OCR

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS 6993

Summer 2004

Monday **21 JUNE 2004** Morning 2 hours

Additional materials:
Answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 6 printed pages and 2 blank pages.

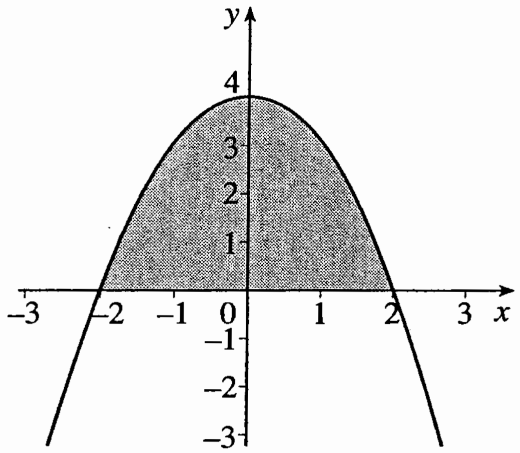
Section A

1 The vertices of a quadrilateral are $A(-2, 0)$, $B(2, 2)$, $C(7, -3)$ and $D(0, -4)$.

(i) Calculate the gradients of the diagonals AC and BD and state a geometrical fact about these lines. [3]

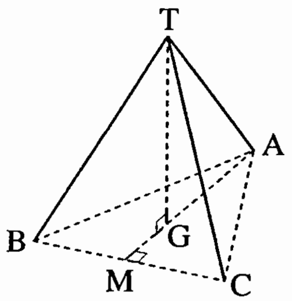
(ii) Show that the mid-point of BD lies on AC. [3]

2 The curve shown is part of the graph of $y = 4 - x^2$.



Calculate the area of the shaded region between this curve and the x-axis, giving your answer as an exact fraction. [4]

3 A tripod with vertex T stands on level ground. The three legs TA, TB and TC are each 60 cm long. The triangle ABC is equilateral with side 50 cm. The point M is the mid-point of BC, and G lies on MA such that MG : GA = 1 : 2. You are given that T lies vertically above G.



Find the angle which the leg TA makes with the ground. [5]

4 Find, by calculus methods, the x-coordinate of the minimum point on the curve

$$y = 2x^3 - 3x^2 - 12x + 6.$$

Show your working clearly, giving the reasons for your answer. [5]

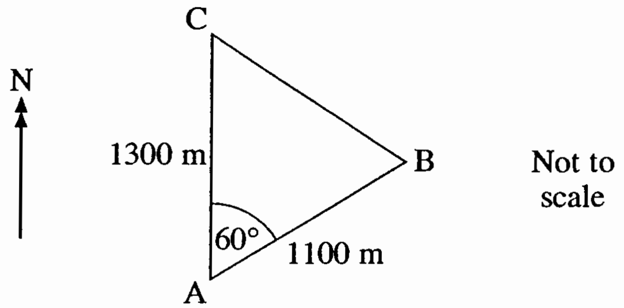
5 **(a)** Find the value of x in the range $0° < x < 360°$ that satisfies **both** $\tan x = 0.75$ **and** $\cos x = -0.8$. [3]

(b) Find all the values of x in the range $0° < x < 360°$ that satisfy $\sin x = -2\cos x$. [3]

6 Express $(2 + \sqrt{3})^3$ in the form $a + b\sqrt{3}$ where a and b are integers. [4]

7 Find the points of intersection of the line $y = 3x + 1$ with the circle $x^2 + y^2 = 12$, giving your answers correct to 2 decimal places. [5]

8 The points A, B, C are three points on an orienteering course. B is 1100 metres from A on a bearing of 060°. C is 1300 metres from A and due north from A.



(i) Show that BC = 1212 metres, correct to the nearest metre. [2]

(ii) Hence find the bearing of C from B. [3]

9 The probability that I am late for work on any given day is 0.1. Being late on one day is independent of any other day.

Find the probability that in a week of 5 working days I am late at least twice. Give your answer correct to 4 decimal places. [5]

10 **(i)** Express $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$ where a and b are integers. [3]

(ii) Hence show that the equation $f(x) = 0$ has no real roots. [1]

You are given that $g(x) = x^3 + 4x^2 - x - 22$.

(iii) Show that $g(2) = 0$. [1]

(iv) Hence show that the equation $g(x) = 0$ has only one real root. [2]

Section B

11 A taxi firm plans to change its fleet of vehicles by buying MPVs (multi-purpose vehicles) and cars. MPVs can carry up to 7 passengers, and cars can carry up to 4 passengers.

MPVs cost £20 000 and cars cost £9000.

The firm can spend up to £180 000.

There is a maximum of 12 drivers available at any one time.

Denoting the number of MPVs to be bought as x and the number of cars to be bought as y, form two inequalities in x and y. [3]

On a graph draw suitable lines and shade the region that the inequalities do **not** allow. [Take 1 cm to represent 1 vehicle.] [4]

From your graph find a pair of values (x, y)

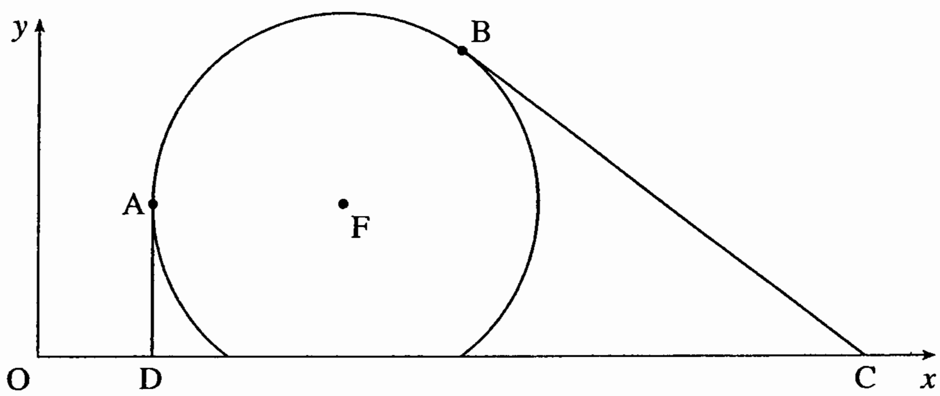
(i) which uses all the £180 000, [1]

(ii) which uses all 12 drivers but minimises the expenditure on vehicles, [1]

(iii) which maximises the number of people that can be carried simultaneously. [3]

12 The shape shown in the diagram is part of a circle. The centre of the circle is F(8, 4) and AD and BC are tangents at A and B respectively. A is the point (3, 4) and B is the point (11, 8).

A wire is stretched from D to A, round the circumference of the circle to B and then to C, where D and C are on the x-axis. Units are centimetres.



(a) Find the equation of the circle. [3]

(b) **(i)** Find the gradient of FB and hence the equation of the tangent BC. [4]

(ii) Given that the length of the wire from A to B in contact with the circle is 11.07 cm, correct to 2 decimal places, find the total length of the wire. [5]

13 I regularly travel a journey of 200 kilometres. When I travel by day I average v kilometres per hour. When I travel at night the traffic is not so bad, so I can average 20 kilometres per hour faster. This means that I am able to complete the journey in 50 minutes less time.

(i) Write down expressions for the journey times during the day and at night. [2]

(ii) Hence form an equation in v and show that it simplifies to

$$v^2 + 20v - 4800 = 0.$$ [5]

(iii) Hence find the times it takes me to complete the journey during the day and at night. [5]

14 A car starts from rest and reaches $20\,\mathrm{m\,s^{-1}}$ in 8 seconds.

(a) Jane models the motion of the car by assuming constant acceleration during the first 8 seconds.

(i) Find the value of the constant acceleration. [2]

(ii) Find the distance travelled during this time. [2]

(b) Paul claims that constant acceleration is not a good model in this situation.

He uses the following formula for the velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds for the first 8 seconds of motion.

$$v = \frac{60t^2 - 5t^3}{64}$$

(i) Show that this formula does give $v = 0$ when $t = 0$ and $v = 20$ when $t = 8$. [1]

(ii) Find the acceleration when $t = 8$. [3]

(iii) Find the distance travelled during the first 8 seconds using this model. [4]

Mark Scheme

June 2004

ADVANCED FSMQ

MARK SCHEME

Maximum mark: 100

Syllabus/component:

6993 Additional Mathematics

Paper set Date: June 21, 2004

Mark scheme

6993 Additional Mathematics. June 2004 Mark scheme

1	**(i)**	Gradient AC = $\frac{-3-0}{7--2}=\frac{-3}{9}=-\frac{1}{3}$	B1	
		Gradient BD = $\frac{-4-2}{0-2}=\frac{-6}{-2}=3$	B1	
		Since $3\times-\frac{1}{3}=-1$ the lines are perpendicular	B1 **3**	
	(ii)	$\text{Mid-point of BD}=\left(\frac{2+0}{2},\frac{2+-4}{2}\right)=(1,-1)$	B1	
		Grad AM $=\frac{-1-0}{1--2}=-\frac{1}{3}$ = Grad AC $\Rightarrow$ points collinear	B1 B 1	
		Alternatively: Equation of AC is $x+3y+2=0$		B1
		This equation is satisfied by (1, -1) as $1-3+2=0$	**3**	B1
2		Area = $\int_{-2}^{2}\left(4-x^2\right)\mathrm{d}x=\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$	M1 A1	Integrate or $2\times\int_0^2$
		$=\left(8-\frac{8}{3}\right)-\left(-8-\frac{-8}{3}\right)=16-\frac{16}{3}=\frac{32}{3}$	M1 A1 **4**	Substitute
3		M is the mid-point of BC $\Rightarrow$BM = 25 cm.		
		By Pythagoras, MA = $\sqrt{50^2-25^2}=25\sqrt{3}=43.3$	M1 A1	For MA
		$\Rightarrow \text{AG}=\frac{50}{3}\sqrt{3}=28.87$ cm	F1	For AG
		$\Rightarrow \text{angle}=\cos^{-1}\frac{28.87}{60}=61.2^0$	M1 A1 **5**	
		Alt: Find angle by cosine rule of TAM Find AM and TM M1 AM correct A1 TM correct A1 Cosine rule used M1 A1		
4		$\frac{\mathrm{d}y}{\mathrm{d}x}=6x^2-6x-12$	B1	
		$=0$ when $x^2-x-2=0$	M1	
		$\Rightarrow(x-2)(x+1)=0\Rightarrow x=2,-1$	A1	
		By considering sign of grad either side of turning point	M1	
		$\Rightarrow$ Minimum at $x=2$	A1	
		Alternatively: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12x-6$: When $x=2$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0\Rightarrow$ Minimum	**5**	M1 A1

5	**(a)**	From calculator, $\tan^{-1}0.75 = 36.9$ There is a second angle in the third quadrant, where the cos value is negative i.e. $x = 180 + 36.9 = 216.9^0$	B1 B1 B1 **3**	 Only 1 answer. Accept anything that is correct.
	(b)	$\tan x = -2$ $\Rightarrow 116.6^0$ and 296.6^0 Alt: square both sides $\Rightarrow \sin^2 x = 4\cos^2 x \Rightarrow \sin^2 x = 4 - 4\sin^2 x$ $\Rightarrow 5\sin^2 x = 4 \Rightarrow \sin x = \pm\frac{2}{\sqrt{5}} \Rightarrow x = 63.4$ Sorting out correct quadrants to give correct angles N.B. any extra angles means no "sorting" of quadrants So M0	B1 B1 B1 **3**	For Tan$x$ -1 extra values in range B1 Pythagoras B1 63.4 B1 two answers
6		$(2+\sqrt{3})^3 = 2^3 + 3.2^2\sqrt{3} + 3.2.3 + 3\sqrt{3}$ $= 8 + 12\sqrt{3} + 18 + 3\sqrt{3}$ $= 26 + 15\sqrt{3}$ **Alt**. Multiply out 3 brackets then mult by third M1 For 7 A1 for $4\sqrt{3}$ A1 Answer A1 **Alt**: Mult everything by everything else (i.e. pick out 8 numbers) M1 4 terms correct A1 Other 4 terms correct A1 Collect up A1	M1 A2 A1 **4**	Binomial, includes coefficients all terms A1 one error collection of integers and surds
7		Substitute for y: $x^2 + (3x+1)^2 = 12 \Rightarrow 10x^2 + 6x - 11 = 0$ $\Rightarrow x = \frac{-6 \pm \sqrt{36+440}}{20} = \frac{-6 \pm \sqrt{476}}{20} = \frac{-6 \pm 21.82}{20}$ $= -1.39$ or $0.79 \Rightarrow y =$ -3.17 or 3.37 i.e.(-1.39, -3.17) or (0.79, 3.37) **Alt:** Sub for x to give quadratic in y: $10y^2 - 2y - 107 = 0$	M1 A1 M1 A1 A1 **5**	Get quadratic Use formula **Alt**: Trial and improvement to 2 dp Both x Both y Alt. A1 one pair, A1 the other pair

8	**(i)**	Cosine rule gives $BC^2 = 1100^2 + 1300^2 - 2.1100.1300.\cos 60 = 147000$ $\Rightarrow$ BC = 1212 metres	M1 A1 **2**	
	(ii)	sin rule gives $\frac{\sin C}{1100} = \frac{\sin 60}{1212} \Rightarrow C = 51.8$ $\Rightarrow$ Bearing = $360 - 51.8 = 308^0$ Or 308.2 or 308.3 (*Do not accept any more decimal places*)	M1 A1 F1 **3**	Or B = 68.2 or 68.3 Alternative methods include cosine rule and splitting triangle into two right angled triangles
9		$P(0) = 0.9^5 = 0.59049$; $P(1) = 5.(0.9)^4.(0.1) = 0.3281$ $P(\text{ at least twice}) = 1 - P(0) - P(1)$ $= 1 - 0.5905 - 0.3281 = 0.0815$ Alt: Add 4 terms: B3,2,1 for the terms. -1 each error or omission. M1 Add the 4 terms. A1 ans Special case P(2) = 0.0729 B2	B1 B2 M1 A1 **5**	 Including coefficient
10	**(i)**	$(x + 3)^2 = x^2 + 6x + 9$. So $x^2 + 6x + 11 = (x + 3)^2 + 2$	 B1 B1 **2**	 For 3, 2
	(ii)	$f(x) = 0 \Rightarrow (x + 3)^2 = -2$ which is never true.	B1 **1**	
	(iii)	$g(2) = 8 + 16 - 2 - 22 = 0$	B1 **1**	
	(iv)	$g(x) = (x - 2)(x^2 + 6x + 11)$ $\Rightarrow g(x) = 0 \Rightarrow (x - 2)(x^2 + 6x + 11) = 0$ $\Rightarrow x = 2$ as quadratic has no roots	M1 A1 A1 **3**	 A comment must be made about the quadratic

Section B

11		Cost: $20\,000x + 9\,000y \le 180\,000$ gives $20x + 9y \le 180$. Drivers: $x + y \le 12$ Graphs [These meet at (6.54, 5.45).] N.B. If inequalities not given but lines and shading correct on graph then give M1 for implied inequalities. If written along lines then give full marks	M1 A1 B1 B4,3,2,1 **7**	B1 axes (whole area on page, axes labelled x,y or by definition B1 B1 each line B1 correct shading
	(i)	(9, 0)	B1 **1**	
	(ii)	(0, 12)	B1 **1**	
	(iii)	The value of $P = 7x + 4y$ at this point is 67.65 So the maximum value of P is 66 or 67 $7x + 4y = 67$ does not pass through any point $7x + 4y = 66$ passes through the point (6, 6) . Alt: Try "near points", e.g. (6, 6), (7, 4) Giving 66 at (6, 6)	M1 M1 A1 **3**	For correct point For integer values required M1 M1 (integer points) A1
12	**(a)**	Radius = 5 $\Rightarrow (x-8)^2 + (y-4)^2 = 25$	B1 M1 A1 **3**	
	(b)(i)	For tangent at B: Grad FB = $^4/_3$ $\Rightarrow$ grad of tangent at B = $-^3/_4$ $\Rightarrow 3x + 4y = 65$	B1 M1 M1 A1 **4**	For gradient For gradient For equation
	(b)(ii)	AD = 4 Tangent at B meets x axis at $(21^2/_3, 0)$ $\Rightarrow$ BC = $\sqrt{\left(21\frac{2}{3}-11\right)^2+(8-0)^2} = 13.33$ AB = 11.07 $\Rightarrow$ Total length = 28.4 cm	B1 E1 M1 A1 F1 **5**	Depends on their equation in (b)(i) Correct application of pythagoras Depends on the M mark

13	**(i)**	Time during the day = $\frac{200}{v}$ Time during the night = $\frac{200}{v+20}$	B1 B1 **2**	
	(ii)	$\frac{200}{v}-\frac{200}{v+20}=\frac{50}{60}$ $\Rightarrow 200(v+20)-200v=\frac{5}{6}v(v+20)$ $\Rightarrow 4000\times 6=5v(v+20)$ $\Rightarrow 5v^2+100v=24000$ $\Rightarrow v^2+20v-4800=0$	M1 A1 M1 A1 A1 **5**	Units correct (i.e. divide by 60) Manipulate
	(iii)	$v=\frac{-20\pm\sqrt{20^2-4.(-4800)}}{2}=\frac{-20\pm\sqrt{19600}}{2}=\frac{-20\pm 140}{2}$ $v=-10\pm 70=60$ $\Rightarrow$ Day speed = 60 $\Rightarrow$ Day time = $\frac{200}{60}=3$ hrs 20 mins $\Rightarrow$ night speed = 80 $\Rightarrow$ night time = $\frac{200}{80}=2$ hrs 30 mins Alt: Night time = day time – 50 mins	M1 A1 A1 F1 F1 **5**	Correct formula For speed Alt: Factorise M1 Corr. brackets A1 For 60 A1 For each time (Allow 3 $^1/_3$ hrs)
14	**(a)(i)**	$v=at\Rightarrow a=\frac{20}{8}=2.5\text{ ms}^{-2}$	M1 A1 **2**	Units required in (a). -1 once if units wrong or missing.
	(a)(ii)	$s=\frac{1}{2}at^2\Rightarrow s=80\text{m}$	M1 A1 **2**	
	(b)(i)	Substitute $t=0$ to give $v=0$ and $t=8$ to give $v=20$	B1 **1**	
	(b)(ii)	Differentiate: $a=\frac{dv}{dt}=\frac{120t-15t^2}{64}$ When $t=8$ $a=0$ Special Case: If 64 left off, M1 A0 B1	M1 A1 B1 **3**	
	(b)(iii)	Integrate: $s=\int_0^8\frac{60t^2-5t^3}{64}\,dt=\left[\frac{20t^3-\frac{5}{4}t^4}{64}\right]_0^8$ $=\left(\frac{10240-5120}{64}\right)=80\text{m}$	M1 A1 M1 A1 **4**	Integrate Substitute

Examiner’s Report

FSMQ Additional Mathematics (6993)   Report, Summer 2004

The regulations for an Advanced FSMQ, as stated clearly in the specification, is that the appropriate starting point is a good grade at Higher Tier. Many candidates started from here and achieved good scores, including some full marks, which was most encouraging. However, as stated last year, it was equally clear that many candidates had not started from the appropriate place. Many candidates not only failed to demonstrate any understanding of the extension material but failed to demonstrate the sort of understanding of some Higher Tier topics. There were a distressing number of candidates scoring very low marks, including single figures and 0. This cannot have been a positive experience for them. Centres are encouraged to seek advice, if necessary, to find the most appropriate course for their students and to seek for further advice on this particular course.
The mean mark was 50.4, down 8 marks from last year, indicating that candidates found the paper this year a little more difficult. The thresholds were reduced accordingly.

Section A
Q1. (Coordinate Geometry)
Weaker candidates usually struggle with this topic, but the basic focus of this question should have enabled all candidates to get started. In fact a number were not able to find the gradient of a line given two points on it. In part (ii) the mid-point was also often wrong. Most candidates found the mid-point, found the equation of AC and showed that one lay on the other. Very few appealed to geometry, or found gradients of, say, AM and AC showing them to be equal.

[Gradients 3 and $-\frac{1}{3}$; lines perpendicular.; Midpoint $(1, -1)$]

Q2. (Area under curve).
In spite of the diagram given not everyone used the correct limits, and then a number got the arithmetic wrong at the end, even offering an answer of 0!!

[Area $= 10\frac{2}{3}$]

Q3. (Trigonometry on 3-D shape)
Often well done, but the greatest error was a failure to use Pythagoras properly taking $AM^2 = 50^2 + 25^2$. A few used other methods, such as the cosine rule and a number did not take enough care over the information given, interchanging 50 and 60.
[61.2^0.]

Q4. (Stationary values).
Most candidates were able to differentiate. Not quite so many were able to deal with the common factor of 6 in the process of solving $\frac{dy}{dx} = 0$. Even fewer were able to justify mathematically the minimum point..
[$x = 2$.]

Q5. (Trigonometrical ratios for angles greater than 90^0.)
Candidates were not comfortable with this question. In part (i) the idea of one ratio being positive and another negative giving the quadrant within which the angle lay was not familiar with most candidates. A number found all the angles satisfying the tan ratio and all the angles satisfying the cos ratio and took the (only) common value. Others gave a number of answers for which they were penalised. In part(ii) the relationship between tan, sin and cos, a specific specification topic, was not well known. Squaring and using Pythagoras was of course an option but no one trying it this way got it right.
[(a) 216.9^0 (b) 116.6^0 and 296.6^0.]

Q6. (Binomial expansion)
A significant number of candidates did not expand using the binomial theorem, choosing instead to multiply out. This was of course acceptable, but long winded. Either way, a number were not able to use the fact that $\left(\sqrt{3}\right)^3 = 3\sqrt{3}$.

[$26 + 15\sqrt{3}$]

Q7. (Intersection of line and curve).
The usual errors were offered, most notably $y^2 = 12 - x^2 \Rightarrow y = \sqrt{12} - x$. Other errors included expanding $(3x + 1)^2 = 3x^2 + 1$.
It is worth noting here that the rubric requires answers to 3 significant figures unless otherwise stated. Candidates ought to be aware of this and be more careful in the way they give their answers. A number lost a mark here by not giving their answers to 2 decimal places as required.
[(-1.39, -3.17) and (0.79, 3.37)]

Q8. (Trigonometry - cosine and sine rules).
This was usually well done except for the calculation of the final bearing, for which a significant number of candidates lost a mark. Given the context of the question, 3 significant figures gives the nearest degree. We condoned one decimal place in this question but no more.
[308^0.]

Q9. (Binomial Probability)
The most significant error in this question was a failure to understand what “at least” means. It was expected that candidates would work 1 - P(0) - P(1). Some worked out the other 4 terms instead but a number gave P(2) as the answer.
[0.0815.]

Q10. (Factor Theorem)
Many found the connections between the parts difficult. Most candidates got part (i) correct but were then unable to use this in part (ii), preferring instead to start again with an attempt to solve a quadratic equation by the formula resulting in a negative discriminant and hence ho roots. This was perfectly acceptable but took rather longer than the award of one mark would warrant.
Likewise, part (iii) was done well but candidates were usually unable to use this in part (iv) to factorise and to find the quadratic that they had been dealing with in the first two parts.
[$(x + 3)^2 + 2$.]

Section B
Q11. (Linear programming)
There were many successful answers to this question, but also many who clearly had not covered this part of the syllabus. Once the inequalities had been derived and the graphs drawn the last part was straightforward, and it is quite possible that most candidates guessed the answer as, on this occasion, the objective function was not asked for.
((i) (9, 0) (ii) (0, 12), (iii) (6, 6)]

Q12. (Coordinate geometry of the circle)
Candidates were not in general comfortable with this topic, and the derivation of the equation of the circle was not always successful. Candidates often also had difficulty finding the equation of the tangent. Many muddled the *x* and *y* axes, finding where this tangent cut the *y* axis.
[(a) $(x - 8)^2 + (y - 4)^2 = 25$ (b)(i) $3x + 4y = 65$, (b)(ii) 28.4 cm.]

Q13. (Algebra)
A significant number of candidates did not answer this question as they were unable to relate speed, distance and time. Many that might have done well then muddled the units, creating an equation involving 50 rather than $^5/_6$. However, many were able to reenter the question for the last part as the quadratic equation was given in the question, and this resulted in up to half marks.
[(i) $\frac{200}{v}$, $\frac{200}{v+20}$ $\frac{200}{v}$, $\frac{200}{v+20}$, (iii) 3 hrs 20 mins and 2 hrs 30 mins.]

Q14. (Calculus)
The question was generally well done except for two rather disappointing errors.
In part (b)(ii), candidates “lost” the denominator of 64. While this still gave $a = 0$ at $t = 8$ the acceleration function was not strictly correct.
In part (b)(iii) candidates “integrated” the denominator, giving the correct integrand divided by $64t$.
[(a) (i) 2.5 ms^{-2}, (ii) 80 m, (b) (ii) 0, (iii) 80m.]