OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2614/1

Statistics 2

Monday **19 JANUARY 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 Ten gymnasts take part in a competition which has two events, floor exercises and parallel bars. The scores for each event are given in the table.

Competitor	A	B	C	D	E	F	G	H	I	J
Floor exercises	9.5	8.8	6.7	6.0	5.9	5.7	5.2	4.2	4.1	3.9
Parallel bars	9.2	9.5	5.8	3.7	5.4	6.3	5.9	4.5	5.6	4.7

(i) Draw a scatter diagram on graph paper to illustrate the data. [2]

A sports journalist wishes to investigate whether there is a positive association between performances on floor exercises and parallel bars.

(ii) By ranking the data, calculate an appropriate correlation coefficient and carry out a suitable hypothesis test at the 5% level of significance. State your hypotheses and conclusions carefully. Comment on the validity of the test. [10]

The product moment correlation coefficient for the two sets of data is 0.837 (correct to 3 significant figures).

(iii) With reference to the scatter diagram, comment on the difference in the values of the two correlation coefficients. Discuss which of the two is more appropriate on this occasion. [3]

2 Soup tins have a capacity of 625 ml. The volume of soup, X ml, dispensed into each tin is Normally distributed with mean 610 and standard deviation 8. If more than 625 ml is dispensed, the tin overflows.

(i) Find the probability that the volume of soup dispensed into a tin is between 600 ml and 625 ml. [4]

The proportion of tins containing at least 600 ml is too low. To increase this proportion to 95%, the dispenser is adjusted in such a way as to reduce the standard deviation of X while leaving the mean unchanged.

(ii) Show that the new value of the standard deviation is 6.08. [4]

(iii) Show that the proportion of tins overflowing is now 0.68%. [3]

Following the adjustment, 1000 randomly chosen tins are inspected.

(iv) Use a suitable approximating distribution to calculate the probability that the soup overflowed on more than 10 occasions when being dispensed into these tins. [4]

3 The number of hits per hour on a website is recorded. Over a long period of time the mean number of hits per hour is 6.9.

(i) Explain briefly why the number of hits per unit time could be modelled by a Poisson distribution. [2]

(ii) Assuming a Poisson model, calculate the probability that the website has

(A) exactly 6 hits in an hour,

(B) exactly 3 hits in each of two successive 30-minute intervals. [6]

(iii) Use a suitable approximating distribution to calculate the probability of between 50 and 60 hits, inclusive, in a period of 8 hours. [4]

(iv) The owners of the website suspect that it has become more popular. In one 8-hour period the site received 75 hits. Using a statistical argument, comment on their assertion. [3]

4 *Cobblers* is a small mail-order shoe business. It makes one style of shoe in several different (British) sizes. Let X represent the shoe size. The demand for sizes of shoe is modelled by

$$\mathrm{P}(X = r) = k(r-3)(12-r) \qquad \text{for } r = 4, 5, 6, 7, 8, 9, 10, 11,$$

$$\mathrm{P}(X = r) = 0 \qquad \text{otherwise.}$$

(i) Tabulate the probability distribution of X and hence show that $k = \frac{1}{120}$. [2]

(ii) Illustrate the probability distribution by a suitable diagram. [2]

(iii) Find $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = 3.85$. [4]

The approximate length of shoe, y cm, may be calculated from the shoe size, x, using the formula $y = \frac{5}{6}x + 21\frac{1}{6}$.

(iv) Deduce the corresponding approximate values for the mean and standard deviation of the length of shoes demanded by customers of *Cobblers*. [3]

One Monday morning *Cobblers* receives 5 orders. It is out of stock of sizes 6, 10 and 11, but has ample stocks of the rest.

(v) Find the probability that *Cobblers* can satisfy at least two of the orders from its stock. [4]

Mark Scheme

Question 1

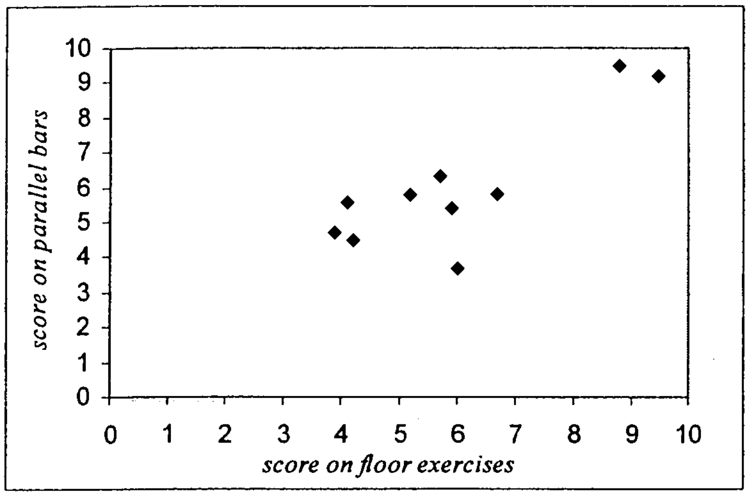
(i)		G1 for all points plotted G1 for linear scaled axes with labels *dep on some points plotted*	2
(ii)	Comp: A B C D E F G H I J Rank x: 1 2 3 4 5 6 7 8 9 10 Rank y: 2 1 5 10 7 3 4 9 6 8 d^2: 1 1 4 36 4 9 9 1 9 4 $r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 78}{10\times 99}$ = 0.527 (to 3 s.f.) [*allow* 0.53 to 2 s.f.]	B1 for ranks (allow all ranks reversed) B1 for d^2 f.t. their ranks s.o.i. M1 for r_s A1 f.t. for $\|r_s\| < 1$ NB No ranking B0B0M0A0	4
	H_0: $\rho = 0$ and H_1: $\rho > 0$ Looking for positive association (one-tail test): critical value at 5% level is 0.5636 Since 0.53 [*or* 0.527] < 0.5636, there is not quite sufficient evidence to reject H_0, i.e. conclude that there is no positive association between performance on the floor exercises and performance on the parallel bars. Test valid provided the data are a random sample [from the underlying bivariate population].	B1 for H_0, B1 for H_1 B1 for ± 0.5636 M1 for comparison with c.v., provided $\|r_s\| < 1$ A1 for conclusion in words f.t. their r_s and sensible cv E1 for explanation	6
(iii)	Product moment correlation coefficient influenced by the two outlying points. The Spearman's rank test is more appropriate, since there is no evidence that the background population of scores is bivariate Normal.	E1 for valid comment relating to outliers on scatter diagram B1 E1 allow reference to elliptical shape	3
			15

Table in (ii):

Comp	A	B	C	D	E	F	G	H	I	J
Rank x	1	2	3	4	5	6	7	8	9	10
Rank y	2	1	5	10	7	3	4	9	6	8
d^2	1	1	4	36	4	9	9	1	9	4

Question 2

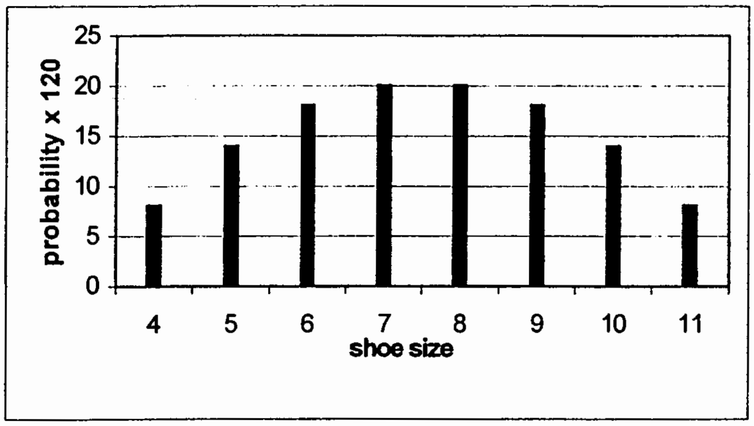
(i)	$P(600 \le X \le 625) = P(-1.25 \le Z \le 1.875)$ $= 0.9697 - (1 - 0.8944)$ $= 0.9697 - 0.1056$ $= 0.864$ (to 3 s.f.)	M1 for standardising A1cao for .9697 or.9696 and .8944 or .8943 M1 for probability calc based on a positive and a negative z value. A1 cao.	**4**
(ii)	Let new standard deviation be σ, then require $P(X \ge 600) = P\left(Z \ge \frac{600-610}{\sigma}\right) = 0.95$ But $P(Z \ge -1.645) = 0.95 \Rightarrow \frac{600-610}{\sigma} = -1.645$ $\Rightarrow \sigma = \frac{10}{1.645} = 6.08$ (to 3 s.f.)	B1 for ± 1.645 M1 for setting up an equation with a z value M1 for solving a correct equation A1 ***answer given*** *NB Max SC3 for verification using σ=6.08*	**4**
(iii)	P(tin overflows) $= P(X > 625) = P\left(Z > \frac{625-610}{6.08}\right)$ $= P(Z > 2.467)$ $= 1 - 0.9932$ $= 0.0068$ (=0.68%)	M1 for $P(X > 625)$ s.o.i M1 for calculation of right hand tail A1 ***answer given***	**3**
(iv)	Let Y represent the number of tins overflowing, then $Y \sim B(1000, 0.0068)$ Since n is large and p is small, a suitable approximating distribution is the Poisson, so approximately $Y \sim$ Poisson (6.8) Hence $P(Y > 10) = 1 - P(Y \le 10)$ $= 1 - 0.9151$ $= 0.0849$ (to 3 s.f.) *or* 0.085 (to 2 s.f.) ***or*** Since n is very large, a suitable approximating distribution could be the Normal, so approximately $Y \sim$ Normal (6.8, 6.75376) Hence $P(Y > 10.5) = 1 - P(Y \le 10.5)$ $= 1 - P(Z \le 1.4237)$ $= 1 - 0.9227$ $= 0.0773$ (to 3 s.f.) *or* 0.077 (to 2 s.f.)	B1 c.a.o. for correct binomial s.o.i. M1 for Poisson with λ = np, s.o.i. M1 for $P(Y \le 10)$ from tables A1 cao ***or*** M1 for Normal with μ=np, σ^2=npq s.o.i. M1 for correct tail (cc not required for M1) A1 c.a.o.	**4**
			15

Question 3

(i)	Number of hits could be modelled by a Poisson distribution because (a) hits occur randomly and independently, (b) there could be a uniform mean rate of occurrence	E1 for first reason E1 for second reason	**2**
(ii)	(*A*) P(exactly 6 hits in an hour) $= 0.4647 - 0.3137$ *or* $e^{-6.9}\frac{6.9^6}{6!}$ $= 0.151$ (to 3 s.f.) (*B*) P(exactly 3 hits in each 30 minute interval) $= \left(e^{-3.45}\frac{3.45^3}{3!}\right)^2 = 0.217265^2 = 0.0472$	M1 for method A1 B1 for $\lambda = 3.45$ M1 for Poisson P(X=3) M1 for squaring A1 f.t. their sensible λ	**2** **4**
(iii)	Using the Normal approximation $X \sim N(55.2, 55.2)$ P(between 50 and 60 hits in 8 hours) $\approx P(49.5 < X < 60.5)$ $= P(-0.767 < Z < 0.713)$ $= 0.7620 - (1 - 0.7785)$ $= 0.5405$ *Alternative solutions which can gain the final A1:* *-omitted continuity correction* $P(50 < X < 60 = 0.7408 - 0.2420 = 0.4988$ NB (z = -0.700,0.646), *-wrong continuity correction* $P(50.5 < X < 59.5) = 0.7186 - 0.2635 = 0.4551$ NB (z = -0.633,0.579),	B1 for Normal approx. with correct parameters s.o.i B1 for continuity corr. M1 for method including use of tables A1 c.a.o. (≥3sf) *NB M0A0 for use of variance*	**4**
(iv)	P(at least 75 hits in 8 hours) $\approx P(X > 74.5) = P(Z > 2.597)$ $= 1 - 0.9953 = 0.0047$ (to 2 s.f.) Since this is so small, it casts doubt on at least some part of the original model (e.g. incorrect λ) – their claim could be justified. *[condone no cont. corr. leading to P(Z>2.665) = 1 – 0.9961 = 0.0039]* *Accept alternative argument based on ≥ 2 sd from the mean, with supporting calculations*	M1 for probability calculation A1 E1	**3**
			15

Question 4

<table>
<tr><td>(i)</td><td>
<table>
<tr><td>r</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>$P(X=r)$</td><td>$8k$</td><td>$14k$</td><td>$18k$</td><td>$20k$</td><td>$20k$</td><td>$18k$</td><td>$14k$</td><td>$8k$</td></tr>
</table>
$(8+14+18+20+20+18+14+8)k = 1$
$\Rightarrow\ k = \frac{1}{120}$</td><td>B1 for tabulation

B1 for equation
(N.B. Answer given)</td><td>2</td></tr>
<tr><td>(ii)</td><td>
</td><td>G1 for linear horizontal scale and attempt at representing data

G1 f.t. for lines in proportion</td><td>2</td></tr>
<tr><td>(iii)</td><td>$E(X) = 4\times\frac{8}{120}+5\times\frac{14}{120}+6\times\frac{18}{120}+\ldots+10\times\frac{14}{120}+11\times\frac{8}{120}$
$= 7.5$ (or by inspection)

$E(X^2) = 4^2\times\frac{8}{120}+5^2\times\frac{14}{120}+6^2\times\frac{18}{120}+\ldots+10^2\times\frac{14}{120}+11^2\times\frac{8}{120}$
$= 60.1$
Hence $Var(X) = E(X^2)-[E(X)]^2$
$= 60.1 - 7.5^2$
$= 3.85$</td><td>B1f.t. for $E(X)$
[provided $\Sigma p = 1$]

M1 for $E(X^2)$

M1 for positive variance

A1 NB Answer given</td><td>4</td></tr>
<tr><td>(iv)</td><td>Mean $= E(Y) = \frac{5}{6}E(X) + 21\frac{1}{6} = \frac{5}{6}\times 7.5 + 21\frac{1}{6} = 27\frac{5}{12}$
$Var(Y) = \left(\frac{5}{6}\right)^2 Var(X) = \left(\frac{5}{6}\right)^2 \times 3.85 = 2.67$
Hence s.d. $= \sqrt{2.67} = 1.64$ (to 3.s.f.)
or
s.d. $= \frac{5}{6}\sqrt{3.85} = 1.64$</td><td>B1f.t.

M1 for $Var(Y)$

A1 cao

M1 for $\frac{5}{6}$ s.d., A1</td><td>3</td></tr>
<tr><td>(v)</td><td>P(out of stock) $= \frac{40}{120} = \frac{1}{3}$
P(satisfy at least 2 orders)
$= 1 - \left[\left(\frac{1}{3}\right)^5 + 5\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)\right] = 1 - \frac{11}{243}$
$= \frac{232}{243}$ or 0.955 (to 3.s.f.)
or using tables
$P(X \le 1) = 0.0453$,
$P(X \ge 1) = 1 - 0.0453 = 0.955$</td><td>B1ft for probability p or q

M1 for binomial expression with n=5
M1 for $1 - [\ldots+\ldots]$
A1 cao
or
M1 for tables with n=5
M1 for $1 - P(X \le 1)$
A1cao</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td>15</td></tr>
</table>

Examiner’s Report

2614 Statistics 2

General Comments

Candidates were generally well prepared for this paper, with relatively few candidates scoring under 20 marks. However, in questions where comments were required, answers were often unconvincing, and even when credit was gained answers often suggested rote learning rather than genuine knowledge. Only a few candidates appeared to be short of time, failing to complete the final question. Both Question 1 on rank correlation and Question 2 on the Normal distribution were frequently answered very well, with only the comment in Question 2(iv) causing much difficulty for most candidates; there was nonetheless a significant number of candidates with very low scores on Question 2. The earlier parts of Question 3 on the Poisson distribution and Question 4 on discrete random variables were also found to be very accessible, but candidates often found problems with the last two parts of both questions.

Comments on Individual Questions

Question 1 (Correlation)

(i) This was usually answered well; nearly all candidates were able to draw the scatter diagram correctly, although a few tried to plot scores in each test against competitor label.

(ii) Most candidates correctly ranked the data and successfully obtained the Spearman's rank correlation coefficient. There were occasionally slips in ranking or squaring. A few candidates tried to calculate r by subtracting scores, not ranks. A few calculated the product moment correlation coefficient, and often then had difficulty in part (iii) where they had to discuss the difference. Occasionally candidates made errors with Spearman's formula such as the omission of 6 or incorporation of 1 in the numerator.

Most test procedures were correctly carried out, and only a few candidates used a two-tailed test or mistook the table from which to take the critical value. Most candidates stated the null and alternative hypotheses correctly. It was often the case, however, that the test was baldly stated as accepting the null hypothesis, without putting this into the context of the question. There is some sympathy for those who have written out the hypotheses in length, in context, but it is preferable to use hypotheses which are succinctly stated in terms of ρ, and to interpret the results in full, rather than the other way round. Few candidates scored the final mark, for explaining that the validity of the test depended on the sample being random; instead many answers related to ellipticity or to the graph showing no sign of curvature; a high proportion of candidates failed to discuss the validity of the test in any way at all.

(iii) The final three marks were rarely earned in full. Candidates were more or less equally divided between those who thought that the pmcc was better and those who favoured Spearman's. Arguments in favour of pmcc mostly said that because the outliers were there the pmcc picked up the correlation that obviously existed. Some candidates were more of the opinion that the outlier was candidate D in the lower centre of the scatter diagram. Candidates were also equally divided between those of the opinion that there was an elliptical shape to the data and those who thought there wasn't. Once again a number of candidates thought that the use of Spearman's is confined to situations where scatter diagrams show clear signs of curvature. Sometimes the reasons given contradicted themselves, but it was good to see that many candidates did know that the pmcc requires an underlying bivariate Normal population.

(ii) 0.527, accept H_0

Question 2 (Normal distribution)

The majority of candidates answered most of this question well, and it is encouraging to note widespread use of correct and clear notation and working in Normal distribution work. However a significant number of candidates are unable to write out correct probability statements.

(i) Most candidates scored all four marks, but some did lose credit for losing the final digit in looking up the probabilities in the tables. Some did not cope with the tail of the distribution and a few introduced spurious continuity corrections.

(ii) It was pleasing to see that most candidates attempted to set up an equation to find the standard deviation rather than using the given answer to verify that the probability was 0.95. However, a large number of candidates produced equations leading to a negative standard deviation, often then ignoring the negative sign, and arriving at the given answer. Candidates should be aware that when an answer is given, their justification needs to be clear and convincing if it is to gain full credit.

(iii) This part was usually done well.

(iv) Most candidates realised that the exact distribution was binomial, although occasionally with $p = 0.68$. Many then used a suitable approximating distribution, usually the Poisson distribution, leading to the correct answer, although use of P $(X > 10) = 1 - $ P (≤ 9) was a common error. Candidates who tried to use the Normal approximation often failed to get the continuity correction right or used $p = 0.68$, leading to unworkable values of μ and σ.

(i) 0.8641; (ii) 6.08 (answer given); (iii) 0.68% (answer given);
(iv) 0.0849

Question 3 (Poisson model)

(i) Many candidates appeared to have learned stock phrases to answer a question of this sort, and this at times led to errors, where they mentioned a 'random number of hits', rather than 'hits occurring randomly' as a reason for a Poisson model. Relatively few commented on the need for hits to be occurring at a uniform mean rate. There was often a mention of large n and small p, which in this question was irrelevant.

(ii)(*A*) This was usually correctly answered, with most candidates using the Poisson probability to obtain P $(X = 6)$, and only a few using tables. Some candidates who used tables gave the value of P $(X \leq 6)$ only.

(*B*) Most candidates found the correct λ for half an hour and the correct probability for 3 hits. Only a few tried to approximate this quantity from tables. Rather more candidates, however, doubled their probability instead of squaring it, at the next stage.

(iii) It was pleasing to see that the Normal approximation was fairly often carried out well, noticeably better than in some previous sessions, although as ever many candidates lost one mark for using an incorrect continuity correction, or none at all. Candidates who could write clear probability statements usually fared best. Some candidates thought that the variance was 55.2^2, quoting this initially, whilst others presumably just forgot to use the standard deviation.

(iv) The final part was not well answered. The instruction to use a *statistical argument* was either ignored or misunderstood by the many candidates who failed to do any probability calculations to support their arguments. Other candidates calculated the probability that exactly 75 hits would be obtained, without realising that this would always be a very small quantity and that a tail probability was required. Even those who made a sensible calculation often failed to make a valid conclusion on what was in effect an informal hypothesis test. Some candidates produced a convincing argument based on the mean plus two standard deviations.

(ii)(*A*) 0.151, (*B*) 0.0472; (iii) 0.5405;
(iv) P ($X > 74.5$) = 0.0047

Question 4 (Probability Distribution)

(i) This was usually well answered, although the second mark was occasionally lost through inadequate demonstration.

(ii) Most candidates drew a correct vertical line diagram, with only a few curves or 'bar charts' without gaps in evidence. However, as mentioned in previous reports, there are still candidates whose vertical line diagram does not have linear scales. Vertical line diagrams must have linear scales with lines whose lengths are in proportion to the probabilities.

(iii) The expected value was usually correct but quite a few candidates failed to show how E (X^2) was obtained and so did not earn all of the available credit. Once again, as in Question 2(ii), it is to be emphasised that candidates must provide clear and correct working to obtain full credit when the answer is given.

(iv) Candidates often found the new mean correctly, but only a few were able to find the new standard deviation, with many adding the constant term on to the new variance or failing to square the coefficient of x before multiplying it by the variance. Other candidates found the mean and variance of the transformed sizes by treating them as eight items of data, making no use whatsoever of the probabilities. Some candidates stopped after correctly finding the variance, presumably forgetting that they were asked to give the standard deviation.

(v) The last part of the question was often done better than part (iv). There was some confusion over whether the probability of success was 1/3 or 2/3, and these probabilities were often replaced with 3/8 and 5/8. Many candidates recognised that a binomial distribution was required, and were successful, although a good proportion subsequently made errors of one sort or another.

(i) $k = 1/120$ (answer given); (iii) 7.5, 3.85; (iv) 27.4, 1.635; (iv) 0.955