x} + C$$, or perhaps $+e^C$.

(iii) Many candidates were able to apply the binomial theorem to obtain a correct expansion of $(1-2x)^{-1}$.Surprisingly, over-enthusiasm for the binomial theorem led a number of candidates to apply it, also, to 2+*x*

$$2+x = 2(1+x/2)^1$$
$$= 2\left(1 + 1.x/2 + \frac{1.(1-1)}{2}(x/2)^2 + \ldots\right)$$
$$= 2 + 2.x/2 = 2+x$$

Multiplication of the series by 2+*x* was almost always correct, although just occasionally the x^3 term was omitted from the binomial expansion leading to an incorrect x^3 term in the product.

Unfortunately a very large number of candidates lost the final mark in this part by failing to state the range of validity or by giving it incorrectly; $|x| \le ½$, $x < ½$,or $½ < x < -1/2$, etc.

(iv) Many candidates differentiated the given f(*x*) and substituted *x* = 0.01 to find f′(0.01) correctly but then failed to find the value of d*y*/d*x* at *x*=0.01, for comparison . Others found f(0.01) and *y*(0.01) both equal to 2.051 and thought they had answered the question. There were, however, many fully correct answers.

3) **Parametric coordinates and equations**

(i) A variety of methods was seen often leading to the correct coordinates for A and B. The work of many candidates, however, was confused, and this question was a good example of one where the work of weaker candidates was very difficult to follow because of the lack of explanation. In some cases a value of θ was given as one of the coordinates.

(ii) Almost all candidates wrote down the equation $1-\cos 2\theta = 2\sin 2\theta$ as a start to this question and realized that it was then necessary to use the double angle formulae in order to prove the given result. The RHS of the above equation presented no problem, and many good candidates gave a very neat solution using ½(1– cos 2θ) = 2 $\sin^2 \theta$, but weaker candidates often used an incorrect formula for cos 2θ or substituted with little regard for bracket or signs.

$\cos 2\theta = \cos^2\theta - 1$ or $1 - \sin^2\theta$,

$\cos 2\theta = 1 - \cos^2\theta - \sin^2\theta$,

$\cos 2\theta = 1 - (\cos^2\theta - \sin^2\theta) = 1 - \cos^2\theta + \sin^2\theta = 1 - 1 = 0$

$\cos 2\theta = 1 - (1 - 2\sin^2\theta) = -2\sin^2\theta$

were all seen.

(iii) Some candidates did the work for the start of this question in part (i) and sensibly referred back to it, others repeated the work here. Many candidates obtained full marks for d*y*/d*x*. Many then went on to find the gradient at C, finding $\tan^{-1}2$ on their calculator and retaining it to obtain an exact value of -1.5 . Others used the double angle formula for tan 2θ, usually correctly.

(iv) Only a small number of candidates obtained the Cartesian equation using the identity $\sin^2 2\theta + \cos^2 2\theta = 1$; most attempted to use $\sin^2\theta + \cos^2\theta = 1$ which involved more work and therefore more likelihood of errors occurring.

4) **Vectors**

(i) The first part of (i) was answered well by the majority of candidates but many failed to go on to find the area of triangle OAB. Those who did attempt this often used an incorrect method; ½ |AB| sin θ, ½ a.b cos θ, ½ OB.BA and ½ OA. AB were all seen, as well as incorrect attempts to find a perpendicular height to go with base OB or OA.

(ii) Many candidates confused the vector equation of AD with the vector AD and this may account for

$$\text{AD} = \begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ -9 \\ 5 \end{pmatrix}$$

instead of r = ……

(iii) The fact that c.a = 0 implies that c is perpendicular to a was well known and usually shown; just a few candidates failed to show the working which was necessary to obtain the mark. However many candidates thought that this was sufficient and did not show that in addition c.b = 0 or c.AB = 0. The equation of the plane OAB was usually found correctly but not always the equation of CDE. A number of candidates wasted time on this question by attempting to use the vector equation of a plane and eliminate the parameters to find the Cartesian equation. Such attempts were only very rarely successful.

(iv) Most candidates obtained the method mark for this final question but many, not having the correct area of the triangle OAB could not get the correct answer.

Section B Comprehension

1) A reasonable number of candidates recognized the problem with units and explained that sending the ratio of the two masses was a way of overcoming this. Other candidates commented on 'the different conditions on earth' or the ratio being 'the same everywhere' without saying why. Some candidates erroneously referred to different number systems or numbers being too small to transmit.

2) Many candidates were able to write down the correct expression for the conversion of the given number in base 5 to a decimal number but unfortunately many of them failed to evaluate it. Many other candidates just wrote down the usual value of π in terms of powers of 10.

3) Very well done indeed, very few candidates were unable to complete the table correctly.

4) Candidates were roughly, equally divided between those who verified that the given values of ϕ satisfied the equation, and those who transformed the equation into standard form and then used the formula to solve it. Unfortunately many of the former used their calculator to evaluate the given values of ϕ and then substituted them into the equation. This approximate solution was not acceptable but a complete justification involving rationalizing was accepted. Those candidates transforming the equation usually applied the formula correctly.

5) Many candidates took the reference to 'the method on page 5' to refer to the demonstration that the sequence there appeared to converge to the limit ϕ, rather than to the proof that it did converge. Some credit was allowed to those candidates who calculated a sufficient number of ratios to draw a reasonable conclusion, but very few did this, and many made errors in their calculations of the early terms of the sequence. Of those who used the algebraic approach some used inconsistent values of the ratio r, but many obtained the appropriate quadratic equation, solved it correctly, gave the positive root as their answer and rejected the negative root.

6) Many candidates failed to attempt this question but others, not necessarily the most able candidates, completed it correctly. There was sometimes some misunderstanding as to which point of bifurcation was required and some candidates calculated both the point of bifurcation from 8 to 16 outcomes and also the point where the number of outcomes changed from 16 to 32. Some candidates gave a range of values of k rather than a specific value.

2604 - Pure Mathematics 4

General Comments

There was a wide range of performance on this paper. About one quarter of the candidates scored 50 marks or more (out of 60), with many of these showing confidence and efficiency in applying the various techniques with apparent ease. On the other hand, about 20% of candidates scored fewer than 30 marks, and some of these appeared to be unfamiliar with the standard topics being examined. Some candidates made very heavy weather of the algebra in questions 2 and 3, and ran out of time, but the great majority were able to complete the paper. Quite a few answered all four questions; in almost every case the time could have been better spent concentrating on three questions and eliminating careless errors. Question 1 was attempted by almost every candidate, and question 4 was by far the least popular.

Comments on Individual Questions

1) **Curve sketching and Inequalities**

This was answered well, with half the attempts scoring 15 marks or more (out of 20).

(i) The equation of the vertical asymptote was almost invariably given correctly, but the oblique asymptote caused some difficulty. Although some gave $y = 3x - 14$ or $y = 3x$, most candidates did attempt a process of division, which was often spoilt by careless sign errors.

(ii) Candidates who started from $y = 3x - 8 - \dfrac{16}{x-2}$ were easily able to differentiate and conclude that the gradient is always positive. However, the great majority applied the quotient rule to the equation in its original form. This is possibly a safer strategy (in case errors had been made in the division), but it involved considerably more work. The gradient was usually found correctly, although errors such as $x(3x-14) = 3x^2 - 14$ and $-(3x^2 - 14x) = -3x^2 - 14x$ did occur frequently. It was then necessary to show that the quadratic expression in the numerator ($3x^2 - 12x + 28$) is always positive. Arguments such as '$3x^2 + 28$ is always greater than $12x$' were often stated but very rarely justified. It was not sufficient just to state 'there are no stationary points' without any justification. The usual approach was to show that the discriminant is negative, although many did not then state that this implies the desired result. Very few answered this by completing the square.

(iii) Much good curve sketching was seen here, although the presentation and clarity varied from excellent to very poor. It should not be necessary to use graph paper, but very many candidates chose to do so. Sketches were expected to include the asymptotes, to identify the point of intersection on the positive *x*-axis, and to show clearly how the curve approaches its asymptotes.

(iv) Most candidates found the critical values which give equality, but many did not know what to do next. Those who simply looked at their graph and wrote down the solution usually obtained the correct answer.

(v) The square root graph was generally well understood, but very many lost a mark for not showing clearly the infinite gradients where the curve crosses the *x*-axis.

2) This was the best answered question, with an average mark of about 15, and about 20% of the attempts scored full marks.

(a) **Summing a series, using standard formulae**

Most candidates obtained a correct expression for the sum of the series, but many were unable to write it as a product of linear factors.

(b) **Summing a series, using the difference method**

The identity at the beginning caused surprisingly many problems, with many writing $3r-(r+1)=3r-r+1$ despite the printed result including $2r-1$, and several tried to use partial fractions. The method of differences was well understood and was usually applied accurately, although some gave the final answer in terms of *r* instead of *n*.

(c) **Proof by induction**

This was well understood, and there were very many fully correct solutions. The main cause of loss of marks was faulty algebra.

3) **Complex numbers**

(a)(i) Finding the moduli and arguments was done well, except that the argument of β was often given as $-\frac{1}{4}\pi$ instead of $\frac{3}{4}\pi$. On the Argand diagram, α and β were usually positioned correctly, but $\dfrac{\beta}{\alpha}$ was less frequently right.

(ii) The obvious approach was to use the modulus and argument and write $4(\cos\frac{2}{3}\pi+\mathrm{j}\sin\frac{2}{3}\pi)$; many did this and obtained the correct answer easily. However, many used much more complicated methods, such as attempting to deal with $\dfrac{-4+4\mathrm{j}}{\sqrt{2}(\cos\frac{1}{12}\pi+\mathrm{j}\sin\frac{1}{12}\pi)}$ directly.

(iii) The line was very often drawn correctly. This was intended to draw attention to the triangle and encourage the use of the cosine rule, and candidates who used this method were usually successful. However, most ignored the hint and either omitted the calculation or evaluated $\alpha-\beta$ and hence found its modulus. This often produced the correct answer of $\sqrt{42}$, but full credit was given only when a fully exact method had been shown.

(b) Most candidates knew that they should substitute $z=a+b\mathrm{j}$ into the equation, and the correct answer was very frequently obtained. The most common error was to equate the imaginary parts as $2a-2b=2$ instead of $2a-2b=-2$.

4) **Vectors and Matrices**

This question was attempted by only about one third of the candidates. It was also the worst answered question, with an average mark of about 12.

(a)(i) Most candidates used the correct method of writing the equations of the two lines with different parameters, equating components, and solving the resulting simultaneous equations to find *k*. This was often carried out accurately, although arithmetic slips were very common.

(ii) Most candidates understood how to find the point of intersection.

(iii) Almost every candidate used the vector product of **AB** and **CD** to find a vector normal to the plane. This was perhaps not surprising, given the work which had already been done, but of course it gave the wrong answer when the value of *k* was incorrect. Candidates could have played it safe by using only the given points A, B and C.

(b)(i) Surprisingly many candidates were unable to describe the transformation defined by the given matrix. Even when it was recognised as a rotation, a full description (including the centre, angle and sense of rotation) was rarely given.

(ii) Correct answers to this part were quite rare. There was a lot of confusion between the object line and the image line, for example finding the image of $(x,\ x-2)$ under the given matrix instead of its inverse. Another common error was equating the image of $(x,\ y)$ to $(x,\ x-2)$ using the same symbol *x* in both the object point and the image point.

2605 - Pure Mathematics 5

General Comments

There was a wide range of performance on this paper, with about a quarter of the candidates scoring 50 marks or more (out of 60), and about a quarter scoring less than 30 marks. Almost every candidate answered questions 1 and 2; then about 80% chose question 3 and only 20% chose question 4.

Comments on Individual Questions

1) **Roots of a cubic equation**

This was by far the best answered question, with half the attempts scoring 17 marks or more (out of 20). For many candidates this question provided a high proportion of their total mark.

Parts (i) and (ii) were almost always answered correctly.

In part (iii), most candidates mentioned the existence of complex roots; but relatively few earned both marks by stating that one root is real and two are complex.

Parts (iv) and (v) were very often answered efficiently and correctly, although some candidates set off on the wrong algebraic track and wasted a lot of time in fruitless effort.

Finding the new cubic equation in part (vi) was well understood, and the product of the new roots was very often found correctly. Many candidates did not realise that they had already found the sum of products in pairs, and calculated this again, often obtaining a value different from their answer to part (v).

2) The average mark on this question was about 13.

(a) **Hyperbolic functions**

In part (i), most candidates were able to show that $x = \ln(c \pm \sqrt{c^2 - 1})$, but only a few then showed correctly that this is equivalent to the desired result $x = \pm\ln(c + \sqrt{c^2 - 1})$.

In part (ii), those who used $\sinh^2 x = \cosh^2 x - 1$ were usually able to obtain $\cosh x = 2$ and hence write x in logarithmic form, but the other solution $\cosh x = -5$ was sometimes not rejected. Those who wrote the original equation in exponential form very rarely made any progress.

(b) **Inverse circular functions and Maclaurin series**

In part (i), the double differentiation of $\arcsin(\frac{3}{5} + x)$ caused a surprising number of problems, notably sign errors.

In part (ii), the Maclaurin series usually followed correctly from the results in part (i), although many forgot to divide $f''(0)$ by 2 when finding q.

Most candidates knew what to do in part (iii), but $0.1\arcsin(0.6)$ was often evaluated as $\arcsin(0.06)$, and degrees were sometimes used instead of radians.

3) **Complex numbers**

The average mark on this question was about 11.

Part (i) was generally answered well, but the responses to part (ii) ranged quite uniformly from very poor to fully correct. Most candidates began by considering $C + \mathrm{j}S$, but some made no progress beyond this. A common stumbling block, when the geometric series had been summed, was the failure to make the denominator real. Careless errors such as $(3\mathrm{e}^{\mathrm{j}\theta})^n = 3\mathrm{e}^{\mathrm{j}n\theta}$, and sign errors, spoilt some otherwise good attempts, and the expression for S often included -3 in the numerator.
In part (iii), the three cube roots were very often given correctly, but a surprising number of candidates had all three arguments wrong.
Part (iv) was also correctly answered by many candidates, although the connection with part (i)(*B*) was not always seen. Some confused w^* with w^{-1}.

4) **Polar coordinates**

This was the worst answered question, with an average mark of about 10.

In part (a)(i), most candidates did not even make the first step of expressing $r\sin\theta$ in terms of θ.

In part (a)(ii), there were some good attempts to sketch the curve, although few earned full marks; the most common error was to draw a cusp at $\theta = \frac{1}{2}\pi$.

In part (a)(iii), there was a lot of good work, and the area was often found correctly. Many made slips in the integration, and the overall factor of $\frac{1}{2}$ was sometimes missing.

In part (b)(i), the ellipse was often drawn correctly, but only a few candidates could answer parts (b)(ii) and (b)(iii).

2606 - Pure Mathematics 6

General Comments

The performance on this paper was generally good; about one third of the candidates scored 50 marks or more (out of 60). However, there was a wide range, and about 20% of the candidates scored fewer than 30 marks. Almost all candidates appeared to have sufficient time to complete the paper. Over half the candidates chose questions 1, 3 and 4; other popular combinations were questions 1, 3 and 5, questions 1, 2 and 3, and questions 2, 3 and 4. Very few candidates attempted more than the three questions required.

Comments on Individual Questions

1) **Matrices**

This question was attempted by most candidates, and it was answered well. The average mark was about 15 (out of 20), and about 20% of the attempts scored full marks.
In parts (i) and (ii), the concepts of eigenvalues and eigenvectors were well understood, and the work was often carried out accurately. A common misconception was to give $(0, 0, 0)$ as an eigenvector corresponding to the eigenvalue 0.
In part (iii), most candidates knew that **P** has the eigenvectors as its columns, but the diagonal matrix **M** was very often given wrongly, usually with the eigenvalues not raised to the fourth power.
In parts (iv) and (v), the use of the Cayley-Hamilton theorem was very well understood.

2) **Limiting Processes**

This question was attempted by about a quarter of the candidates, and the average mark was about 12.
Part (a) was generally answered well, although the derivative of $\mathrm{G}(x) - \mathrm{G}(2)$ was quite often given as $\mathrm{G}'(x) - \mathrm{G}'(2)$.
In part (b)(i), most candidates drew rectangles of width 1, and fully correct explanations were quite common. Some were not sufficiently precise in specifying which set of rectangles were being considered for each inequality, and some did not mention that the definite integrals give the area under the curve.
In part (b)(ii), most candidates found the bounds correctly by evaluating infinite integrals, although the deduction that the series is convergent was often not made.
In part (b)(iii), many candidates did not realise that the series should be split as $\sum_{r=1}^{60} \frac{1}{r^2} + \sum_{r=61}^{\infty} \frac{1}{r^2}$; those who did were usually able to use the previous result to obtain bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. However, very few candidates appreciated that the given value of 1.6284 (correct to 4 decimal places) implied a true value between 1.62835 and 1.62845.

3) **Multi-variable Calculus**

This question was attempted by almost every candidate, and it was the best answered question. The average mark was about 15, and about a quarter of the attempts scored full marks.
Parts (i), (ii) and (iii) were very often answered correctly, although the *z*-coordinates of the stationary points were sometimes omitted.

In part (iv), most candidates tried to solve $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 27$, and many obtained the correct values of k, although careless errors were much more frequent here than in the earlier parts of the question.

4) **Differential Geometry**

This question was attempted by about two thirds of the candidates, and the average mark was about 13.
In part (a), the arc length was usually found correctly.
In part (b), many candidates were unable to write down an integral giving the surface area. Those who did usually went on to make the substitution $u = 2\cos\frac{x}{a}$, but the change of limits was not always carried out correctly.
In part (c), the principles for finding the radius and centre of curvature were well understood, although the work was often spoilt by minor errors in differentiation and sign errors.

5) **Abstract Algebra**

This question was attempted by less than a quarter of the candidates. It was the worst answered question with an average mark of about 11.
In part (a), the inverses and orders of the elements were usually given correctly. Most candidates gave some of the subgroups, but the list was not often complete.
In part (b)(i), most candidates gave a satisfactory definition for a basis of a vector space, but parts (b)(ii) and (b)(iii) were rarely seriously attempted.

2607 - Mechanics 1

General Comments

This paper was found to be far more accessible by the majority of candidates than those of previous sessions. Questions 1, 2 and 3 were done completely correctly (or very nearly so) by many of the candidates. Question 4, however, caused problems due to lack of knowledge of the properties of an acceleration-time graph.

Generally the quality of mathematics offered was high, however many candidates did not know how to *show* displayed results properly. It was quite clear at times that their knowledge of mechanics was not deficient but their skill in demonstrating a given answer or result was. In these situations many candidates would probably have fared better had the result not been displayed at all.

The proportion of candidates who were seemingly totally unprepared for this examination was far lower than in previous years.

Comments on Individual Questions

1) **The motion of a car and a trailer and the force in the tow-bar**

This question was generally answered very well. Many completely correct solutions were in evidence.

(i) Almost always correct.

(ii) Many demonstrated the given result effectively and sufficiently. However it is a concern that there are a significant number of candidates who treat a numerical "show" as an invitation to play a numbers game. Their "solution" merely consists of a sequence of arithmetical operations involving the given values and it seems reaching the "target number" is their only concern. It was not unusual to see such arithmetical listing without any indication whatsoever of the mechanical principles involved, or even an indication of which physical quantities were being considered.

(iii) As in (ii), much good work. Many candidates were able to demonstrate the given result – others "played" with the given values until the target value was reached. Some candidates (usually successfully) found the force in the tow-bar first.

(iv) Well answered although some of the number players had their bluff called for the first time here as the value was not displayed. Errors common with similar past questions were made: missing forces, extra forces and sign errors.

(v) Once again, very well done. There were many completely correct solutions given. Common errors mentioned in (iv) were also evident here. A number of candidates used the wrong mass when attempting to find the force in the tow-bar, many thinking this could be found by applying Newton's second law to the entire system.

2 A projectile problem

Generally extremely well done. The majority of candidates found this question very accessible and, with the possible exception of (ii), allowed them to display their knowledge effectively.

(i) Usually correct. Incorrect methods were normally due to equations of the form $10\cos\theta = u$ and $12\sin\theta = u$ being used.

(ii) The displayed result (vertical displacement of particle A above the ground) was perhaps too helpful and worked to the candidates' own detriment as this trivial result was almost always written straight down by candidates whose answers were generally good. It seemed as though they were unable to decide what exactly needed to be written down to convince the examiner of their knowledge. The short answer to this is to absolutely "spell it out" to the Examiner; in that way the Examiner – and candidate - cannot be left in any doubt about the completeness of the solution. Omission of the reason for the first term (9) of the given expression was very common.

The expression for the horizontal displacement was usually correct although a number seemingly overlooked the request for it.

(iii) Often correct although a common oversight was to find the maximum height of particle A above ground level (rather than the point of projection). Some candidates took an indirect route by finding the time taken to the greatest height first. Nevertheless this was well done and sign errors that have occurred in past projectiles questions were far less evident in this session.

(iv) Many demonstrated that the horizontal displacements were equal. However a large number thought it was sufficient to show the horizontal components of projection speeds were the same. Unless the other initial conditions (position, time) were mentioned this was deemed insufficient.

(v) Usually correct.

(vi) Again, very well done. Many equated the given expressions and then solved successfully. Some did not give evidence that their equation led to a solution of 1.7 seconds correct to two significant figures - they merely wrote 1.7 seconds as their answer without mention of a more accurate value. A significant number of candidates simply substituted the given time into the two given equations for vertical displacement; this, of course, did not show that the solution was correct to the stated degree of accuracy.

3 A block in equilibrium & a vector statics problem

This was perhaps not as well done as the first two questions. Nevertheless the majority of candidates scored high marks.

(a)(i) Almost always correct. 5 kg was seldom misread as 5 N. Some candidates, clearly unprepared for even the simplest calculation, took the tension to be a combination of 5(g) with 20(g).

(ii) Diagrams were normally clear and correctly labelled. Usual errors involved duplication of labels (e.g. *T* and *T*), extra forces (for example, friction) and arrows missing.

(iii) Many candidates were able to find the tension correctly. sin/ cos muddles were pleasingly uncommon and very few candidates included extra forces.

(iv) This was handled correctly by a large number of candidates. Few seemed to have any real difficulties although there remain a few who maintain that the normal reaction is equal to the weight of the block. As with (iii) it was pleasing to see that this candidature seemed to experience few problems with resolution.

(b)(i) About half of the candidates correctly found the missing force vector; the majority of the remainder thought it was the sum of the two given forces i.e. the force equal and opposite to the correct one. The remainder combined the given forces in a variety of astounding ways.

(ii) Almost all were able to follow through correctly to gain full marks for the magnitude of their vector found in (i). Many knew how to find the direction of a vector but overlooked the instruction to find the angle between ***R*** and the **i** direction (an obtuse angle) – many gave the acute angle between ***R*** and the **–i** direction for which part credit was given.

4 **The use of an acceleration-time graph**

This was by far the least well done question on the paper. A highly significant number of candidates did not appreciate that the area beneath an acceleration-time graph represented the change in velocity; because of this most of the marks in parts (i), (ii) and (iii) were not awarded. The common misconception was that the gradient represented change in speed. Also many thought the constant acceleration formulae applied when, of course, they didn't.

(i) Almost all were able to read off the acceleration from the graph. Only those who knew about the area beneath the graph were able to find the speed at $t = 4$. The common errors/ misconceptions were to find the gradient of the line segment or to use constant acceleration formulae.

(ii) A number of candidates identified the correct time as $t = 7$ but were unable to explain fully why it was the time at which the speed was greatest. A highly popular incorrect response was $t = 5$ (or $t = 4$- 5) presumably because of the maximum acceleration there.

(iii) Those who knew how to obtain change in speed had few problems. The majority, however, did not and thus scored zero with the same mistakes/ misconceptions discussed earlier.

(iv) The majority of candidates were able to write the correct expression for *a* (some, no doubt, by differentiation of the given expression for *v*). Integration was then normally and successfully used to prove the given result. The vast majority however forgot to include a constant of integration and show this was zero which deprived them of the final mark.

(v) Despite some excellent solutions, performance on this part was quite disappointing. Many candidates applied the constant acceleration formulae throughout irrespective of the strong hint given in (iv). Others integrated correctly but used limits from t = 0 to t = 4 or, even worse, from t = 1 to t = 5 (this was indeed quite common). The majority who knew how sensibly to tackle the problem made silly mistakes; for example, using wrong values when applying the constant acceleration formulae over the 1 second interval.

2608 - Mechanics 2

General Comments

This paper appeared to be accessible to all of the candidates, with the majority able to obtain at least some credit on some part of each question. A large number of excellent scripts were seen. As in previous sessions there were some candidates who did not seem to appreciate that a diagram assists in finding a solution and can help to clarify the solution to the examiner. The main difficulties that arose related to giving reasons for a calculated answer or in establishing given answers. There was, from some candidates, a lack of rigour with relevant steps in working being omitted and/or insufficient explanation as to the principles being employed. A small number of candidates penalised themselves by premature rounding of answers leading to inaccuracies in final answers.

Comments on Individual Questions

1) **Impulse and Momentum**

(a) Problems arose in this part for those candidates who did not appreciate the vector nature of the information given, and hence did not give full enough details about the direction of the velocities requested.

(i) This part was almost always successfully answered.

(ii) (A) Many candidates obtained the correct speed for Sheuli but did not specify direction. Others set up a correct equation for Roger's speed but then quoted $-12\mathbf{i}$ ms^{-1} as the solution to it or obtained the answer $12\mathbf{i}$ but then failed to convert to $\mathbf{v_s} = -12\mathbf{i}$.

(B) This part was done more successfully with many obtaining a complete solution in terms of **i** and **j**.

(b)(i) Unfortunately many candidates did not draw a diagram for this part of the question and so errors in signs and inconsistencies in equations were quite frequent. Candidates could help themselves by stating which principle is being applied and specifying the meaning of the variables being employed.

(ii) This part of the question was poorly attempted by almost all of the candidates. While many of them could state that the speed would be unchanged and that the angle of reflection would be the same as the angle of incidence, few could give clear and unambiguous reasons as to why this was so. Most merely stated that the collision was perfectly elastic without expanding on what this would affect. Very few candidates seemed to appreciate the need to investigate directions parallel and perpendicular to the wall and of those that did, only a small number mentioned that there would be no impulse in the direction parallel to the wall and hence no change in that component of the velocity.

2) **Work and Energy**

Candidates either scored well on this question or very poorly

(i) This part posed few difficulties for the majority of candidates although a small number of them failed to give any indication of the principles being employed and merely wrote down a set of numbers that produced the required answer.

(ii) Most candidates gained full credit for this part. A small minority did not appreciate that, for constant speed, the resultant force must be zero and hence could not get very far with the solution.

(iii) A sizeable number of candidates ignored the method requested in the question and attempted a solution using Newton's second law and the constant acceleration equations, obviously not appreciating that if both the power and the resistance are constant, the acceleration cannot be. Of those who used the requested method, most obtained some credit but many omitted the work done term associated with the power.

(iv) Candidates who used work-energy methods for this part were on the whole more successful than those who opted for Newton's second law and *uvast*. As in previous sessions common errors were usually the omission of one of the terms in the work-energy equation or in the sign of the acceleration in *uvast*.

3) **Centres of Mass**

This question gave few difficulties to the majority of candidates. Almost all of them understood the method required to find a centre of mass and could present their working clearly. Some excellent answers were seen.

(i) A high proportion of candidates obtained the correct answer to this part of the question. However, there were a small number of candidates who treated the shape as if it were in three parts, a lamina and two squares formed by rods and other candidates treated it as if the whole shape was a lamina.

(ii) A large number of candidates also scored highly on this part of the question. The main errors were in the sign of the z component of the centre of mass. The majority of the candidates understood that the use of Pythagoras in 3D was required to find the distance of the centre of mass from A. However, a few merely quoted the calculated co-ordinates as the distance or added them together and presented this as the distance required.

4) **Moments and Resolution**

Some excellent responses to this question were seen but the quality of the diagrams in some cases was disappointing.

(a)(i) Those candidates who resolved horizontally and vertically and then took moments about A (or C) or vice versa were usually successful in showing the given results. However, a number of candidates chose to take moments about B without first establishing that $U = 0$ and omitted the moment of U.

(ii) It was pleasing to see a large number of correct responses to this part of the question. Almost all of the candidates appreciated the need to resolve at a pin-joint although some did not appreciate that A was the best place to do this and therefore did more calculations than were absolutely necessary. Those candidates who drew a diagram showing all of the internal and external forces with clear labels were generally more successful than those who either did not draw a diagram or who drew a poor and inadequately labelled one.

(b)(i) The standard of diagrams in many cases was less than helpful to the candidate; forces were omitted or unlabelled; others showed the weight and both components of it as if they were three separate forces. The most frequently omitted force was the frictional force at A and many thought that the normal reaction forces at A and B would be the same.

(ii) This part of the question gave few problems to the majority of the candidates with almost all of them appreciating the need to take moments. A very small number apparently did not understand that ‘normal reaction’ meant the reaction at right angles to the plank.

(iii) Many candidates gained significant credit on this part of the question. However, some very creative working was seen from the few who were determined to find that $\mu = \tan\theta$ come what may.

2609 - Mechanics 3

General Comments

There seemed to be a wide range in the ability of the candidates. Also, although many produced reasoned solutions to every question, others did not seem to be familiar with all of the topics and produced good answers to only one or two questions. Some candidates were unable to do much on any question.

As in earlier sessions, although there were many very well presented scripts, quite a few candidates suffered from their poor presentation, lack of diagrams and lack of indication of methods. For instance, in Q1 many candidates wrote $2h^2$ when they meant $(2h)^2$ and, although they might well claim that 'I know what I mean', quite few deceived themselves into using $2h^2$ instead of $4h^2$. Candidates should know that when asked to *show* a given result, a single step is rarely sufficient.

Unfortunately, Q1 presented rather greater problems to many of the candidates than were intended or anticipated. The accumulated effect of their errors seemed to cause some candidates to spend too long on this question, which may have made it hard for them to finish the paper. Account was taken of this when setting the grade thresholds.

There were some very pleasing solutions to every question and the general standard on the volume and centre of mass question was particularly high.

Comments on Individual Questions

1) **The tension and energy in a stretched elastic band**

Although many candidates made fundamental mistakes, some of them doing so in more than one part, quite a few candidates scored at least half marks despite their errors. There were a number of very neat complete solutions. Quite a common error was to confuse *stiffness* with *modulus of elasticity*.

(i) Most candidates scored all the marks for this section, showing that they understood the initial situation. Some used an unnecessary division of the band into parts corresponding to each side of the square.

(ii) Very many candidates failed to establish the given expression for the extension in the band because they made no reference to its unstretched length; it was not clear whether this was because they (wrongly) thought it was too obvious to mention or whether having found the given result was the length of the band below the lower pegs they thought they had finished.

There were many poor attempts at finding the vertical distance h. Many candidates thought that the equilibrium position could be found by equating the elastic potential energy gained to the gravitational potential energy lost. Many others tried to equate the weight of the particle to some vertical force but took this vertical force to be T or $2T$, where T is the tension in the band. Only a minority drew a clear diagram and realized that T had to be resolved. Many of these errors gave expressions that were obviously wrong and some candidates seemingly spent a lot of time trying to find out why.

(iii) Most candidates rightly used an energy method here. Some took the equilibrium position to be the position of the particle with $h = 0$. Relatively few candidates included all the terms in the work – energy equation. A few omitted both elastic potential energy terms but many omitted the gravitational potential energy and/or one of the elastic potential energy terms.

2) (a) **Dimensional analysis**

(i) Most candidates could write down the required dimensions but by no means all could establish the dimensions of ω and quite a lot of poor notation was seen. The most common error was to argue that the term in brackets had dimensions $L^2 - L^2$, which was dimensionless. Taking a to be acceleration was quite common.

(ii) Some candidates omitted this part. Most who answered it correctly gave frequency or angular speed. Some interesting specific examples were seen such as pulse rate.

(b) **The simple harmonic motion of a fisherman's float**

(i) Quite a few candidates could not come up with a complete argument to establish the constant of proportionality in this example of direct proportion, even with the answer given. Many found plausible combinations of the given values that came to 1.5 but could not say why they were relevant.

(ii) Many candidates knew exactly what to do and did it well. Some did not take x to be positive downwards and they mostly obtained a wrong expression for y in terms of x; these candidates, and many with x correctly defined, made mistakes with the sign of at least one term. Many candidates with a sign error slipped in the necessary 'adjusting' sign change without comment in order to obtain the given result. Quite a few candidates did not use Newton's second law properly and tried to establish the equation of motion without reference to the weight of the float.

(iii) There were many good answers to this part, especially for v. There were some sign errors in the working for t and many candidates who avoided that error still went for the time when the float was going *upwards* at the required height. Some candidates tried an energy approach for v but they typically forgot the work done against the force F.

3) **Motion in a horizontal circle**

There were some very good answers to this question and quite a few candidates (often whole centres) did well at all parts except (i) and the last part of (v). However, on the whole this was the least well answered question on the paper with many candidates (often whole centres) obviously not being familiar with a situation of this sort. It also seemed that many candidates were not familiar with the acceleration towards the centre being expressed in terms of the angular speed.

(i) There were relatively few clear correct answers to this part. Some candidates may have realized that their statements depended on constant angular speed but they did not say so. A common reason given was that there was no friction because the particle was not slipping.

(ii) Many candidates correctly found the normal reaction as the force towards the centre of the circle. One quite common error was to take the acceleration to be v^2/r with v given the value of the angular speed.

(iii) Many candidates obtained full marks for this. The most common error was to take the frictional force to be the force towards the centre and the normal reaction to be the weight instead of the other way round.

(iv) Many candidates could see what to do but not many of them could argue efficiently that $0.3 \times 0.4 \times (10 + 5t)^2 = 3(2 + t)^2$, with most electing to expand the bracket first and then factorise later.

(v) Only a minority of candidates realized that the frictional force could be found by applying Newton' s second law to the transverse motion and a common error was to assume that the frictional force was the weight of the particle. Relatively few candidates could argue the last part properly. Those who saw that μ takes its least value when $t = 0$ and argued from there often scored both marks; those who spotted that the given result could be obtained by putting $t = 0$ often did this without much or any explanation and rarely scored marks.

4) **A volume of revolution and centre of mass obtained using calculus**

There were many complete or almost complete solutions to this question. Relatively few candidates thought they were dealing with areas and the general standard of the working was high. A few candidates (often whole centres) had obviously not prepared this topic and scored very few marks, making elementary mistakes such as integrating the constants as if they were variables. The following comments apply to the majority of the candidates who understood essentially what should be done.

(i) There were a few slips with the constants and the limits of integration and the arithmetic. Many candidates lost the final *show* mark because they did not properly establish how their expression produced the given result.

(ii) Many candidates stated the given result but did not say that this must correspond to $h = r$ or did not show that it worked.

(iii) Again, most candidates knew what to do and did it but others did not do enough to establish the given result.

(iv) Some candidates omitted this part. Surprisingly, many candidates elected to substitute new limits into their expressions in (i) instead of using the results of parts (ii) and (iii). Most who tried to use parts (ii) and (iii) produced a correct expression as long as they had the correct volume for solid B. A very common mistake was to leave the final answer as a distance from O instead of subtracting ½ r to find the required distance from the plane face of B.

2610 - Differential Equations

General Comments

Many candidates showed a good understanding of the techniques required for this unit. The standard of work shown was generally good. Questions one and four were the most popular choices.

The standard of graph sketching was very variable and centres are asked to give candidates the following advice with regard to sketching solution curves. In this unit, sketches are expected to show the basic features of the solution (e.g. oscillating, increasing, decreasing, decaying, growing, asymptotes), and detailed calculations are not required, unless the question specifically asks for them. However, sketches are expected to show the initial or boundary conditions given in the question and any results found in the course of answering the question.

Comments on Individual Questions

1) (i) This was usually correctly answered, although some candidates differentiated with respect to *x* rather than *t*.

(ii) Most candidates knew how to solve the differential equation for *x*, although some candidates omitted the particular integral. Many candidates used a linear or quadratic form for the particular integral. Although this usually led to a correct answer, it was inefficient. When the right hand side of an equation such as this is constant, candidates are expected to use a constant for the particular integral. Indeed the particular integral can be simply stated as the ratio of the right hand side over the coefficient of *x*.

(iii) Candidates generally were able to find the particular solutions.

(iv) The sketches were very variable. Often candidates' sketches did not oscillate. Another common error was for the sketches not to start at the origin, despite the given initial conditions. Some candidates omitted the final request to explain how the long-term values could be found without solving the equations.

2) (i) The sketches were often adequate, although some candidates did not show the initial values on their sketches.

(ii) This was often well done. However some candidates were muddled in the sign of their constant of proportionality. It was surprising that some candidates tried to find the solution with no attempt to set up or solve a differential equation.

(iii) This was often not done well. Some candidates did not realise that *r* needed to be integrated, and those who did often omitted the constant or did not see how to calculate it from the initial conditions.

(iv) Many candidates knew the method required, but accuracy was usually a problem here. Despite the direction given in the question, a sizeable minority of candidates tried to use the integrating factor method, resulting in an integral which few candidates were able to find.

3) (a)(i) The calculations were often done well, but some candidates produced a string of wrong numbers with no evidence of method.

(ii) Most candidates showed some understanding of why the calculation gave an overestimate, but some sketches were unclear, and some did not refer to the significance of the decreasing values of y'.

(b)(i) The solution was often done well, but some candidates omitted the constant of integration, or assumed its value with no method shown.

(ii) This part was rarely done well. Candidates usually did not consider the effect on dy/dx, but concentrated on the effect on y, ignoring the derivative.

4) (i) This was usually done very well, but some candidates only used two of the roots in their complementary function.

(ii) Candidates generally were able to produce two equations using the initial conditions, but some were unable to make use of the condition on the behaviour as t tends to infinity.

(iii) Some candidates did this very well. However some claimed that the expressions for y and dy/dt in terms of u were not zero with little justification. The sketches were generally good, but some omitted the initial value of y.

2611: Mechanics 5

General Comments

The standard of work varied widely, but most candidates were able to demonstrate some understanding of the principles involved in this unit.

Comments on Individual Questions

1) (i) Most candidates integrated successfully, but a surprising number omitted the arbitrary constants. Although they are zero in this case, they must not be simply ignored, particularly when the answer is given.

   (ii) The path was often found successfully, but many of the sketches were incomplete.

   (iii) Some were unable to make any progress here, but others knew the relevant formula and were able to apply it successfully.

   (iv) Many candidates knew how to find the work done, but algebraic slips were common.

2) (a)(i) Most candidates attempted this with Cartesian vectors rather than geometric methods. It was usually done correctly.

   (ii) Again this was usually done correctly.

   (b) Candidates generally knew how to find the closest approach, but many did not show that it occurred just after Y passes vertically above X as requested.

3) (i) This standard result was usually done well.

   (ii) This was also often done well, but some struggled to get the correct expression for the second derivative.

   (iii) Most were able to get the general solution for the differential equation. Most candidates did not know how to use the given condition to find the position of the initial line and just assumed its position. Many candidates were not able to correctly substitute for h in their expression.

4) (i) Most candidates were able to derive the moment of inertia correctly.

   (ii) Again, most candidates were able to do this correctly.

   (iii) Some candidates were unable to calculate the value of λ.

   (iv) Most candidates were able to find the moment of inertia.

   (v) Errors in the energy equation were common, so correct answers were rare.

   (vi) Few candidates were able to correctly find the angular acceleration, either from the equation of motion or from the energy equation.

2612 - Mechanics 6

General Comments

Questions one was not a popular choice, with most candidates attempting the other three questions. The standard of work varied widely, but most candidates were able to show some competence at three questions.

Comments on Individual Questions

1) (i) Most candidates were able to find the angular speed and deduce that slipping occurred.

(ii) The velocity and angular velocity expressions were often well done, although some confusion of signs occurred with the angular velocity.

(iii) Most candidates knew the condition for slipping to stop.

(iv) Most candidates assumed that the frictional force was zero without any explanation, and then used this to show that the velocity was constant.

2) (a) This part was generally done well, except few candidates pointed out that the total moment was not zero, and hence this was a couple (rather than equilibrium).

(b)(i) There were many good solutions, but some tried to work with scalars rather than vectors.

(ii) The word ‘hence’ indicated that candidates should use the angular momentum, which some did not. However, there were many good solutions to this part.

3) (i) Most candidates knew what to do here, but algebraic slips were common. Some candidates made very heavy weather of finding the gravitational potential energy, using very complicated geometry.

(ii) This was often well done except for the $\lambda = 2mg$ case. In this case it was very surprising how many candidates wrongly thought that a zero second derivative guaranteed a point of inflection. Also surprising was that they often then deduced that the equilibrium was stable on one side and unstable on the other!

(iii) Again, most candidates knew what to do, but algebraic errors hindered some.

4) (i) Deriving the relevant differential equation was often done well, but some candidates confused the signs. Most candidates found v correctly.

(ii) Most candidates knew how to find the distance, but many struggled with the integral, even with the result that was given in the question.

(iii) Some candidates produced excellent concise solutions to this part, but some thought that integration was required.

2613 - Statistics 1

General Comments

The examination attracted a broad range of candidates from across the ability spectrum. There were many who were comfortable with the content of the paper and were able to score highly but equally there was a cohort for whom the paper was beyond reach.

Generally, the responses to question 1 were sound but there were many protracted solutions seen in the attempt of the last part of the question. Question 2 was, without doubt, the most successfully answered question with many gaining full or close to full marks.

The work on sampling methods in question 3, with the related probability calculations, caused a noticeable dip in performance by almost all of the candidates. Whilst a sizeable proportion of the candidates made good headway in question 4 there were many erroneous methods and misconceptions prevalent. It was disappointing to see that candidates had trouble with the hypothesis test on the Binomial distribution. Only a few years ago the examiners felt that candidates were beginning to improve in this part of the specification. There are still too many candidates using point rather than tail probabilities to construct their argument.

The presentation of the solutions was generally pleasing with only a small handful of scripts resembling battlefields. Probabilities expressed as percentages have all but now disappeared. As a matter of protocol, candidates should be made aware that they risk losing marks by showing no working. It was not uncommon to see the incorrect answers, with no working, to a question e.g. mean =14.87, standard deviation = 6.7 followed by the words …. (Calc used). The examiners cannot be expected to unpick such a response to find hidden method marks.

Comments on Individual Questions

1) **Estimates of the mean and standard deviation of a discrete grouped data set. Reliability of the calculations. Outlier testing. Linear coding of the mean and standard deviation.**

(i) Almost all candidates were able to find and use the mid-points of the classes to estimate the mean. There was a small minority who thought they had to use the class widths instead of the mid points. Many were able to continue successfully to find the standard deviation but there were a worrying number who calculated f^2x, or even worse, $(fx)^2$ and tried to use these in the standard deviation formula. Candidates who used $f(x-\bar{x})^2$ often made careless errors along the way.

(ii) A little more than the response 'because the data are grouped' was required to earn the mark here for explaining why the mean and standard deviation were not exact values. Some indication that the original raw data had been absorbed into the table or that mid points were being used to represent a class width was needed to clinch the mark.

(iii) There were some very sensible attempts to this part of the question. Most were able to calculate the mean $\pm$ 2 standard deviations and identify the outliers but the examiners did see 1.5 and even 3 in place of 2. The statements and consequent reasoning were usually correct but some insisted that there **had to be** at least 1 outlier above 26.9 rather than there **may be** values above 26.9. Some candidates thought erroneously that they could round the lower outlier of 2.5 to 3 and tried to argue that 'this value was now inside the data'. Candidates did lose marks for (a) not using 14.7 and 6.1 as requested in the question, preferring instead to use the original mean of 15 and standard deviation of 6.6 or (b) attempting to answer the question qualitatively without recourse to any numerical evidence or calculation. The latter group suffered the most penalties.

(iv) Those who realised that they had to substitute the mean of 14.7 into $p = 20\bar{x} + 15$ and the standard deviation of 6.1 into $sd_y = 20\ sd_x$ had the solution out in two lines. However, many proceeded along protracted lines and tried to convert the original data (given as chapters) into pages by using the formula, thus wasting an inordinate amount of time. Often the calculations faltered due mainly to not realising that the original frequencies were required. It was not uncommon to see nearly a page of work with, alas, the incorrect final answers.

2) **Probability question on football scores including conditional probability and the solving of an inequality in relation to the game.**

(i) Very well answered with only a small number giving 0.3 instead of 0.12 as the answer.

(ii) Well attempted. Almost all achieved 0.16 as the answer with a small number multiplying the answer by 2, believing that the order was germane to the question.

(iii) Once again, a very positive response with 0.265 seen regularly.

(iv) Invariably correct but occasionally one of the terms was curiously missing.

(v) Most recognised the need for a conditional probability calculation but many solutions stopped short of this with only the 0.12 being calculated (required for the numerator). A generous follow through from part (iv) was allowed for the denominator.

(vi) Very few candidates were able to set up the initial inequality of $0.4^k > 0.01$ with alternatives of $0.4^k = 0.01$ or $0.4k = 0.01$ or even $0.2^k > 0.01$ being regularly seen. For those using trial and improvement it was essential that they tested 0.4^5 **and** 0.4^6 in order to gain the method mark. Some only went as far as 0.4^5 and then declared unequivocally that k = 5 **must** be the answer. Such faltering logic was penalised. There was a fair minority who thought that the question was asking for $p(X \geq k) > 1\%$.

3) Systematic Sampling of components. Comparison of sampling procedures. Using random numbers to select a sample. Calculation of the number of selections with associated probability methods.

(i) Most candidates scored at least one of the two marks available here. Whilst most realised that a selection of every 10th component was necessary, fewer appreciated that a random starting value between 1 and 10 was needed for the selection of the first component. For those deciding to choose a starting point (above 10) there had to be a clear indication that the cycle was being completed if every 10th component was mentioned. A small minority of candidates thought that the systematic sample was to do with the times of the day being split up before the components were selected.

(ii) Many candidates gave sensible answers to this part of the question, realising that for the advantage a response along the lines of cheaper/simpler or less time consuming was required. For the disadvantage, many realised that such a form of sampling on one day only was not necessarily representative of the rest of the week.

(iii) There was a variety of responses to this part. At a simplistic level some candidates thought they could select the 200 components by using the random numbers 000 to 999 without any further ado or consideration of random numbers greater than 200. This gained no credit. At the next level, a sizeable majority stated that if the random number generated was 001 to 200 (and discarding numbers greater than 200) then the components that had been allocated these numbers could be selected. This deserved 1 out of the 3 marks available. The more discerning candidates realised that they had to do something with the random numbers greater than 200. Various acceptable methods were **either** allocating blocks of numbers to each component e.g. 000 – 004 corresponded to component 1; 005 - 009 corresponded to component 2 …… 995 – 999 corresponded to component 200 **or** dividing each generated random number by 5 and rounding up/down to create a number back in the range 1 to 200 **or** even slicing layers of 200 from the generated random number e.g. if the random number generated was 201 – 400 then subtract 200; if the random number generated was 401 – 600 then subtract 400 etc. The final mark, that very few earned, was for realising that repeated numbers must be discarded. One wonders about the definition of a random sample that one candidate gave: 'A random sample is the random a sample can get but it will never always be 100% random'.

(iv) Invariably answered correctly with $^{15}C_5 = 3003$ being seen.

(v) This part of the question proved to be difficult for many candidates with many believing that a binomial probability calculation was required which, of course, it was not. The correct response to (A) of either $\dfrac{\binom{13}{5}}{\binom{15}{5}}$ or $\dfrac{13.12.11.10.9}{15.14.13.12.11}$ or more succinctly $\dfrac{10}{15}\cdot\dfrac{9}{14} = \dfrac{3}{7}$ was seldom seen. Instead, the examiners saw on many occasions $\dfrac{13}{15}\cdot\dfrac{12}{14}$ or $\binom{15}{0}\left(\dfrac{2}{15}\right)^0\left(\dfrac{13}{15}\right)^{15}$ or marginally better $\binom{5}{0}\left(\dfrac{2}{15}\right)^0\left(\dfrac{13}{15}\right)^5$. Only the latter solution gained some credit under 'special case'. Similarly, in part (B) the expected working of $\dfrac{\binom{13}{4}\binom{2}{1}}{\binom{15}{5}}$ or $\dfrac{5.13.12.11.10.2}{15.14.13.12.11}$ or more succinctly $\dfrac{5}{15}\cdot\dfrac{10}{14}\times 2$ was often replaced by the incorrect versions of $\dfrac{2}{15}\cdot\dfrac{13}{14}\times 2$ or $\binom{15}{1}\left(\dfrac{2}{15}\right)^1\left(\dfrac{13}{15}\right)^{14}$ or marginally better $\binom{5}{1}\left(\dfrac{2}{15}\right)^1\left(\dfrac{13}{15}\right)^4$. Again, only the last showing gained some credit under 'special case'. A fair proportion of candidates omitted this part of the question.

4) **Use of the Cumulative Binomial tables or formula in the context of examination passes. Expectation of the Binomial distribution. One tailed hypothesis test on the Binomial distribution.**

(i) A surprising number of candidates fell at the first hurdle. Many were unable to use the binomial tables correctly to find *p*(X = 13). Some mistakenly believed that *p*(X =13) was found from
p(X $\leq$ 14) – *p*(X$\leq$ 12).

(ii) Again, many errors were seen in the calculation of *p*(X$\geq$ 8) with many believing it was found from 1- *p*(X$\leq$ 8) or even 1 – *p*(X = 7). On occasions the examiners wondered whether some candidates had access to the binomial tables particularly when candidates resorted to protracted methods by calculating $\sum_{9}^{20} p(X = x)$ or even
1 - $\sum_{0}^{7} p(X = x)$.

(iii) Invariably correct but curiously many went on to calculate *p*(X = 11) which was **not** asked for in the question.

(iv) The statements for H_0 and H_1 were usually given in the correct form but there are still candidates who squander valuable marks by using a sloppy notation. As it has been mentioned in almost every previous report it is **NOT** acceptable to write H_0 = 0.55 or even H_0 : *p*(x = 0.55). Such notations are penalised. The explanation of 'why the alternative hypothesis took the form it did' was usually well answered by most but this year's howler must go to the candidate who wrote 'because the **police** meant to increase the average number of pupils passing at grade C or above'. Certainly one alternative to present educational methods! The subsequent work on the hypothesis test was quite depressing with an inordinate amount of candidates favouring an argument involving point probabilities rather than a tail probability. Even those who knew they had to find *p*(X$\geq$ 16) often faltered by giving 1 – *p*(X $\leq$ 16) instead of 1 – *p*(X$\leq$ 15).

2614 - Statistics 2

General Comments

Overall the performance of candidates was slightly better than in the June 2004 paper. There were fewer very weak scripts and a number of outstanding submissions. The candidates showed, on the whole, a good grasp of the basic methods including accurate and structured solutions.

The stronger candidates scored highly on all the questions with only the final comments in question 1 and the final probability calculation in question 2 causing regular problems. Weaker candidates tended to gain the majority of their marks in calculating Spearman's rank correlation coefficient and carrying out the associated hypothesis test in question 1 and working though standard Normal calculations in question 2.

Most answers were well presented and generally supported by sensible working and explanations.

Comments on Individual Questions

1) **Bivariate data: Spearman's rank correlation: calculation, hypothesis test, comments: comparison of marks of two judges of shops in a retail chain.**

This was a good starting question for most candidates. The first two parts were usually well answered. The final two parts discriminated well between stronger and weaker candidates. It was pleasing to see that the majority of candidates set out their working for the hypothesis test in a clear and logical fashion.

(i) Most candidates demonstrated that they knew how to calculated Spearman's rank correlation coefficient proficiently. There were only occasional arithmetic slips; common errors were forgetting the " 1 – " in the formula and numerical slips in squaring *d*. The weakest candidates attempted calculations based on the difference in the marks, rather than the ranks, for which no credit was given.

(ii) Generally candidates set out their hypotheses and subsequent calculations and explanation very well. However, a prevalent error was to write the alternative hypothesis as a two-tailed test. Most candidates compared the test statistic with the critical value and expressed their conclusion in context. Candidates using a two-tailed test were able to gain all but one of the marks for this part of the question.

(iii) Only the most able candidates gained both marks for their comment. The required answer was that "the background population should be *bivariate Normal*". Credit was then given for the candidate's evidence in discussing whether or not the scatter formed an ellipse, and hence whether or not the product moment correlation coefficient was valid. Often only one mark was gained by discussing the elliptical nature of the scatter, with no mention of bivariate Normality. Weaker candidates missed the point completely, referring only to the linearity of the data.

(iv) This part found most candidates wanting. The modal mark was 1 out of 3. Most candidates did not use ranks, but preferred to compare the performance of shops G and J using marks, often concluding that "shop G was best because it gained the highest total (or average) marks from the two judges". A more subtle analysis was required. Since there was such a discrepancy in the spread of marks of the two judges, then ranks would be better to compare the shops' performance. To gain full marks candidates were expected to compare ranks given to the shops by both judges: shop J came 1st and 3rd, whereas shop G came 5th and 1st. Whilst shops G and J were both awarded 1st place by one of the judges, shop J had a better aggregate ranking than shop G.

2) **Normal distribution: sketch diagram, Normal and binomial calculations: modelling the distribution of lengths of men's trousers**

This probability question turned out to be accessible by even the weakest of candidates. The majority of candidates scored full marks in parts (i), (ii) and (iv). However, only a small minority gained any credit in part (v).

(i) Nearly all candidates gained both marks for the sketch. Occasionally a marked was dropped because of poor labelling.

(ii) Most candidates demonstrated a good understanding of using and applying a Normal probability calculation, with many gaining full marks. Occasional errors were usually in manipulation of the probabilities, e.g. using '0.9192 – 0.7976', which lost the last two marks.

(iii) The binomial calculation was often carried out correctly, but some candidates either misinterpreted this part of the question or omitted it altogether. A large number of candidates failed to gain credit for the assumption, some missing it out completely. Even when attempted, the required answer of 'a random sample from the population' was rarely seen.

(iv) Most candidates knew how to use the Normal distribution 'backwards' and gained all three marks for finding the shortest length for *Extra Long* trousers.

(v) This final part of the question was rarely attempted, presumably because of lack of understanding of what was required. Even when a solution was given, it was rarely correct. Only the strongest candidates were able to express the condition in terms of solving the inequality "$1 - 0.98^n > 0.9$". Even fewer successfully used logarithms, or very occasionally trial and improvement, to find the required value of n (114). A very small number of candidates successfully applied a Poisson approximation (expecting a 'large' n with the small p).

3) **Poisson distribution: calculations and comments, Normal approximation: modelling the distribution of the number of items of junk mail received daily.**

This question proved a good mark earner for most candidates. The main error was the lack of precision in the answer given for the descriptive part.

(i) Very few candidates scored both marks in identifying two features for which a Poisson distribution would be suitable. The required answers were "uniform average rate of occurrence" and "junk mail is likely to arrive randomly and/or independently". Quite often the second response was not put in context, thus losing the mark available.

(ii) Most candidates successfully explained why the mean was 1.3, and many followed this by concluding correctly that the variance was 1.25. However, there seemed to be much confusion here between a statistical calculation – which was required – rather than a probability calculation – which was condoned. Prevalent errors in calculating the variance included 'forgetting to divide by 100'and 'forgetting to subtract 1.3^2'.

(iii) This was part usually well done, with a correct conclusion that another good reason for using the Poisson distribution as a model was that the sample mean and variance were approximately equal.

(iv) In part (A), nearly all candidates calculated the Poisson probability correctly using the formula. Very rarely were tables used, despite being a perfectly good method.
In part (B), the correctly value of λ (7.8) was identified by nearly everyone, with many going on to gain full marks for this part of the question. However, a disturbingly large minority of candidates interpreted 'P(X > 10)' as '1 – P(X ≤ 9)' rather than '1 – P(X ≤ 10)', this losing the final two marks.

(v) It was good to see many completely correct responses to the "Normal approximation to the Poisson distribution". Candidates seemed to be well prepared for this type of calculation. Prevalent errors, which have occurred on previous occasions, included the omission of, or incorrect, continuity correction, and incorrect use of the extrapolated variance from part (i) in the Normal approximation.

4) **Discrete random variables: calculations and explanations, expectation and variance: matching pictures with locations.**

Most of this question was successfully attempted by the majority of candidates. However, parts (i) (C) and (iii) (calculation of the variance) proved a pitfall for many.

(i) Reponses to part (A), explaining why P(X = 3) = 0, were often very good, as was the derivation of the constant k in part (B). The modal mark for part (C), using a probability argument to show that P(X = 4) = $\frac{1}{24}$, was 0. Only the most able candidates were able to provide a suitable explanation, usually in terms of a product of probabilities.

(ii) Most candidates were able to score full marks in evaluating the expectation and variance for this discrete random variable. Occasionally some used decimals and lost at least one accuracy mark. On rare occasions candidates forgot to subtract $E(X)^2$ from $E(X)^2$.

(iii) Most candidates obtained the correct expectation (£ 100), but failed to find the correct variance ($£^2$ 10000), for getting the rule $\mathrm{Var}(aX) = a^2\mathrm{Var}(X)$.

(iv) Many candidates used the binomial distribution correctly in part (A), but slightly fewer correctly found the expected prize money in all six rounds (£5400). A prevalent error was to forget that the first round did realise a prize of £400 and use the expected value (£100), thus realising a total of £5100. This answer gained the method mark, but not the accuracy mark.

2615 - Statistics 3

General Comments

There were slightly fewer than 800 candidates for this paper, compared with about 1000 in June 2004. Once again the overall standard of the scripts seen was pleasing: many candidates appeared well prepared for this paper. However, as in the past, comments and explanations were a consistent weakness.

Invariably all four questions were attempted. However, Questions 1 and 2 were well answered, with many candidates scoring full or nearly full marks. On the other hand the marks scored in Questions 3 and 4 seemed to be more uniformly spread across the range. There was evidence to suggest that candidates found themselves short of time at the end: in many cases Question 4 appeared rushed or unfinished.

Comments on Individual Questions

1) **Continuous random variables; sales of petrol.**

(i) On the whole this part was well answered, although there were a number of candidates who appeared less familiar with how to find the mode than they were with other parts of the question.

(ii) The quality of sketching was felt to be quite poor. Many candidates' curves were sloppy and careless. The most common failing was neglecting to show a gradient of zero at $x = 0$, a feature that should have been obvious from a careful analysis in part (i).

(iii) The mean and variance were found correctly in the vast majority of cases, but the examiners would have liked to see better presentation and attention to detail, and correct notation.

(iv) There were many good, completely correct answers to this part too. The errors that occurred were usually to do with the variance. Some candidates tried to work in litres or millions of litres but they inevitably came unstuck because they could not get the variance to agree. As above, correct and consistent notation (such as using $52X$ when they mean $X_1 + X_2 + \ldots + X_{52}$) was in fairly short supply.

2) **Combinations of Normal distributions; confidence interval for the population mean using the *t* distribution; the times taken to complete components of a fitness training programme.**

In this question some candidates appeared not to understand the context: their answers seemed to suggest that they thought that they were dealing with the *manufacture* of components. Also it was very widespread to see candidates using A, B and C as the random variables rather than X, Y and Z given in the question.

(i) This part was usually correct, although a few candidates added the standard deviations rather than the variances.

(ii) This part was often correct too. The difficulties encountered resulted from an incorrect formulation of the requirement of the question (leading to the complement of the right answer) or from the wrong variance for the difference in times used. Once again the use of notation left much to be desired: it seemed that many candidates do not handle inequalities well, sometimes preferring to omit them altogether. A surprising error which happened sufficiently often to draw comment was "21·4 – 20·4 = 1·4".

(iii) There were many correct answers for the confidence interval. It was pleasing to see so many candidates identify correctly the appropriate percentage point from the *t* distribution. But there were those who used 1·96, from the Normal distribution, instead, and/or the wrong standard deviation.
The greatest difficulty in this part of the question was the interpretation of the interval. Some candidates ignored the interval altogether, arguing that 19·5 is less than 20·4 therefore there must have been a reduction in the training time. Others came to the same conclusion by saying that 20·4 was in the upper half of the interval. Others simply omitted to make any comment.
Some candidates set up their entire answer to this part of the question as a hypothesis test.

(iv) This was badly answered. Candidates had not read the preamble to parts (iii) and (iv) carefully enough, and their answers failed to address the question of whether *these* (first) 8 recruits could be regarded as a *random* sample. Two common misconceptions were that sample size was a relevant issue and that for "random" one could substitute "representative".

3) **Hypothesis test for the population mean using the Normal distribution; Type II error; efficiency measures for electric fans.**

(i) The hypotheses were usually stated correctly but many candidates neglected to define the symbol μ.
The test statistic was often worked out correctly. Most, but not all, appreciated that they were given the standard deviation for the population and that it did not require any adjustment. However the small sample size caused some to use the *t* distribution.
Despite the fact that they had given a correct alternative hypothesis earlier, the sign of the critical value quoted by many candidates did not always agree with it. One wondered if they properly understood that they were (or should have been) carrying out a 1-tail test at the lower tail.

(ii) On the whole a greater proportion of candidates than in the past showed that they understood something about Type II errors. However significant numbers of candidates worked out their critical point using the sample mean and/or used the distribution $N(530, 14^2)$ even when they had used the correct standard error in part (i).

4) **Chi-squared hypothesis test for the goodness of fit of a Poisson model; confidence interval for the population mean using the Central Limit Theorem and the Normal distribution; monitoring radiation levels.**

As mentioned above, many of the answers to this question contained careless errors or were incomplete, suggesting that candidates were running out of time at this point.

(i)(A) Hardly any candidates failed to earn the mark for this part, though, worryingly, when a sample mean other than 2 was found the candidate concerned was likely to persist into part (B) with his/her incorrect mean.

(B) Most candidates found the correct expected frequencies using the model, although, despite the prompt in the table, many neglected to either include the class "more than 6" or to check that their expected frequencies added up to 100. There then followed some uncertainty about the criterion for combining classes: there were those who decided to combine on the basis of low observed (rather than expected) frequencies. Nonetheless the correct test statistic was obtained in the majority of cases.
Some candidates identified the wrong number of degrees of freedom and hence the wrong critical value. This was usually because they did not allow for the estimated parameter (the mean) and/or for having combined classes.
After stating a conclusion to the hypothesis test carried out, almost all candidates omitted to go beyond and "comment briefly".

(ii) There were many good answers to this question, but also there were many that showed signs of being rushed. Most realised, even if they did not say so, that the CLT allowed them to use the Normal distribution here. However some wanted to use a percentage point from t_{99} or t_{100}. Quite a few candidates were unable to cope with the summary information in the form supplied, particularly when trying to estimate the standard deviation (many thought that 1216·68 *was* the variance).

2616 - Statistics 4

General Comments

There were 93 candidates from 20 centres (June 2004: 82 from 20). The overall standard of the scripts seen was pleasing: many candidates were clearly well prepared for this paper. Routine calculations were carried out well but the candidates' ability to comment and interpret were a little disappointing at this level.

Question 1 was by far the least popular question with only about 15 candidates attempting it. Every candidate attempted Question 2; Questions 3 and 4 were equally popular.

Comments on Individual Questions

1) **Estimation theory**

Although this was the least popular question it seemed to have the highest mean mark, with most of those attempting it scoring full or almost full marks. Those who were prepared to try it were likely to be successful as long as their algebra was up to the task. Sometimes the algebra arrived at the correct destination by brute force rather than elegance.
There were just two places where marks seemed likely to be lost: part (iv) where some neglected to verify that the required value of k did indeed give a minimum and part (vi) where there was a temptation for some to use the converse argument.

2) **Two sample *t* test and confidence interval; the strengths of steel rods**

This was the most popular question being attempted by all candidates. It was also a very high scoring question: about half of the entry scored full or almost full marks.

(i) The hypotheses were usually stated correctly but there was rather less care in providing verbal definitions of the population means. Similarly, the required assumptions were sometimes less than ideal.

(ii) Most candidates carried out the test competently. There was rarely any problem over finding and using the pooled variance. The critical value was almost always correct but on a number of occasions the conclusion was badly expressed.

(iii) As in part (ii) most candidates had little difficulty here. Just occasionally the standard error (which had been correctly constructed in part (ii)) became "pooled $s \times \frac{1}{\sqrt{17}}$".

(iv) This part was almost always correct.

3) **Paired sample *t* test and one-sided confidence interval; comparing fertilizers**

(a) The hypotheses were usually stated correctly but candidates were not as careful about defining the symbol μ. Nor were they sufficiently careful when it came to the distributional assumption.
However there were only a very few candidates who did not realise that they should carry out a paired test. The vast majority made good progress with the test itself, and only the final conclusion left room for improvement.

(b) As above, most realised what to do here and the correct value for the lower bound was usually found. A small minority tried to construct the confidence interval using the information from the paired test. There was some uncertainty again with the distributional assumption.
The main area of difficulty was with the interpretation of the interval. Very many comments revealed a flawed understanding of a confidence interval to quite a worrying extent.

4) **Wilcoxon rank sum test for the median; Chi-squared test for goodness of fit; waiting times in an airport**

(a) This part of the question was almost always answered well. Many fully correct solutions were seen.

(b) (i) This part was frequently done correctly.

(ii) Most candidates calculated a correct value of X^2 (with or without grouping) but relatively few were able to identify the correct Chi-squared distribution to look up. Most of those who got this second aspect wrong made no allowance for estimated parameters while a few thought that there were 200 degrees of freedom. Hardly any commented on the fact that the test statistic was significant at **any** level available to them in the tables.
Disappointingly few candidates took the trouble to comment at all on the reasons for the poor quality of fit.

(iii) In this part of the question very few candidates realised that they could refer back to the previous part for evidence that the assumption of background Normality was not viable. They knew that Normality was required, but often chose to look at the sample data in part (a), sometimes with the aid of a dot plot. Hardly any candidates included in their discussion the small sample size which might prompt the use of a *t* test.
No more than a handful of candidates picked up on the fact that a *t* test examines the population mean whereas the Wilcoxon test in part (a) examined the median.

2617 - Statistics 5

General Comments

There were only 13 candidates, from 7 centres – including some unfamiliar ones, which it was nice to see among such a small entry for the last regular sitting of this module.

In view of the small number of candidates, this report is couched in very general terms so as to avoid any possibility that individuals are identifiable.

Comments on Individual Questions

1) This was on probability generating functions, based on the Poisson distribution and the sum of Poisson distributions. Candidates were able to do the technical work in the first three parts of the question, as far as and including using the pgf to find the distribution of the sum. However, in the last part there was considerable insecurity in use of conditional probability, so that several candidates were left with a struggle to try to manipulate incorrect expressions so as to achieve the given result.

2) This question was based on moment generating functions, leading to a proof of the central limit theorem for the case of an exponential distribution. Most candidates met with considerable success here, perhaps helped by the substantial number of intermediate steps given in the question.

3) This question was based on the chi-squared test for variance. The initial test was usually done correctly. Most candidates could then integrate the given pdf of the chi-squared distribution with 4 degrees of freedom so as to obtain the cdf, and most candidates then knew how to use this to obtain the level of significance of the data. Not all, however, grasped the point about the relation of this to entries in the chi-squared table. The next part of the question was concerned with setting up the acceptance region for the test in a general way. Most candidates seemed to know what to try to do, but there were some difficulties in doing it. However, the given result was used well in the final part in deriving values of the operating characteristic of the test, usually with sensible interpretations of the rather poor nature of the test in this (very small sample) case.

4) This was a composite question covering a confidence interval for a difference between two proportions and a test for the equality of two variances. Mostly it was done quite well. The *F* distribution with 9 and 7 degrees of freedom is not tabulated in the MEI tables; candidates were expected to overcome this and did so in a variety of ways. A fairly common error was to work with upper-tail 5% points whereas, as the test is two-sided, upper-tail 2½% points should have been used.

2620 - Decision and Discrete Mathematics 1

General Comments

This paper was a subset of the paper set for 4771, and this report overlaps greatly with that of 4771.
Candidate performances on 2620 were generally good – much better than has been the case in the past. This was to be expected since most AS candidates will have been taking 4771.

Comments on Individual Questions

1) **Graphs**

(i) Part (i) asked for the number of connections which the electrician has to make. However, some candidates gave the number of arcs in their network.

(ii) Those making the error referred to in part (i) usually added 1 to their answer.

(iii) Examiners do not expect candidates to show any detailed knowledge of the scenarios presented. Nothing is required beyond that which is given in the question. Thus they should not have been looking to their knowledge of domestic electricity circuits, nor bemoaning their lack of such knowledge, in attempting to answer part (iii). The issue here is that which has been considered in past examination papers – that introducing a new vertex into a network can have the effect of reducing the weight of the minimum connector.

2) **Algorithms**

(i) Most candidates were successful with this question. Those that failed mostly allowed themselves to get stuck in a dead end.

(ii) That the algorithm does **not** leave one stuck in a dead end was not a sufficient answer to this question – that alone does not guarantee a route from entrance to exit. What was required was the recognition of the existence of two continuous connections between entrance and exit, the "northeast" wall (plus protuberances) and the "southwest" wall (plus protuberances).

3) **CPA**

Most candidates were very successful with this question. Performance was much better overall than is usually the case on longer CPA questions set in context.

4) **Networks**

(i) This was a very discriminating question. Good candidates started their Dijkstra from C. Less good candidates started from P.

(ii) Kruskal is arguably the conceptually easiest algorithm on the syllabus. It might be expected that only the very weakest candidates would be unable to answer this question. However, rather more candidates then expected were not able to.

(iii) Very many candidates failed to score this mark by not providing an adequate answer. Noting that there will be a reduction in length is not an adequate answer to a question asking for the effect of a change. By how much, or to what, is required.

(iv) As per part (iii).

(v) Most candidates recognised the semi-Eulerian issue, if usually implicitly. Unsophisticated students gave a route as justification. Others noted the two odd nodes or pointed out that, since there was such a route from P to C before the bridge, a route is now given by crossing the bridge and then following that original route.

5) **LP**

(i) Candidates exhibited all the usual weaknesses. At the worst extreme some identified variables (sometimes explicitly and sometimes implicitly) to do with fibre and nutrient, rather than with Flowerbase and Growmore. Less disastrously very many candidates failed adequately to define their variables (e.g. "Let x = Flowerbase and y = Growmore"), and many failed to note that the problem is a maximisation problem.

(ii) Too many candidates assumed that the optimal solution would be represented by the intersection of the two non-trivial constraint lines.

(iii) Not everyone who answered (B) correctly was able to provide an adequate justification.

6) **Simulation**

(i)(ii) Most candidates scored all 4 of these marks.

(iii)(iv) Not many failed at this next fence.

(v) This was answered quite well. Mistakes were easy to make, and were made, but most candidates showed a good understanding of what was needed.

(vi) Many candidates attempted to answer this question as per part (v), but with returns generated by the new distribution. In fact, the new distribution only comes into play after the number of laptops in stock drops to 2 or fewer. Thus the start of this simulation should be the same as the start of the simulation in part (v). It often was not.

2621 - Decision and Discrete Mathematics 2

General Comments

This paper was a subset of the paper set for 4772, and this report overlaps greatly with that of 4772.
Very few candidates scored high marks and very few performed badly. Marks were fairly evenly distributed across the middle range. Candidates generally seemed to have been well prepared for the paper, but there was some evidence that some candidates were short of time.

Comments on Individual Questions

1) **Logic**

This question was answered extremely well – maximum scores were not uncommon.

2) **Decision Analysis**

Most candidates were able to complete part (i) and gain at least some credit on part (ii). Few gained much on part (iii) however, the concept of utility completely passing most of the candidates by.
This question also revealed a significant difficulty in work on Decision Analysis. Alternative approaches are possible to the accounting, but some have the potential for causing problems. The safest is to work with final payoffs. Thus in part (i) candidates who worked with profits came to the correct answer with effectively the same computations as those using payoffs, but that was not the case in part (ii). The problem here is that, if r(t) is the exchange rate and v(t) is the value of the investment, then

$$r(t) \times (v(t) - v(0)) \neq (r(t) \times v(t)) - (r(0) \times v(0))$$

The left hand side of the above expression is what many candidates used – it results from working with profits. The right hand side is correct, and is consistent with the answer obtained by working with payoffs.
Whilst this error is not obvious, working with profits rather than payoffs in part (iii) is a fundamental mistake. Utility functions give the utilities of positions not changes.
The definition of the utility function itself created some problems (…**thousands** of euros”), but those using euros instead of thousands of euros were not penalised.

3) **Networks**

Most candidates found some success with this question. Typically, they began well with parts (i), (ii) and (iii). However, by part (iv) errors began to creep in and by parts (v), (vi) and (vii) it was common to see candidates using the wrong algorithms to answer the various parts. When calculating the lower bound in part (vi) a common error was to identify (correctly) the various arcs, finally writing 1 + 3 + 1 + 2 + 3 = 9!
In part (iv) a few went into knee-jerk routine and produced a full-blown working of Floyd, inevitably wasting a fair amount of time.

4) **LP**

This question was generally answered well, but there were two distinct causes of problems.
In part (ii) a clear majority chose the wrong pivot. This leads to a negative element appearing in the last column, something avoided by a correct application of the ratio test. Candidates making this error seemed unaware that it was causing a problem and carried on, often producing solutions that were patently incorrect.
The second difficulty came in part (iv). The first mark here was asking why it is that Theo's formulation, though incomplete, leads to the correct solution. The answer looked for was that the constraints he omitted are (clearly) not active in the solution. Candidates did not recognise the issue.

2622 - Decision and Discrete Computation

General Comments

This paper was substantially the same as the paper set for 4773, and this report overlaps greatly with that for 4773. On this paper each question was marked out of 20 and candidates were required to attempt 3 out of 4 questions. The questions were reduced slightly in content for 4773, and were worth 18 marks each, but candidates were required to attempt all 4.

Candidate performances on 2622 were good generally good. A small number of candidates appeared not to use Lindo at all, which consequently affected at least one question. In a few other cases, Lindo appeared to have been used to generate a solution, but no evidence was included with the script.

Candidates should take care in labelling their computer printout pages, ensuring that they have the correct question number on them and that they are assembled in the correct order.

Comments on Individual Questions

1) **Recurrence relations**

(i) Most candidates got this right, although some computed u_2 as their answer.

(ii) This was a little more difficult than part (i), and a significant number of candidates failed on it.

(iii) A large proportion of candidates managed to find their way completely successfully through this intricate calculation, though some did not understand the use of a particular solution and tried to insert the constant from the recurrence relation.

(iv)(v) Most candidates succeeded in building correct spreadsheets. Not all of those achieved full marks, failing to make simple observations about convergence and limits.

2) **Networks**

(i) to (v) This work on network theory was completed on the insert. It was generally well done. Some candidates had difficulty with part (iii) which, for them, made part (iv) more difficult than it should have been. Nevertheless, they were able to recover since any flow pattern giving a total flow of 6 was acceptable.

(vi) & (vii) Most showed a good idea of how to construct the LP model, even if mistakes were made along the way. Again, there was a weakness in extracting results from the output.

3) **Simulation**

(i) Most could do this in principle, but a significant minority made one of two mistakes –

- using 0, 0.15 and 0.75 instead of 0, 0.15 and 0.9 in their lookup table
- failing to accumulate the service times.

(ii) A significant minority of candidates failed to compute the standard deviation of their 10 accumulated times. In some instances candidates tried to do the computation longhand, instead of using the spreadsheet function.
The majority of candidates could do the computation to determine approximately how many repetitions are required. However, it was quite common to see answers in error by a factor of 4.

(iii) Almost all candidates succeeded with this.

(iv) Most candidates could do this simulation, but many failed to compute the queuing times.

(v) A majority could build this two-server simulation, but again, many failed with the queuing times. A number of candidates treated the barriers as being in series rather than parallel – this of course extends rather than reduces the exit time and thus defeats the purpose of this section of the question. Others had errors in their formulae and did not appear to check their computed values for reasonableness.

4) **LP modelling**

(i) It was expected that there would be errors made in this part of the question. In fact, many candidates got it completely right.

(ii) It is sometimes the case that simple, single-mark questions elude candidates. The obvious is not spotted. This was the case here. All that was required was the realisation that minimising the number of schedules minimises the number of pilots required. The question was intended to help candidates build their LP model in part (iii).

(iii) & (iv) It is possible that part (ii) did its job, even for candidates who gave wayward answers, for most succeeded in building and running the LP model. Unsurprisingly though, many of those who produced output failed to interpret it to say how many pilots were needed. A number of candidates missed one or two schedules from constraints and did not appear to check back through their work.

(v) Very few candidates scored any marks on this. All that was required was a systematic suppression of each of the three solution schedules in turn. In each case more than 3 pilots are required, showing that there is no alternative solution. A few candidates produced alternative, logically reasoned arguments based on permutations of 4-flight schedules, whilst others tried to base their answers on their original Lindo output.

2623 - Numerical Methods

Comments on Individual Questions

1) **Errors and approximations**

In part (a) there was some confusion between absolute and relative error. Many candidates did not appreciate that to maximize relative error it is necessary to make the denominator as small as possible.

Many of the explanations in part (b) were not as clear as they might have been. The calculations were mostly accurate, though some candidates confused pounds and pence.

Part (c) was worth 5 marks, but many answers made only one or two points. The question contains several quite distinct requests and, as a matter of examination technique, candidates would be advised to respond carefully to each one in turn. There were a small number of excellent solutions to this part.

2) **Secant and fixed point methods to solve an equation**

Parts (i) and (ii) were generally well done, though some candidates did not use the methods specified. There can be no credit for using an alternative method even if it gives the correct numerical solution.

In part (iii), many of the sketches were of poor quality and the accompanying explanation was frequently inadequate. The point here is that the curve has a shallow gradient to the left of the root but is steep to the right of the root. This makes the secant method slow to converge.

3) **Numerical integration**

The numerical work was very well done in parts (i) and (ii), with many candidates appearing to be comfortable using the relationships between the trapezium rule, the mid-point rule and Simpson’s rule to minimize labour. In part (iii) the extrapolation defeated some, but for many it proved no problem.

4) **Difference table, Newton’s forward difference method**

The missing values in the difference table were found correctly by most, though some made sign errors. Demonstrating the value of *a* presented little difficulty. The algebra required to obtain the cubic was more of a challenge, however, and there were many errors. In part (iii), a significant number did not think to find the minimum by differentiation.

2624 - Numerical Analysis

General Comments

There was only a small candidature for this paper in its final session, but the standard was gratifyingly high.

Comments on Individual Questions

1) **Errors and accuracy**

In the first four parts of this question the calculus and the numerical work were handled confidently by almost all. The final part, moving from the error in $\cos\varphi$ to the error in φ, defeated a few.

2) **Taylor series**

The calculus in part (i) was done well by everyone, and almost everyone was able to explain the approximation near the origin in terms of x^3 and y^3 being negligible. The rest of the question was generally done well, though there was some confusion in relating x to h at the end.

3) **Summing series**

Though this was the least popular question, it presented little difficulty to those who tackled it. Adding in the first correction term in part (ii) gives a dramatic increase in accuracy. Adding in the second correction term in part (iii) gives complete agreement to 7 significant figures.

4) **Divided differences**

Yet again, almost all candidates performed very well on this question. In part (iii) a rough sketch of the data shows that the two quadratic approximations have opposite curvature; it is this that makes the estimates of the root so different. In part (iv) candidates had no difficulty producing the required cubic. Perhaps surprisingly, some were not sure what to do in the final part. The intention was that they should show a change of sign by substituting 2.55 and 2.65 into the cubic.

4751 - Introduction to Advanced Mathematics

General Comments

This was the second time this paper has been sat. The candidature was smaller than in January, with fewer year 13 students transferring to the new specification.
There were many excellent scripts, but also a long tail of very weak candidates who appeared to have gained little from the course.

A calculator is not allowed in this paper and, as in January, some candidates found this a considerable disadvantage, making errors in basic arithmetic as well as when they used longer methods such as using the quadratic formula when an equation factorises. In question 8, for instance, some were unable to divide 112 by 7 or 8, in spite of the fact that they had already established $7 \times 8 = 56$ and that short division and other options were available to them. In general, time was not a problem, but for candidates who used long methods, or who re-worked questions in an attempt to eliminate their errors, there was some evidence of 'rushing' on the last question.

As in January, examiners found that many candidates wasted time by plotting graphs on graph paper when a simple sketch was requested. Teachers are asked to note that in a sketch, relevant points such as the intersections with the axes should be labelled appropriately. It is quicker and better for both candidates and examiners if sketches are done in the examination booklet rather than on graph paper.

Presentation was generally good, but some were careless with algebraic notation and language. For instance, in question 12 many omitted brackets for the quadratic factor when writing expressions such as $(x + 1)(x^2 - 7x + 10)$, and many gave an expression not equation as requested for the answer to the last part.

Although the content for this paper does not include calculus, some students were able to use their knowledge of calculus to answer some questions using alternative methods, most frequently in question 10 part (i). However, some used their knowledge of the C2 content inappropriately, for instance in question 4 to find the sum of 3 consecutive integers using the formula for the sum of an arithmetic progression and making the question rather harder for themselves in the process.

Comments on Individual Questions

Section A

1) Most candidates attempted a long division, with many gaining a method mark for the start of the long division but being unable to cope with the subtraction or with the lack of an x term. Nearly all those who used the simpler remainder theorem method were successful and gained the simple two marks intended for the start of the paper.

2) Many gained a method mark for collecting x and y terms on different sides of the equation. However, many errors were made after this, with weaker candidates often trying to divide by m and making further errors. Many failed to factorise as required, or did not realise that $y + 5y$ could and should be simplified at this level.

3) Some omitted this question, not knowing what consecutive integers were. Most candidates who attempted it managed to write down $n + 1$ and $n + 2$, although some gave $2n$ and $3n$. However, many then used numerical examples as 'proof' rather than following the hint of the algebra. Poor algebra abounded from weaker candidates, such as $\frac{3n+3}{3} = n$.

4) Many candidates did this well, although there were errors in rearranging, especially by weak candidates, with some failing to divide by 5 and giving the gradient as 3 and the y-intercept as 12. Some sensibly and elegantly used the given form to find the intercepts and some then used to find the gradient. It was fairly common for candidates to miss out one or more of the three things they were asked to find.

5) The majority used the binomial theorem and substituted in, although carelessness with brackets and signs led to errors. Some completely ignored the minus sign. Few of those who multiplied out the brackets did so successfully.

6) The first two parts were often correct, although some gave the wrong answers of 0 and a^4 or a^{-3}. The last part was found to be much more challenging, with the 9 causing most problems although weaker candidates also made errors such as $a^6b^2 = ab^8$.

7) The second part was often attempted with more success than the first, with candidates recognising more easily what was required in rationalising the denominator. $\sqrt{30}$ rather than the correct answer was probably the most common answer for the first part, whilst $\sqrt{24} = 4\sqrt{6}$ was another common error. Some who correctly obtained $2\sqrt{6} + \sqrt{6}$ then went on to equate it to $2 + 2\sqrt{6}$. In the second part, many found the answer competently but others realised they should be multiplying numerator and denominator by something but could not remember what that something was, or made errors in their multiplying or in dividing only one term by 18.

8) Many candidates omitted the first part completely and went straight into the factorisation. Those who did attempt it sometimes wrote down two equations in two variables and did not know how to proceed. Those who realised that the length was $30 - 2x$ or $\frac{112}{x}$ usually then went on to successful completion. A number of candidates found the factorisation difficult, not recognising that $56 = 7 \times 8$. A few used the quadratic formula to find the roots and then found the factors. Relating the values of x back to finding the dimensions of the rectangle was found to be more difficult than expected.

9) As in the January paper, many candidates knew what to do here and many did so successfully, although some attempted to factorise or did not remember the quadratic formula accurately. A few attempted completing the square, with most being unsuccessful. Mercifully few attempted to substitute for x – only one or two candidates did so successfully. Some wasted time by going on to find the y-coordinates.

Section B

10 (i) Many candidates were successful in this simple example of completing the square, although there were the usual errors in the value of *b*, with 41 being a common wrong answer.

(ii) Many candidates started again, with calculus being a popular method, to find the coordinates of the minimum point. With only two marks available for the sketch graph, candidates who did not mark the minimum point were not penalised, but those who did not mark the *y*-intercept were.

(iii) Most candidates attempted to solve the inequality, with a variety of standard methods being employed. A number of candidates simply solved the equation and stopped there. Only a minority sorted out the direction of the inequalities satisfactorily. Many attempted to combine them to get $1 > x > 7$. A few candidates drew a sketch graph or referred to the one they had already sketched, to help them to draw a correct conclusion.

(iv) Many candidates gained this mark but some were unsure as to whether they should add or subtract 20. Some gave the answer without the '$y =$ '. A few candidates attempted to multiply some terms by 20.

11 (i) Candidates were usually successful in calculating the length of AC, but were less rigorous in showing the right angle. The gradient and the Pythagoras methods were equally common, and equally successful, though some candidates did not draw a clear conclusion from their calculations.

(ii) Most candidates knew how to write down the equation of a circle, but were less sure about showing that they the understood why the centre was at (3, 6) and the radius 5. Better candidates did refer to the diameter, but hardly any referred to the reason why AC was the diameter.

(iii) Many candidates attempted this well though they often got lost in the arithmetic at the end, thus gaining method marks but not accuracy marks. Some candidates only found one of the intercepts. A few candidates tried to use calculus methods to determine the gradient of the tangent, but these were generally unsuccessful. A small minority thought that BC would be the tangent and found the equation of that line.

12 (i) Those who knew how to write the equation in factorised form then usually were successful in multiplying the factors out, with the given answer enabling them to spot and correct their errors. A few worked backwards by attempting to find factors for the cubic.

(ii) Attempts at the sketch of the graph were reasonable. Candidates usually gave the values of the intersections with the *x*-axis, but some omitted that for the *y*-axis.

(iii) Candidates who determined the value of f(4) were generally successful, though a few did make an arithmetic slip. Many found the quadratic factor correctly, but very few candidates wrote down the required quadratic equation, most choosing to leave it as a quadratic expression. Quite a number of candidates thought that they were required to solve this equation. A number of candidates only did the long division in this part of the question and some did not make it clear that they could conclude from their results that they had shown $x = 4$ to be one root of the equation. A few divided $(x - 4)$ into f(x) rather than $f(x) + 10$ but were rarely able to interpret the implications of the result.

4752 - Concepts for Advanced Mathematics

General Comments

There were the usual number of excellent scripts produced by candidates with good grounding in arithmetic, algebra and calculus. Others showed serious weaknesses in these basic skills; $12 – 40 + 12 = 40$ or -40, $3^n – 1 = 728$ therefore $3^n = 727$, derivative of $x = 0$. Good candidates set out their work in a logical fashion and were well prepared for all topics covered, many scoring over 90%. A small fraction of the entry was totally unprepared and scored less than 10%. In Q.10 all parts can be solved quite simply if the appropriate moves are picked out; time could be well spent planning the work rather than spotting a side or angle that can be found leading to a long chain of operations culminating, possibly, in the right answer.

Comments on Individual Questions

1) Some had difficulty converting the second term to a power of x, so this question did not yield many marks to weaker candidates. The derivative of x was often wrong. A small minority attempted to integrate.

2) The question states that $6 + 5n$ is an A.P. so it would have been sensible to put $n = 1,2,3$ just to have a look at it in its simple form. A great many took the first term to be 6 and managed to score half marks, but many were confused and scored 0 or 1.

3) Not by design, but inevitably each session, candidates are given the opportunity to make some variations on the mistake $(a + b)^2 = a^2 + b^2$. In this question, the very large number who used $\sin^2 \theta + \cos^2\theta = 1$ could not resist taking the square root of all three terms. Some did it immediately with $\sin\theta + \cos\theta = 1$, not even bothering to write the correct form first; some did it later, even as late as $\cos^2\theta = 1 – 3/16$. It is puzzling that the arithmetic on the right-hand side was found so daunting. Those who put the information onto a right-angled triangle still had trouble squaring 4 and $\sqrt{3}$ and subtracting the results. The successful ones found $\sqrt{13}/4$. It was disappointing to see $\sqrt{13}/\sqrt{16}$ so often. Very few indeed gave both values of $\cos\theta$.

4) Many produced an excellent response here. Many had difficulty handling $x + x^{-1}$ and its derivative.

5) Some achieved full marks in fine style, others stumbled and fumbled. Expressing 1/9 as a power of 3 was found difficult. In part (iii) many failed to simplify $\log x + 5\log x$ to $6\log x$.

6) Many produced good looking sketch graphs, no doubt aided by their calculators. Marks were lost by cutting short the curve at the y-axis so that nothing appeared in the second quadrant. For full marks it was necessary to indicate in some way that the y intercept was 1.
$2^x = 50$ was solved by most, usually by using $x\log 2 = \log 50$ or by trial and improvement; this latter method often left them with insufficient accuracy.

7) Many weaker candidates thought that dy/dx was a gradient that could be used as m in $y = mx + c$, a fundamental misunderstanding. Others had difficulty integrating $6/x^3$. Many coped well and scored full marks.

8) (i) Nearly all produced the first quadrant solution and many went on to give the correct second solution.

   (ii) The name “stretch” was essential here, together with the direction and stretch factor; and many gave all this information very briefly for 3 marks. Many unfortunate candidates did not know of the existence of a stretch and they wasted a lot of time sketching both graphs and describing in great detail the relationships between the two shapes discussing amplitudes, periods, squashes, oscillations and so on.

9) This yielded good marks, even for the poorer candidates. Most could differentiate the cubic and use their result to find the required tangent. Common errors were arithmetical and many could not resist converting their gradient of -16 to 1/16 before using their procedure for a straight line. In part (iii) some recognised that the factors $(x + 2)(x – 6)^2$ were important and they scored full marks if they expanded them correctly to fit the cubic or conversely, used a valid method to obtain them from the cubic. Those who did not could score marks for showing
$f(-2) = 0$ of for $f^1(6) = 0$. In part (iv) the method was generally well known but some integrated their derivative, some simply used the cubic as their integrand and some differentiated the cubic before evaluating using their limits. Although the arithmetic was quite involved most good candidates got it right, weaker candidates got it wrong.

10) (i) The sine rule and the formula A = ½ ab sin C were well known and most scored 3 of the 4 marks; many forgot to double the area of triangle ABC to get the area of the logo. Some dropped a perpendicular from A to BC and if they proceeded correctly they achieved the marks; if they assumed that line bisected BC, and quite a few did, they did not score. A few joined DB and produced CA and went through a long chain of calculations finding all angles, sides and areas.

(ii) Those who used right-angled triangle OST could score 4 of the 8 marks immediately. The work with the sector and arc, which they did not find difficult, gave them the other 4 marks quite quickly. Alas, SR and OT were joined and again many angles, sides and areas were found, with maybe, the side and the area needed.

11) (i)&(ii) Most candidates recognized the progression 1, 3, 9 and its continuation 27, 81 and so even the weaker ones found 81. Many also realised it was a GP and gave the n^{th} term $ar^{n-1} = 3^{n-1}$.

(iii) Although the question gives a broad hint that the number of stems in year n is the sum of the above G.P. to that year, many did not find it obvious and substituting a = 1, r = 3 into $a(r^n – 1)/r – 1)$ was not always seen. Many simply tested the printed expression for n = 1,2,3.

(iv) Most wrote down $364 = (3^n – 1)/2$ and many got to $3^n = 729$. This was solved using logs or by trialling and the answer 6 was often achieved. Having got year 6, it was no problem to get the number of flowerheads.

(v) It is generally better to solve inequalities as such rather than as equations with an appropriate symbol inserted in the final line, so there was a penalty of 1 mark for those who used a wrong symbol anywhere. Those who began $3^{y-1} > 900$ very often proceeded with the next three moves correctly and scored 3 marks. It was surprising that not all who got to $y > 7.19$ converted to 8.

4753 - Methods for Advanced Mathematics

General Comments

There was a pleasing response to the first C3 paper in the new specification. We saw very sound work from many candidates, and most centres appeared to prepare candidates well for the new specification.

The paper discriminated well across the whole ability range. Full marks were scored infrequently, but there were plenty of candidates scoring over 60 out of 72. Equally, even the weakest candidates managed to find some accessible questions and score over 20 marks.

The work was quite well presented, although there was some evidence of poor use of notation in questions1, 5, 8 and 9. A few candidates ran into time trouble – usually caused by inefficient methods or making several attempts at the same question. However, virtually all students attempted all the questions. Candidates should be advised not to spend too long on low tariff section A questions, and to allow ample time to tackle the high tariff section B questions.

Comments on Individual Questions

Section A

1) A fair number of candidates found this an easy three marks. However, there was a noticeable amount of poor notation in solutions, with statements such as $|3x| = -1$. Nearly all the candidates scored the B1 for $x = -1/3$; far fewer achieved $x = -1$. Some candidates treated the question as if it were an inequality. The 'squaring' method was seen relatively infrequently.

2) This three mark question is technically very easy, but was on the whole poorly done, even by quite competent candidates. The concept of inverse trigonometric functions was not well understood.

3) The concept of composite functions seems to be well understood, and this question was well answered. Virtually all candidates found $\mathrm{fg}(x) = \ln(x^3)$, although $(\ln(x))^3$ was seen occasionally; The next A1 for $3\ln x$ was achieved by over half of candidates, and the description of the transformation was good – we required 'stretch', 'scale factor 3' and 'parallel to O*y*'.

4) The first and last demands were very well answered – the solution of the exponential equation by taking logarithms is a routine in which candidates are usually well drilled. However, the second part on the initial rate was poorly done, and often omitted. Quite a few candidates differentiated correctly but then failed to substitute $t = 0$.

5) Integration by substitution is found difficult by weaker candidates, who often fail to substitute for the du – the omission of dx often causes this. However, many candidates successfully achieved the first three marks, but then failed to divide through by u (or did so with errors). Some tried integration by parts here, which works after a couple of applications. When substituting limits to achieve a result given on the paper, it is important that sufficient working is shown.

6) The quotient rule was generally well known, with *u*s and *v*s in the right places. Plenty of candidates achieved 3 marks for a correct expression for the derivative. Thereafter, marks were squandered by faulty algebraic simplification of the numerator(e.g. spurious cancelling), or failing to evaluate the coordinates exactly in terms of e. Setting the denominator to zero and pursuing this value was penalised.

7) Most candidates solved $y^2 + y - 12 = 0$ correctly, sometimes by inspection or formula rather than by factorising. In the implicit differentiation, $2y\,dy/dx + 1 =$ … was quit a common error. Good candidates scored full marks here with little difficulty, however.

Section B

8) This question was found to be the most challenging on the paper, with good solutions to part (iv) being quite rare. Students lost marks at various stages through working in degrees rather than radians.

(i) This part was intended as a simple warm-up for three marks. In the event, it was rather poorly done by a lot of candidates. Many candidates could not get further than $x\sin 3x = 0$; $x = 60$ was quite common.

(ii) Candidates had to conclude in radians before achieving this mark.

(iii) The derivative of sin 3x was well known, and many achieved 3 marks for the correct product rule. Candidates had to mention (or imply) that the gradient of $y = x$ was 1 to achieve the E mark. Substituting degrees instead of radians into the derivative was *not* condoned.

(iv) This was the hardest question on the paper. Many recognised the integration by parts, but then got into sign 'muddles' , or integrated sin 3x as 3cos 3x or cos 3x / 3. Some also got the limits wrong, or failed to find and subtract the area under the line.

9 There were quite a few fairly easy marks to be gained in the question, which was often done quite well.

(i) The property of even functions was well understood. Some equate it with 'even powers' of x; some verify using a single value. However, many gave fully correct algebraic proofs, with the brackets in the correct place. Symmetry about Oy was also well done.

(ii) This was a pretty easy 4 marks for candidates who knew how to differentiate lns. $1/(1 + x^2)$ was a common error, however.

(iii) The condition for an inverse was quite well understood, although 'one to many' instead of 'many to one' was seen on occasions.

(iv) A generous method mark was awarded for recognisable attempts to reflect in $y = x$. To achieve the 'A' mark, candidates needed to discount the negative x domain, and have a 'reasonable' shape, including an inflection and correct gradients at the origin.
Finding the inverse function was generally well done. Weaker candidates tried $1/f(x)$ or expanded $\ln(1 + x^2) = \ln 1 + \ln x^2$.

(v) Differentiating the inverse function proved to be easier than the original, and then substituting to find the gradient of 5/4 was often successfully negotiated. For the final 'B' mark, candidates need to mention or specify the idea of 'reciprocal' – 'inverse' was not enough.

4754 - Applications of Advanced Mathematics

General Comments

This was the first time that this examination has been set. The examination for this new specification was longer than the previous one with more questions and the Comprehension was slightly longer.

The standard of work was quite good although there was a wide variety between centres. Some achieved an excellent standard with high scores and well reasoned solutions but, equally, there was some disappointing work from other centres both in content and presentation.
The Comprehension was the least successfully answered question. It scored, on average, approximately one third of the marks available and generally had a negative overall effect on the candidates' scores.
Candidates should be encouraged to:

- consider coefficients when using partial fractions
- use a logarithmic constant when integrating when all other terms are logarithms
- show fully the scalar product of the normal vector with two vectors in the plane to show that it is, in fact, perpendicular to the plane
- use the approach from $ax+by+cz=d$ when finding the Cartesian equation of a plane.
- realise that they should attempt the Comprehension questions as they represent a significant proportion of the marks.

Comments on Individual Questions

1) The first three marks were usually scored. Almost all candidates successfully found that R=5. The angle was usually found correctly but the answer was often given in degrees rather than radians. The second part of the question was less successful. Most candidates failed to realise that they were expected to use the fact that cos x must vary between -1 and +1 and either missed this part out or tried to solve an equation such as $f(\theta)=0$. Good candidates were, however, successful here.

2) There were some good answers for the binomial expansion. The binomial coefficients were usually correct. Many candidates found it difficult to factorise the 4 out of the bracket successfully. They often used a factor of 4 or ½ instead of $\sqrt{4}$ or 2. Further errors often arose as a result of brackets being missing around the terms in $(\frac{1}{2}x)$.There were also errors in cancelling fractions.
Many candidates-including many good candidates- failed to give the range for which the expansion was valid. Others gave the answer as $-2 \le x \le 2$ instead of inequalities or answered $-1/2 < x < 1/2$.

3) Appropriate trigonometric identities were usually used but the majority of candidates failed to obtain the negative root and hence missed the second solution of $2\pi/3$. Some failed to give their solutions in terms of π as required. Most candidates scored three out of the possible four marks available.

4) The majority of candidates used the correct formula here. Most scored well. The most common error in the integration was to multiply by 2 instead of dividing when integrating the exponential. A disappointing number of candidates failed to substitute the lower zero limit into the integrand. Another frequent error was failing to leave the answer in its exact form.

5) This question saw candidates scoring both the highest number of zero marks and the highest number of full marks. Candidates who did not attempt to use a double angle substitution for cos *2x* were generally unable to proceed. Many tried, incorrectly, to use cos $2x=\cos^2x$ -1. Some made an incorrect substitution but understood the general method. They therefore achieved some marks by forming a quadratic equation and attempting to solve it. In general further marks were rarely scored except by those that chose a relevant initial substitution. For those candidates that started correctly full marks were usually scored. Occasionally additional incorrect solutions were given.

6) (i) Although a few candidates failed to attempt the first part, the majority of those that did achieved at least the first method mark for substituting for y and x in terms of t into y^2-x^2=4. The most common error being incorrect expansions of $\left(t+1/t\right)^2$ and $\left(t-1/t\right)^2$ -usually omitting the middle term.

(ii) This part was approached in a variety of different ways. The most common involved finding dy/dt and dx/dt and dividing. Although the method was generally understood there were errors in simplification to the given result. The most common error was $\frac{dy}{dx} = \frac{1-1/t^2}{1+1/t^2} = \frac{1-t^2}{1+t^2}$.

Other errors included the incorrect use of dx/dt=1+1/t^2 so dt/dx =1+t^2.

Similar errors occurred when the question was approached via implicit differentiation. Using the explicit differentiation of $y=\sqrt{(4+x^2)}$ and substitution was seen less often.

The final part of the question was missed out many times with the loss of some relatively easy marks. Of those that did attempt it there were too many simple arithmetical errors often leading to only one set of co-ordinates being correct. Many did, however, complete this successfully.

7) (i) Often fully correct but some failed to realise this was a logarithm. Most candidates obtained the answer directly rather than by substitution.

(ii) Many good marks were scored here. Some candidates failed to include brackets around the Bt+C term. There were some long solutions involving substitutions whereas comparing coefficients was quicker and less prone to error.

(iii) The first step here should have been to separate the variables. Several candidates retained the M on the right hand side of the equation as if it were a constant. Other candidates omitted the Ms and only proceeded with the right hand side. The substitution of partial fractions was usually correct as was the following integration but many candidates omitted the constant at this stage. Although candidates were usually able to demonstrate that they knew how to combine logarithms successfully, there were very few instances where the constant was dealt with correctly. Using a logarithmic constant should be encouraged in such questions as fewer errors are incurred with this approach. There were a few excellent complete solutions to this part but they were rare. Weaker candidates often omitted this integration.

(iv) Nearly all found K correctly and many obtained full marks but M tending to 0 or infinity was a fairly common answer.

8) (i) Usually high scoring and well answered.

(ii) Candidates should be reminded that it is necessary to show a vector is perpendicular to **two** independent vectors in a plane in order to establish that it is perpendicular to the plane. One is not sufficient.
Too few candidates showed the numerical evaluation of their scalar products resulting in the unnecessary loss of marks.
When finding the Cartesian equation of a plane the candidates should be advised to use the approach via *2x+3y+2z= a constant.* Those that started using a vector equation and eliminating parameters made more errors and their method took longer.

(iii) The co-ordinates were often found correctly. Although some candidates did give good clear solutions in finding the co-ordinates of T, many gave unclear, confused solutions with poor notation and reasoning that was difficult to follow. As the answer was given in the question there was a tendency to try to acquire the correct solution from incorrect figures.

(iv) The vector equation of the line of the drill hole was usually given correctly although some candidates confused the position vector and the direction of the line. Both vectors were given in the question but some candidates still made errors.
Thereafter, those that made a serious attempt produced reasonable answers but there were numerical errors in the final stage. The majority chose to show that the point C did not lie on the vector equation of the line although a few gave an equally reasoned argument that the direction of the vector **CT** was not in the same direction as that of the drill hole.

Section B: Comprehension

1) Candidates were required to give an answer that showed an understanding of the fact that the units we use would not be understood in other civilisations but that ratios which are dimensionless would be understood everywhere. There were some good solutions but many others missed the point.
Some said that mass would be different on other planets or that gravity would be different. Others wrote of the intelligence of other civilisations.

2) The number bases proved difficult. Many candidates correctly obtained the method mark by stating $3.03232=3+0/5+3/5^2 +2/5^3+3/5^4 +2/5^5$ or equivalent but then failed to obtain the answer mark-often quoting 3.14159 instead.

3) This was the mark that was most frequently scored in the comprehension. Some candidates failed to give the requested full calculator display.

4) A large number of candidates found the quadratic equation and hence the solution. In a number of cases the solution was just stated rather than being established and then the E mark could not be given as the answer was given in the question.
Several candidates tried to use a numerical value of ϕ.

5) There were several methods of approaching this but good solutions were rare. Candidates needed to discount the negative root and then simplify their expression.

6) There were a few very good solutions here but some tried to use an incorrect expression for r such as r = $\frac{a_{n+1}}{2a_n}$ and r = $\frac{2a_n}{3a_{n-1}}$.

Many others found successive terms and their ratios but either made numerical errors when calculating the terms or did not calculate as far as the a_{10}/a_9 ratio or both. Others did not attempt the question.

7) This was not attempted by many. For those with the right approach there were good solutions. In general either 0 or 4 marks were scored. Several candidates found the required value of *k* and, unnecessarily, the subsequent one too.

4755 - Further Concepts For Advanced Mathematics

General Comments

This was a successful paper, which performed according to design. Many clearly talented students scored highly but this was not true of the weaker candidates and there was a realistic tail of low marks.

The general standard of work was high and the majority of candidates showed a good level of competence, including in handling algebra, though weaker candidates lost marks through poor algebraic manipulation.

There was evidence, in the form of a fair number of fragmentary answers to the last question, that some candidates found the paper rather long. The paper was very appropriate for its candidature.

Comments on Individual Questions

1) Many candidates got this question fully right. Among those who did not get full marks, the most common cause was not knowing how to use matrices to solve simultaneous equations; some used Gaussian elimination instead and they were awarded no marks

2) While many candidates got this question fully right, there were also many who lost marks through careless mistakes. A particularly common error was to say that $\frac{8 \pm \sqrt{-4}}{2} = 4 \pm 2\text{j}$. There were also some weaker candidates who did not seem to know the meaning of modulus-argument form and so did not attempt the second part of the question. Several candidates calculated modulus and argument but failed to get full marks because they did not give their answers in modulus-argument form, even though this was specifically required by the question. A few candidates gave a negative modulus for one of the roots.

3) Most candidates either got this question fully right or did not know the appropriate method. A common error was to equate $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ instead of to $\begin{pmatrix} x \\ y \end{pmatrix}$.

4) On the whole this question was well answered.

   Nearly all candidates got part (i) right.

   Part (ii) produced more errors including a distressing number of answers in which it was claimed that $\alpha^2 + \beta^2 = (\alpha + \beta)^2$.

   Part (iii) was rather better answered than part (ii) but several candidates lost a mark by giving a quadratic expression rather than a quadratic equation.

5) Many candidates got this question fully right.

In part (i) almost all candidates drew a circle of radius 2, but several had the wrong centre.

Part (ii) was less well answered with several candidates drawing a circle rather than a line. Among those who knew it should be a line, several drew a full line instead of a half line.

A common error in part (iii) was to give the answer as the sector formed by the half-line and the circle, rather than as the two points of intersection.

6) This question was generally well answered. Many candidates got it fully right. However a significant proportion lost marks by missing out logical steps in the argument, or by not stating them adequately. There was also a significant proportion who failed to manipulate the algebra to establish the result for $k + 1$ terms.

7) Many candidates answered this question well up to the last line, where many candidates lost a mark by failing to give it in a fully factorised form. However, a significant minority failed to separate the summation correctly and several tried to multiply standard sums rather than add them.

8) Virtually all candidates were able to obtain some marks on this question but only the very best achieved full marks.

(i) Almost all candidates got the vertical asymptote right but many failed to get the horizontal one. Assuming $y = 0$ was a horizontal asymptote was a very common error.

(ii) This was not well answered; many candidates did not realise that they were expected to show from which sides the curve approaches the horizontal asymptote.

(iii) Many candidates drew a curve that was at least nearly right but the approaches to the horizontal asymptote were often incorrect. Some candidates did not appreciate that they are expected to indicate the asymptotes and to give the points where the curve cuts the co-ordinate axes.

(iv) Candidates were required to solve an inequality and while most knew what to do, many spoilt their answers by making careless mistakes in their algebra.

9) There were many fully correct answers to this question.

Almost everyone got part (i) right.

There were three possible strategies for part (ii). Those who used the pairs of conjugate roots to form two quadratic factors, and then multiplied them together, were mostly successful. A more common approach was to find $\sum\alpha$, $\sum\alpha\beta$, $\sum\alpha\beta\gamma$ and $\alpha\beta\gamma\delta$ and use them to find coefficients of the quartic; those using this method often lost marks from careless mistakes. The third method involved substituting two roots into the equation and so forming real and imaginary equations for the coefficients; those who chose this method were seldom successful.

10) Most candidates knew what to do in part (i) of this question and a very encouraging number got it fully right. The most common errors were not correctly identifying the terms left over after the summation, and failing to demonstrate the last step combining the algebraic fractions into the form in the given answer.

A few candidates 'simplified' the terms and so could not see the cancelling pattern.

Few candidates were successful in part (ii). Most did not consider what happened to the fraction as $n \to \infty$ and many of the very best still failed to spot that the required sum was half of the sum in part (i), so they gave a final answer of $\frac{1}{2}$ rather than $\frac{1}{4}$.

4761 - Mechanics 1

General Comments

Most candidates seemed to be able to do a substantial amount of the paper with quite a few doing well on every question. There were relatively few candidates who could not make any real progress with any question. Most candidates did well on Q2, 3, 6, and 7. The responses to the two section B questions were especially pleasing with many essentially complete solutions to each. Many candidates had major problems with one or more of Q1, 4 and 5. Perhaps it was the case that these questions somehow did not allow some candidates to show what they knew but there was an impression given that many of them were not familiar with the techniques required.

As always there were many beautifully presented scripts with clear, accurate working and full accounts given of the methods used but many other candidates lost marks because of slips, poor (or no) diagrams and lack of adequate explanation, especially of *given* results that had to be *shown*.

Comments on Individual Questions

1) **The use of an acceleration-time graph**

This question gave a bad start to many candidates as it seems they did not realize that the area under an acceleration – time graph represents change in velocity. These usually scored the first mark (showing that they did understand it was an acceleration – time graph) and could also get the third mark for writing down that $a = 2t$. Otherwise, they mostly either tried to use the constant acceleration results or argued that velocities were connected to gradients so they needed the gradients of the lines on the graphs.

(i) The correct acceleration was found by almost every candidate but even some who realized they should be finding an area miscalculated to get 32 instead of 16 m s^{-1}.

(ii) Answered correctly by the majority of the candidates.

(iii) Even some candidates who realized that the area under the curve represented velocity change thought that the answer was $t = 5$, where the acceleration is greatest, instead of $t = 7$ where the acceleration changes from positive to negative. Some who correctly wrote $t = 7$ gave as their reason 'the acceleration is zero' instead of it noting that it changes sign.

(iv) Only those who understood that the area represented velocity change could score marks here and those that understood did well with few slips.

2) **Kinematics in 1 dimension using calculus**

Very many candidates scored full marks on this question, including some who did not do well overall. There was no pattern to the few errors that were not slips except, of course, for the minority who tried to use the constant acceleration results to find the distance travelled.

3) **Equilibrium of forces given in terms of unit vectors; the magnitude and direction of a vector**

(i) Most candidates found **R** correctly, including the sign. This is pleasing as a sign error has been more common in recent sessions.

(ii) Most candidates obtained the correct magnitude of **R** but only a few realized that its direction is an angle in the second quadrant and so they obtained the wrong angle with **i**.

4) **A heavy block in equilibrium on an inclined plane**

Although quite a few candidates worked through this problem accurately and efficiently, many others scored few marks at all.

(i) Many of the diagrams were poor. Some candidates did not even show four forces, as requested, usually omitting the normal reaction. Many diagrams failed to show the weight and/or the tension in the string to be vertical and others introduced an extra force marked F (for friction?) down the plane. Labelling was often incomplete and in many cases arrows were missed out.

(ii) By no means all the candidates followed the instruction to resolve parallel to the slope. Those that did not do this were not penalized in this case but usually forgot the component of the normal reaction. Many candidates omitted at least one force and/or failed to resolve both the tension and the weight. The resolution attempted was, in many cases wrong.

(iii) All the mistakes seen in part (ii) were seen here also. By far the most common error (seen in many sessions in the past) is to take the normal reaction to be that component of the weight perpendicular to the plane.

5) **The kinematics of a particle moving in 2 dimensions, given in vector form**

Although some fully correct solutions were seen, few candidates managed part (iii) satisfactorily.

(i) Most candidates knew what to do to find t but a surprisingly large minority argued that $0.5t = 2$ implies that $t = 1$.

(ii) Only about half of the candidates seemed to know that they should solve the linear equation for t and then eliminate t from the quadratic equation. Some who did failed to give enough working to *show* the result.

(iii) Very few candidates realized that direction of the movement is the direction of the velocity; instead, they mostly tried to equate the **i** and **j** components of the position vector. The successful attempts were approximately evenly split between those who differentiated the cartesian equation of the path to find where the gradient of the tangent is 1 and those who differentiated **r** and then found where the **i** and **j** components of **v** are the same.

6) **The motion of a car and a trailer and the force in the tow-bar**

Very many candidates did well on all parts of this question.

(i) Almost all the small number of errors were miscopies or slips

(ii) Most candidates knew what to do but some solutions based on showing all the figures given were consistent with Newton's second law did not properly *show* the given result – some candidates wrote 2000 – 600 = 1000 x 1.4 without any indication of method or comment.

(iii) Again in this part, most candidates knew what to do but some failed to *show* the result as in part (ii). Quite a few candidates analysed the motion of the car and trailer separately and so found the force in the tow-bar requested in part (iv).

(iv) I felt that a higher proportion of the candidates knew what to do than in some recent sessions but there were still attempts that incorporated the weight of the car and trailer and attempts not based on the application of Newton's second law at all. As mentioned above, some candidates had already found the required value because of their approach to part (iii).

(v) I was pleased to see so many complete solutions to this part. Most candidates knew how to find the new acceleration but many made mistakes with the signs of some of the terms. A common error was to omit the resistance of the trailer or the car. Lack of a clear sign convention added to the problems, especially when moving to the use of the new acceleration to find the new force in the tow-bar so that, for instance, a value of acceleration with backwards positive was used with forces signed as if forwards were positive. A minority of candidates do not look to their calculations to decide on whether the force is a tension or a thrust but seem to think the answer is to be found from qualitative arguments.

7 A projectile problem

There were many complete solutions to this question and many more almost complete.

(i) Almost every candidate correctly obtained the required values

(ii) Most candidates knew what to do but many gave too little explanation to *show* a given result. Commonly, the + 9 of the expression for the vertical height appeared with no or inadequate explanation. Many candidates forgot to give an expression for the horizontal distance travelled.

(iii) Most candidates knew what to do, although many gave the height above the ground – this was not penalized in this case. As always when using the result $v^2 = u^2 + 2as$, there were some sign errors seen. Quite a few candidates rather inefficiently used a method requiring finding the time to the greatest height first.

(iv) A pleasing number of candidates did this part very well. Those who tried to consider the flight in sections tended to forget the section from the height of projection to the ground or failed to find correctly the time for this part of the flight.

(v) Many candidates did not give a complete argument and so lost one of the marks.

(vi) This was usually done well.

(vii) There were many good answers to this part. Again, marks were lost because the given result was not completely established. A quite common and surprising error in the light of parts (v) and (vi) was simply to show that the horizontal displacements were the same at the given time.

4762 - Mechanics 2

General Comments

This paper appeared to be accessible to all of the candidates, with the majority able to obtain at least some credit on some part of each question. A large number of excellent scripts were seen. There were some candidates that did not seem to appreciate that a diagram assists in finding a solution and can help to clarify the solution to the examiner. The main difficulties encountered related to giving reasons for a calculated answer or in establishing given answers. There was, from some candidates, a lack of rigour with relevant steps in working being omitted and/ or insufficient explanation as to the principles being employed. A small number of candidates penalised themselves by premature rounding of answers leading to inaccuracies in final answers.

Comments on Individual Questions

1) **Impulse and Momentum**

(a) Problems arose in this part for those candidates who did not appreciate the vector nature of the question and hence, did not specify the direction of the velocities requested.

(i) This part was almost always successfully answered.

(ii) (C) This part posed few problems for the vast majority.

(D) Many candidates obtained the correct speed for Sheuli but did not specify direction. Others set up a correct equation for Roger's speed and obtained the answer 12**i** but then failed to convert to $\mathbf{v}_s$ = -12**i**.

(E) This part was more successful with many obtaining a complete solution in terms of **i** and **j.** There were errors with signs in a number of cases.

(b)(i) Unfortunately many candidates did not draw a diagram for this part of the question and hence, errors with signs and inconsistent equations were quite frequent. Candidates could help themselves by stating which principle is being applied and specifying the meaning of the variables being employed.

(ii) This part of the question was poorly attempted by almost all of the candidates. While many of them could state that the speed would be unchanged and that the angle of reflection would be the same as the angle of incidence, few could give clear and unambiguous reasons as to why this was so. Most merely stated that the collision was perfectly elastic without expanding on what this would affect. Very few candidates seemed to appreciate the need to investigate directions parallel and perpendicular to the wall and of those that did, only a small number mentioned that there would be no impulse in the direction parallel to the wall and hence no change in that component of the velocity.

2) **Work and Energy**

Candidates either scored well on this question or very poorly.

(i) This part gave few problems to the majority of candidates although a small number of them failed to give any indication of the principles being employed and merely wrote down a set of numbers that produced the required answer.

(ii) Most candidates could gain full credit for this part.

(iii) The majority of candidates gained some credit for this part. Errors that occurred were usually due to the omission of the resistance term in the Newton's second law equation.

(iv) A sizeable number of candidates ignored the method requested in the question and attempted a solution using Newton's second law and the constant acceleration equations, obviously not appreciating that if both the power and the resistance are constant, the acceleration cannot be. Of those who used the requested method, most obtained some credit but many omitted the term involving power.

(v) Candidates who used work-energy methods for this part were on the whole more successful than those who opted for Newton's second law and *uvast*. Errors were usually the omission of one of the terms in the work-energy equation or in the sign of the acceleration in *uvast*.

3) **Centres of Mass**

This question was well done by the majority of candidates; many of them scoring highly on it. Almost all of the candidates understood the method required for finding a centre of mass and some excellent answers were seen.

(i) A high proportion of candidates could obtain the correct answer to this part of the question. However, a small number of candidates treated the shape as if it was composed of three parts, a lamina and two squares formed by rods.

(ii) A large number of candidates scored highly on this part of the question. The main errors were in the sign of the z component of the centre of mass. The majority understood that the use of Pythagoras in 3D was required to find the distance of the centre of mass from A. However, a small minority of candidates omitted this part altogether.

(iii) This part of the question gave problems to many of the candidates with only the more able candidates achieving well. Unfortunately very few candidates drew a diagram that was helpful to them. Those that drew a diagram were usually more successful in identifying the lengths necessary to calculate the requested angle and could gain some credit for their work. A significant minority did not seem to appreciate that the centre of mass of the shape had to lie directly below A.

4 **Moments and Resolution**

Some excellent responses to this question were seen but the quality of the diagrams in many cases was poor.

(a)(i) Those candidates who resolved horizontally and vertically and then took moments about A (or C) or vice versa were usually successful in showing the given results. However, a number of candidates chose to take moments about B without first establishing that $U = 0$ and omitted the moment of U.

(ii) It was pleasing to see a large number of correct responses to this part of the question. Almost all of the candidates appreciated the need to resolve at a pin-joint. Those candidates who drew a diagram showing all of the internal and external forces with clear labels were generally more successful than those who either did not draw a diagram or who drew a poor and inadequately labelled one. Without a diagram, sign errors and inconsistent equations were common. Some candidates confused tension and thrust.

(b)(i) Many poor diagrams were seen here with forces omitted or unlabelled in many of them. The most frequently omitted force was the frictional force at A and a significant minority of candidates thought that the normal reaction forces at A and B would be the same. It was common to see the weight represented as Wg.

ii) This part of the question gave little difficulty to the majority of candidates with almost all of them appreciating the need to take moments. A very small number apparently did not understand the meaning of 'normal reaction' and attempted some complex algebra to find a reaction that acted vertically upwards.

(iii) Many candidates gained significant credit on this part of the question. However, some very creative working was seen from the few who were determined to find that $\mu = \tan\theta$. This included some of the candidates that had obtained full credit for the previous part.

4766 - Statistics 1

General Comments

The majority of candidates appeared to be well prepared for this paper and were able to have a good attempt at all the questions. However a significant number of candidates struggled with even the very straightforward material in questions 1, 2 and 5. The work of these candidates was also characterised by poor explanations, a lack of clear working and general carelessness. There was little evidence that candidates did not have sufficient time to complete the paper.

Comments on Individual Questions

1) (i) Most candidates were able to calculate the mean correctly. Any errors tended to be pure carelessness. The sample variance proved to be a greater challenge, with candidates confusing variance with standard deviation, divisor 20 with 19, $\sum x^2$ with $(\sum x)^2$ and $\sum (x-\bar{x})^2$.

(ii) Most candidates used the two standard deviation definition method and did so successfully. A minority of candidates used the 1.5 interquartile range method and received full credit.

2) (i) A majority of candidates did not show their calculated values of the cumulative frequencies. This was not a problem unless the points were plotted incorrectly, in which case no method marks could be gained. A significant number of candidates plotted points in the middle of class intervals rather than at the end.

(ii) Most candidates knew how to obtain values for the median and the quartiles from their graph, and almost without exception were ale to calculate the interquartile range.

(iii) The majority of candidates correctly described the skewness as positive, but a significant number, possibly confused by the shape of the cumulative frequency graph, gave the opposite response.

3) Most candidates did well on this question.

(i) This part of the question was almost always answered correctly.

(ii) Most candidates were able to calculate the mean, although a few calculated $\sum p$ rather than $\sum rp$. There were more errors in the calculation of the variance, including forgetting to subtract $(\mathrm{E}[X])^2$, or getting lost in a method based on $\sum (x-\bar{x})^2$. A small number of candidates did not attempt this part of the question.

(iii) This part of the question proved to be accessible even to those candidates who were unable to attempt part (ii). A significant number of candidates felt that the answer needed to be an integer, and so gave the answer 7 weeks. A smaller number of candidates converted the answer to days.

4) (i) Almost always answered correctly.

(ii) Although most candidates correctly obtained the correct three values of 20, 35 and 56, a considerable number of candidates then proceeded to add them, rather than multiply.

(iii) Despite being led by the previous part, most candidates were unable to make much progress with this part. Those attempting a solution using a product of fractions were, virtually without exception, doomed to failure. Often seen was $\frac{3}{6} \times \frac{4}{7} \times \frac{5}{8}$, and even those candidates who successfully obtained a string of 12 correct fractions failed to include a combination term.

5) (i) Virtually all candidates were able to complete the table correctly.

(ii) Parts *A* and *B* were often done correctly, but in part *C,* the majority of candidates assumed independence and simply multiplied their answers to parts *A* and *B.* Naturally, this gave them a problem in part (iii). Many other candidates also simply gave an answer with no supporting working. Simple annotation of the table could have earned these candidates marks for method.

(iii) Of those candidates who had not assumed independence earlier in the question, a significant number confused independence with mutual exclusivity and stated that the events could not be independent because some values were both greater than 6 and multiples of 5. Finally, some candidates who knew the definition for independence gave insufficiently clear answers such as $\frac{1}{3} \times \frac{11}{36} = \frac{11}{108}$ so independent.

6) This question proved a good source of marks for most candidates and also gave the opportunity for the very best candidates to shine in the final part.

(i) Virtually all candidates were able to complete the insert correctly.

(ii) Almost always correct.

(iii) Almost always correct.

(iv) Usually well done, but a significant minority of candidates failed to realise that conditional probability was involved and simply gave the answer of 0.05.

(v) Those candidates who took the approach of 1 – "the probability that no-one is asked to leave" were by far the most successful. Those who took an additive approach often omitted the required factors of 3.

(vi) This was probably the most difficult part of the paper and it prompted some very good solutions from a small number of candidates. A pleasing number of candidates were also able to gain some credit for being able to show that they had some understanding of the structure of the situation. Many candidates, however, based their answer on $B(9, 0.7)$.

7) The response to this question was not as good as in previous sessions, particularly in terms of hypothesis testing. The use of point probabilities was seen extensively.

(i) Almost always correct.

(ii) Usually correct, but some candidates omitted the combination term.

(iii) Often correct, but a significant number of candidates gave $P(X > 3)$ to be equal to either $1 - P(X \le 2)$ or $1 - P(X = 3)$. Some also took an additive approach which rarely succeeded.

(iv)*A* Most candidates failed to define p in the hypotheses. Most candidates were able to calculate the correct probability, compare this with 10% and then reject the null hypothesis. However, only a minority then went on to explain this rejection in the context of the situation, i.e. Conclude that there is sufficient evidence at the 10% level that the dice are biased against sixes.

(iv)*B* This part was done much less well than the previous part. Many candidates calculated $P(X = 5)$. Many others were unable to calculate $P(X \ge 5)$ correctly.

(v) There were some good answers here which mentioned the fact that the results were contradictory, that different decisions would have been made at the 5% level and that these events could have occurred purely by chance.

4767 - MEI Statistics 2

General Comments

On the whole candidates scored well on this paper, many probably being Further Maths students taking this A2 unit in Year 12. Most candidates demonstrated a good level of knowledge and understanding of all of the topics and there were many scripts in which candidates gave very good responses to all four questions. Very few candidates appeared to have been inappropriately entered for the paper. Question 4 which examined the new topics in the specification (contingency tables and the hypothesis test for the mean of a Normal distribution) was answered well, with many candidates gaining nearly full marks. Most parts of the first three questions also elicited good responses, although candidates struggled to give two valid assumptions in Question 1 part (i). Question 2 part (v), although not exceptionally demanding, did prove to be beyond the majority of candidates. Hypothesis testing was generally well done, except for a failure to define the parameter used in the hypotheses (very frequently seen) and a failure to give the final conclusion in context. It appeared that most candidates had adequate time to complete the paper, with the possible exception of a few who adopted extremely time consuming methods, such as the calculation of ten separate Poisson probabilities, rather than the use of tables in Question 1 part (v).

Comments on Individual Questions

1) (i) This standard request was the least well done part of Question 1, even by very high-scoring candidates. In this case independence (of events) and a uniform mean rate of occurrence were the correct assumptions. Many candidates quoted the former but fewer quoted the latter, sometimes instead mentioning a ‘known’ mean rate, but more often randomness or mention of large *n* and small *p* were suggested. Randomness rather than a deterministic situation is a requirement of every statistical distribution, not specifically of the Poisson. Many candidates who mentioned independence were able to make a suitable comment which indicated that they understood the meaning although this was true of less of those who mentioned the second assumption.

(ii) Most candidates scored either full marks or lost just one mark due to the use of divisor n rather than $n - 1$ in the sample variance. In the new specification a divisor of n is used in finding *msd*, not variance.

(iii) This was usually correct.

(iv) Once again this was well answered with only a few candidates rounding λ to 1.6 and then using tables, which is not acceptable. Many were able to go on to compare their result with the frequency of $x = 2$ in the table. Some candidates thought that ‘the table’ referred to the cumulative Poisson probability tables.

(v) Most candidates correctly multiplied 1.62 by 5 to find the new mean and then used tables, but at this stage a few made errors of the form $P(X \geq 10) = 1 - P(X \leq 10)$. Some used entirely spurious methods, or occasionally did not use tables but instead calculated ten separate point probabilities and then subtracted their sum from one, usually making some calculation error on the way.

(vi) Most candidates realised that a Normal approximation was required and found the parameters correctly. Continuity corrections were often omitted and sometimes the wrong correction, 549.5 instead of 550.5, was used. Relatively few candidates miscalculated the parameters.

2) (i) This was well answered, with just a few candidates using variance instead of standard deviation or giving an inaccurate final answer due to premature approximation. A few of the candidates who used graphical calculator built-in probability functions did not appear to know how to do this correctly since their answer was wrong and thus they could be given no credit since there was no working shown.

(ii) Most candidates realised that an inverse Normal calculation was required, but many did not realise that a negative z-value was appropriate and so obtained a final answer which was on the wrong side of the mean. As has been stated in reports on the legacy specification 2614, candidates are advised to draw a sketch if there is any doubt in their mind as to which tail is involved. Alternatively a mental check of their final answer in relation to the value of the mean should indicate if they have made an error in the sign of z.

(iii) This was very well answered. Most candidates scored full marks, although a number lost one mark by rounding 0.6745 to 0.675, which then does not lead to the given answer. Candidates should realise that given answers are correct to the number of decimal places stated and if they get a different answer then they have made an error. Some candidates, having gained credit for a correct equation in σ, then failed to show any working whatsoever to simplify their equation and simply quoted the given value of σ.

(iv) Few fully correct sketches were seen. In some cases both curves were shown centred around the same mean, or just one curve was drawn. In other cases the means were clearly different but the standard deviations were not. However some candidates produced very clear sketches, including the more subtle point that the maximum height of the diesel curve should be lower than that of the petrol, since both areas are equal to 1.

(v) Only a small proportion of candidates answered correctly. Most started off by finding the probability that the diesel model is above 45.0, gaining one mark. However candidates then either stopped at that, multiplied by the probability for petrol, or in many cases found the sum P(diesel > 45) + P(petrol > 45) + P(both > 45), whereas P(both > 45) should of course be subtracted from the sum of the former two.

3) (i) Most candidates found the equation of the regression line correctly and many of those who made errors appeared to have made a slip rather than not knowing what to do.

(ii) In past sessions many candidates have had little knowledge of residuals. Happily this situation has improved, and the vast majority scored full marks here.

(iii) Most candidates realised and were able to explain that the recalculated equation is preferable as it excludes the result which is not representative of the triathlete's usual performance. A few felt that this was a genuine result and therefore should be included. This argument was not worthy of credit, since the result may have been genuine but is not representative.

(iv) The hypothesis test was generally done well with most candidates scoring 4 marks out of 5. However, despite regular reference to this in examiners reports for the legacy 2614, a correct definition of ρ as the 'population correlation coefficient' was very rarely seen. Pleasingly, most candidates gave the concluding statement in context, rather than simply stating that 'there is no correlation'. Few candidates thought that a two-tailed test was appropriate, although such candidates could follow through and lose just one mark.

(v) Many candidates failed to quote the required assumption of a bivariate Normal distribution. As in the legacy 2614 this failing was again often strongly linked to centres, with many centres in which no candidates gained this mark, and others where almost all did so. The fact that an elliptical scatter diagram can be used as an indication that the test is valid was better known, although by no means universally so, and again the knowledge thereof was strongly linked to centres. Following the removal of coursework from the Statistics 2 assessment, centres need to place more emphasis on ensuring that candidates learn these assumptions, given that they no longer meet them as part of their coursework.

4) (a)(i) Many hypotheses were given in words or occasionally in terms of $\bar{x}$ or ρ rather than in terms of μ as is required. Those candidates who did use μ rarely defined μ as the mean of the population (ie of all houses on the large estate) thus losing credit.

(ii) It is pleasing to report that many correct responses were seen on this new topic. The majority of candidates who were successful found the test statistic in the form of a z-value and then compared this to the critical z-value. A much smaller number compared two probabilities. However many candidates failed to divide the standard deviation by $\sqrt{6}$, thus in effect simply using the distribution of X and this error was heavily penalised. There was a variety of other errors, the most common of which was to calculate a probability and then compare it with a z-value or vice versa.

(b) Once again this new topic was generally dealt with very well. In a contingency table test of association, hypotheses should be given in words and most candidates did so, although some mentioned correlation rather than association. A few candidates had no idea how to proceed and some others knew they had to calculate expected frequencies, but not how to do so. However most knew what to do and did it correctly, with surprisingly little evidence of premature approximation. Having calculated the test statistic, most candidates went on to complete the comparison and conclusion correctly, but a few lost marks, either by making an error in the calculation of the number of degrees of freedom, or by using the wrong figure from the tables, or by making the comparison based on correct figures, but coming to the wrong conclusion. Some candidates failed to give their result in context.

4771 - Decision and Discrete Mathematics 1

General Comments

This paper was a slightly extended version of the paper set for 2620, and this report overlaps greatly with that of 2620.

Candidate performances were generally good – much better than has been the case in the past.

There was some evidence to suggest that some candidates spent far too long on question 5 and consequently ran out of time.

Comments on Individual Questions

1) **Graphs**

(i) Part (i) asked for the number of connections which the electrician has to make. However, many candidates gave the number of arcs in their network.

(ii) Those making the error referred to in part (i) usually added 1 to their answer, which was allowed.

(iii) Examiners do not expect candidates to show any detailed knowledge of the scenarios presented. Nothing is required beyond that which is given in the question. Thus they should not have been looking to their knowledge of domestic electricity circuits, nor bemoaning their lack of such knowledge, in attempting to answer part (iii). The issue here is that which has been considered in past examination papers – that introducing a new vertex into a network can have the effect of reducing the weight of the minimum connector.

(iv) Many candidates realised this was the case but found difficulty justifying it.

2) **Algorithms**

(i) Most candidates were successful with this question. Those that failed mostly allowed themselves to get stuck in a dead end.

(ii) That the algorithm does **not** leave one stuck in a dead end was not a sufficient answer to this question – that alone does not guarantee a route from entrance to exit. What was required was the recognition of the existence of two continuous connections between entrance and exit, the "northeast" wall (plus protuberances) and the "southwest" wall (plus protuberances).

(iii) Most said 'yes'.

(iv) Most said 'yes'. However too many answers concentrated on 'both sides of the walls' rather than routes. One was left with the impression that many had not realised that the maze was different from part (i).

3) **CPA**

(i) Most candidates were very successful with this question. Performance was much better overall than is usually the case on longer CPA questions set in context. A small number of candidates used activity on node (poorly) – the specification is clear that activity on arc is to be used.

(ii) Again done fairly well – most errors occurred when candidates had multiple end networks.

(iii) Generally disappointing. A significant number seemed to think that 'total' implied that floats, usually calculated incorrectly, had to be added together. Too many could not distinguish between 'total float' and 'independent float' conceptually, and/or failed to clarify what float they were actually evaluating.

4) **Networks**

(i) This was a very discriminating question. Good candidates started their Dijkstra from C. A significant minority started from P or V.

(ii) Kruskal is arguably the conceptually easiest algorithm on the syllabus. It might be expected that only the very weakest candidates would be unable to answer this question. However, rather more candidates then expected were not able to.

(iii) Very many candidates failed to score this mark by not providing an adequate answer. Noting that there will be a reduction in length is not an adequate answer to a question asking for the effect of a change. By how much, or to what, is required.

(iv) As per part (iii).

(v) Most candidates recognised the semi-Eulerian issue, if usually implicitly. Unsophisticated students gave a route as justification. Others noted the two odd nodes or pointed out that, since there was such a route from P to C before the bridge, then a route is now given by crossing the bridge and then following that original route.

5) **Simulation**

(i)(ii) Most candidates scored all 4 of these marks

(iii)(iv) A mixed response. Many recognised the need to discard some random numbers but choices of numbers discarded included various groups of numbers in the late 90s, several omitted 84-99 and a few 73-99. However too many used the whole range 00-99.

(v) This was answered quite well. Mistakes were easy to make, and were made, but most candidates showed a good understanding of what was needed.

(vi) Many candidates attempted to answer this question as per part (v), but with returns generated by the new distribution. In fact, the new distribution only comes into play after the number of laptops in stock drops to 2 or fewer. Thus the start of this simulation should be the same as the start of the simulation in part (v). It often was not.

6) **LP**

(i) A significant number of students had clearly run out of time when they started this question. Candidates exhibited all the usual weaknesses. At the worst extreme some identified variables (sometimes explicitly and sometimes implicitly) to do with fibre and nutrient, rather than with Flowerbase and Growmore. Less disastrously very many candidates failed adequately to define their variables (e.g. "Let x = Flowerbase and y = Growmore"), and many failed to note that the problem is a maximisation problem.

(ii) Too many candidates assumed that the optimal solution would be represented by the intersection of the two non-trivial constraint lines. It was disappointing to find a significant minority of candidates drawing graphs in their lined answer books – in several cases it appeared that centres did not make graph paper available to their students.

(iii) Not everyone who answered (B) correctly was able to provide an adequate justification.

4772 - Decision Mathematics 2

General Comments

This was the first presentation of the new unit. Questions were extended versions of those set for 2621. Performances were clustered towards the mean, with few very poor performances and few very high scores. Whilst most candidates attempted all 4 questions there was some evidence of time pressure.
It was clear that candidates had been well prepared for the paper.

Comments on Individual Questions

1) **Logic**

This question was answered well. Only part (iii) caused any difficulty. Some candidates thought it so obvious that they could not see what needed to be written down.

2) **Decision Analysis**

Most candidates were able to complete part (i) and gain at least some credit on part (ii). Few gained much on part (iii) however, the concept of utility completely passing most of the candidates by.
This question also revealed a significant difficulty in work on Decision Analysis. Alternative approaches are possible to the accounting, but some have the potential for causing problems. The safest is to work with final payoffs. Thus in part (i) candidates who worked with profits came to the correct answer with effectively the same computations as those using payoffs, but that was not the case in part (ii). The problem here is that, if r(t) is the exchange rate and v(t) is the value of the investment, then

$$r(t) \times (v(t) - v(0)) \neq (r(t) \times v(t)) - (r(0) \times v(0))$$

The left hand side of the above expression is what many candidates used – it results from working with profits. The right hand side is correct, and is consistent with the answer obtained by working with payoffs.
Whilst this error is not obvious, working with profits rather than payoffs in part (iii) is a fundamental mistake. Utility functions give the utilities of positions not changes.
The definition of the utility function created some problems ("… **thousands** of euros."), but those using euros instead of thousands of euros were not heavily penalised.
Part (iv) was answered by few, and part (v) by very few.

3) **Networks**

Most candidates found some success in this question. A few went into knee-jerk routine in part (iii) and attempted to apply Floyd, wasting quite a lot of time in the process.

4) **LP**

As in Q2, this question revealed a fundamental flaw in candidates' approaches to part of the question. A majority chose the wrong first pivot in part (ii). Choosing the wrong pivot always leads to a negative element appearing in the last column. It is that which the ratio test, when applied correctly, avoids. Candidates making this error carried on with their negative RHS, blissfully unaware that there was a problem.
Another slight difficulty in the question occurred in part (iv). The first mark here was asking why it is that Theo's formulation, though incomplete, leads to the correct solution. The answer looked for was that the constraints he omitted are (clearly) not active at the solution. Candidates did not recognise the issue.
Apart from those two difficulties, and the fact that some candidates were short of time for this last question, it was answered well.

4773 - Decision Mathematics Computation

General Comments

This paper was substantially the same as the paper set for 2622, and this report overlaps greatly with that for 2622. On that paper each question was marked out of 20 and candidates were required to attempt 3 out of 4 questions. The questions were reduced slightly in content for 4773, and were worth 18 marks each, but candidates were required to attempt all 4.

Candidate performances on 4773 were good. There was evidence of some candidates running out of time. In a few other cases, Lindo appeared to have been used to generate a solution, but no evidence was included with the script.

Candidates need to take great care in labelling their computer printout pages, ensuring that they have the correct question number on them and that they are assembled in the correct order.

Comments on Individual Questions

1) **Recurrence relations**

(i) Most candidates got this right, although some computed u_2 as their answer.

(ii) This was a little more difficult than part (i), and a small but significant number of candidates failed on it.

(iii) A large proportion of candidates managed to find their way completely successfully through this intricate calculation.

(iv)(v) Most candidates succeeded in building correct spreadsheets. Not all of those achieved full marks, failing to make simple observations about convergence and limits.

2) **Networks**

(i) to (v) This work on network theory was completed on the insert. It was generally well done. Some candidates had difficulty with part (iii) which, for them, made part (iv) more difficult than it should have been. Nevertheless, they were able to recover since any flow pattern giving a total flow of 6 was acceptable.

(vi) & (vii) Most showed a good idea of how to construct the LP model, even if mistakes were made along the way. Again, there was a weakness in extracting results from the output.

3) **Simulation**

(i) Most could do this in principle, but a significant minority made one of two mistakes –
- using 0, 0.15 and 0.75 instead of 0, 0.15 and 0.9 in their lookup table
- failing to accumulate the service times.
-

(ii) A significant minority of candidates failed to compute the standard deviation of their 10 accumulated times. In some instances candidates tried to do the computation longhand, instead of using the spreadsheet function.
The majority of candidates could do the computation to determine approximately how many repetitions are required. However, it was quite common to see answers in error by a factor of 4.

(iii) Almost all candidates succeeded with this.

(iv) Most candidates could do this simulation, but many failed to compute the queuing times.

(v) A majority could build this two-server simulation but many failed with the queuing times. A number of candidates treated the barriers as being in series rather than parallel – this of course extends rather than reduces the exit time and thus defeats the purpose of this section of the question. Others had errors in their formulae and did not appear to check their computed values for reasonableness.

4) **LP modelling**

(i) It was expected that there would be errors made in this part of the question. In fact, many candidates got it completely right.

(ii) & (iii) Most succeeded in building and running the LP model. However, many of those who produced output failed to interpret it to say how many pilots were needed. A number of candidates missed one or two schedules from constraints and did not appear to check back through their work.

(iv) Very few candidates scored any marks on this. All that was required was a systematic suppression of each of the three solution schedules in turn. In each case more than 3 pilots are required, showing that there is no alternative solution. A few candidates produced alternative, logically reasoned arguments based on permutations of 4-flight schedules, whilst others tried to base their answer on their original Lindo output.

(v) Examiners do not expect candidates to show any detailed ab initio knowledge of the scenarios presented. Nothing is required beyond that which is given in the question. Thus they should not, for instance, have been worrying about the mechanics of refuelling and preparing aircraft. Indeed, there is no mention in the question about the aircraft which are used, so that can form no legitimate part of the answer. However, "turning around" a pilot certainly was relevant.

4776 - Numerical Methods

General Comments

The increase in numbers taking this paper was welcome. Most were reasonably well prepared, though as usual conceptual understanding was not as strong as arithmetical facility.

Comments on Individual Questions

1) **Numerical differentiation**

This question proved accessible to almost everyone. The extrapolation in part (ii) was generally well done, though judging the appropriate level of accuracy was found more difficult.

2) **Errors and accuracy**

Part (i) on rounding and chopping was very easy. In part (ii), finding the maximum possible relative error proved more difficult. In some cases answers were given with no explanation or with an explanation that was difficult to follow.

3) **Secant method to solve an equation**

This question was generally well done, though some candidates did not use the method specified. There can be no credit for using an alternative method even if it gives the correct numerical solution.

4) **Numerical integration**

The numerical work was well done, though a surprising number of candidates did not give answers to the required precision. The extrapolation defeated some, but for many it proved no problem.

5) **Errors in summing a series**

This question was worth 5 marks, but many answers made only one or two points. The question contains several quite distinct requests and, as a matter of examination technique, candidates would be advised to respond carefully to each one in turn.

6) **Difference table, Newton's forward difference method**

The missing values in the difference table were found correctly by most, though some made sign errors. Demonstrating the value of a presented little difficulty. The algebra required to obtain the cubic was more of a challenge, however, and there were many errors. In part (iii), a significant number did not think to find the minimum by differentiation. In part (iv), a number of candidates did not recognize the need to use Lagrange's formula. Those who did sometimes confused the x and $f(x)$ values.

7) **Fixed point iteration and the Newton-Raphson method**

This was found to be a challenging question. In part (i) the graphs defeated some, while others drew correct graphs but said nothing about the roots. Part (ii) was generally successful, but in part (iii) a good many seemed to think that the ratio of differences should have been 0.25. The algebra in part (iv) was difficult for some and there were many dubious manipulations of signs. Part (iv) was too much for many. All that is required here is to show that the ratio of differences decreases substantially.

7850,7852,7853,7856,7857, 3850-3857 AS and A2 MEI Mathematics
June 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
2601	Raw	60	49	42	36	30	24	0
2602	Raw	60	46	41	35	29	23	0
2602/02	Raw	15	12	10	9	8	7	0
2603	Raw	75	54	46	39	32	25	0
2604	Raw	60	45	39	33	28	23	0
2605	Raw	60	42	37	32	27	22	0
2606	Raw	60	46	40	34	29	24	0
2607	Raw	60	48	42	36	31	26	0
2608	Raw	60	46	41	35	29	23	0
2608/02	Raw	15	12	10	9	8	7	0
2609	Raw	60	37	32	27	22	17	0
2610	Raw	60	47	41	35	29	23	0
2610/02	Raw	15	12	10	9	8	7	0
2611	Raw	60	38	33	28	24	20	0
2612	Raw	60	39	34	29	24	20	0
2613	Raw	60	47	41	35	30	25	0
2614	Raw	60	49	44	38	32	26	0
2614/02	Raw	15	12	10	9	8	7	0
2615	Raw	60	47	41	35	30	25	0
2616	Raw	60	48	41	35	29	23	0
2617	Raw	60	46	40	34	28	23	0
2620	Raw	60	45	40	34	29	24	0
2620/02	Raw	15	11	9	8	7	6	0
2621	Raw	60	39	35	30	25	21	0
2621/02	Raw	15	12	10	9	8	7	0
2622	Raw	60	42	36	31	26	21	0
2623	Raw	60	49	43	37	31	25	0
2623/02	Raw	15	12	10	9	8	7	0
2624	Raw	60	44	39	33	27	22	0

2624/02	Raw	15	12	10	9	8	7	0
2625	Raw	60	44	38	32	27	22	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	**A**	**B**	**C**	**D**	**E**	**U**
7850/7852/7853/ 7856/7857	600	480	420	360	300	240	0
3850-3857	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	**B**	**C**	**D**	**E**	**U**	**Total Number of Candidates**
7850	42.6	62.6	77.8	89.5	96.3	100.0	5332
7852	0	0	100.0	100.0	100.0	100.0	1
7853							0
7856	63.2	78.3	87.6	95.1	97.9	100.0	940
7857	77.4	80.7	90.3	90.3	93.6	100.0	31
3850	51.4	65.7	78.4	88.5	95.8	100.0	504
3851	19.9	34.4	55.1	72.4	93.4	100.0	604
3852	0	50.0	50.0	100.0	100.0	100.0	2
3853	9.8	23.5	45.1	66.7	86.3	100.0	51
3854	75.0	75.0	75.0	75.0	75.0	100.0	4
3855							0
3856	54.8	70.9	81.4	91.3	96.5	100.0	484
3857	73.2	83.9	96.4	100.0	100.0	100.0	56

7895,3895-3898 AS and A2 MEI Mathematics
June 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	57	49	41	34	27	0
4752	Raw	72	55	47	40	33	26	0
4753	Raw	72	56	49	42	34	26	0
4753/02	Raw	18	14	12	10	9	8	0
4754	Raw	90	65	56	47	38	29	0
4755	Raw	72	54	47	40	33	27	0
4761	Raw	72	56	49	42	35	28	0
4762	Raw	72	54	47	40	33	26	0
4766	Raw	72	52	46	40	34	28	0
4767	Raw	72	56	49	42	35	28	0
4771	Raw	72	51	45	39	33	27	0
4772	Raw	72	46	40	35	30	25	0
4773	Raw	72	50	43	37	31	25	0
4776	Raw	72	54	48	42	35	28	0
4776/02	Raw	18	13	11	9	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	30.4	53.0	71.8	86.7	95.8	100.0	3420
3895	25.3	41.8	56.4	69.1	80.8	100.0	10530
3896	42.8	60.3	75.9	86.5	94.6	100.0	428
3898	18.8	34.4	46.9	56.3	65.6	100.0	32

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