OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2602/1**

Pure Mathematics 2

Wednesday **14 JANUARY 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.
- An **INSERT** is provided for Question **2**.

This question paper consists of 3 printed pages, 1 blank page and an insert.

[Turn over

1 (a) Write down any function $f(x)$ which is even, and explain how this property relates to its graph. [2]

(b) Find $\int \frac{1}{1+2x}\,dx$. [3]

(c) Given that $f(x) = x^2$ and $g(x) = f(x-3) + 2$, sketch the graph of $y = g(x)$.

Express $g(x)$ in the form $x^2 + ax + b$. [5]

(d) Given that $y = \ln(1 + x^2)$, find $\frac{dy}{dx}$.

Show that $\frac{d^2y}{dx^2} = \frac{2 - 2x^2}{(1 + x^2)^2}$. [5]

2 **Use the insert provided to answer this question.**

Chris bought his house in 1975 for £10 000. The following table gives the market value of his house at five-year intervals.

Year	1975	1980	1985	1990	1995	2000
Market value (£)	10 000	18 000	33 000	61000	90 000	200 000

He decides to model the market value using the equation $V = ab^t$, where £V is the market value t years after 1975.

(i) Using the insert provided, complete the table and plot the values of $\log_{10} V$ against t. Hence show that the model fits the data well, except for the value in 1995. [4]

(ii) By fitting a straight line through the remaining 5 points, use the graph to estimate the market value given by the model for 1995. [3]

(iii) Use the straight line you have drawn to estimate the values of a and b. [5]

(iv) Chris wants to estimate when his house will be worth £1 000 000. When does the model predict that this will happen? [3]

3 A geometric progression u_i is defined by

$$u_i = e^{-i}, \qquad i = 1, 2, 3, \ldots$$

where e is the base of natural logarithms.

(i) Calculate u_1 and u_2, and show that $u_3 = 0.05$, correct to 2 decimal places. Write down, in terms of e, the common ratio of the progression. [3]

(ii) Find the least value of i for which $u_i < 10^{-12}$. [4]

(iii) Show that the geometric series $u_1 + u_2 + u_3 + \ldots$ is convergent, and that the sum to infinity is $\dfrac{1}{e - 1}$. [3]

(iv) The sequence v_i is defined by $v_i = \ln u_i$. Show that v_i is an arithmetic progression, stating its first term and common difference. Hence calculate $\sum_{i=1}^{100} v_i$. [5]

4 Fig. 4 shows a sketch of the graph of $y = x\sqrt{1 - 2x}$. This meets the x-axis at the origin and the point P. The point Q is the maximum point on the curve.

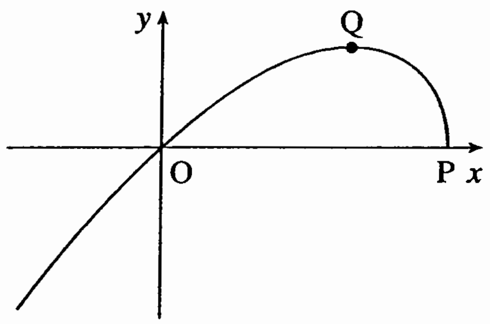

Fig. 4

(i) Write down the coordinates of P. [1]

(ii) Show that $\dfrac{dy}{dx} = \dfrac{1 - 3x}{\sqrt{1 - 2x}}$. Hence find the exact coordinates of the point Q. [7]

(iii) Given that $u = 1 - 2x$, show that the area A enclosed by the curve and the x-axis is given by

$$A = \tfrac{1}{4}\int_0^1 (u^{1/2} - u^{3/2})\, du.$$

Evaluate A, giving your answer as an exact fraction. [7]

Candidate Name	Centre Number	Candidate Number

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Pure Mathematics 2

INSERT

Wednesday **14 JANUARY 2004** Morning 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **2**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Attach the insert securely to your answer booklet.

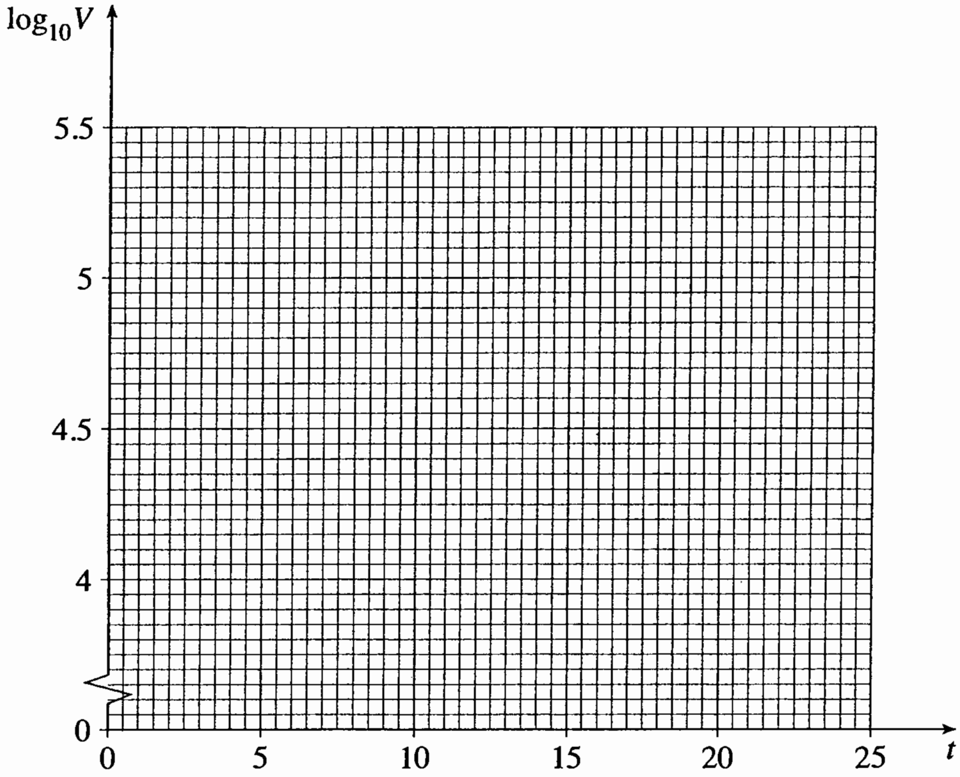
This insert consists of 2 printed pages.

HN/4

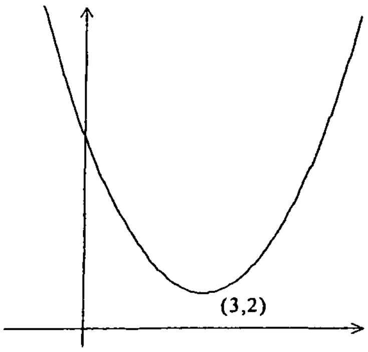
[Turn over

2

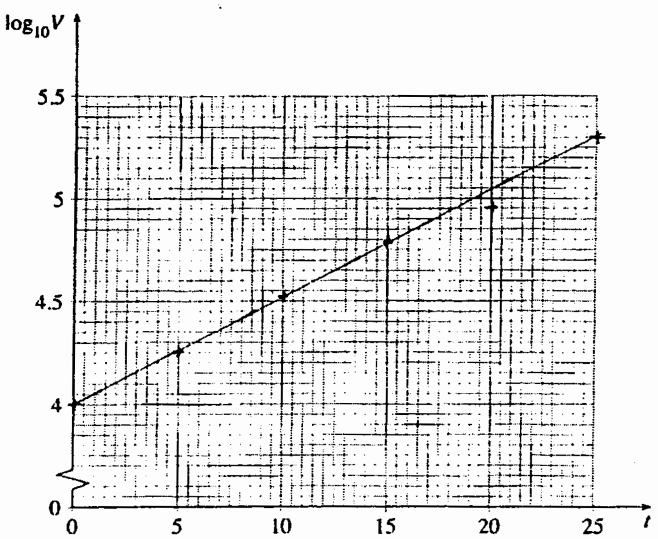
t (years after 1975)	0	5	10	15	20	25
V (£)	10000	18000	33000	61000	90000	200000
$\log_{10}V$ (to 3 sig. fig.)	4.00		4.52		4.95	



Mark Scheme

1(a) Any even function; symmetrical about y axis	B1 B1 [2]	e.g. $y = x^2, x^2 + x^4, 0, \cos x$, etc…
(b) $\int \frac{1}{1+2x} dx$ let $u = 1 + 2x$, $du = 2dx$ $= \int \frac{1}{2}\frac{1}{u} du$ $= ½ \ln(1 + 2x) + c$	M1 A1 B1 [3]	Substitution to get $\int \frac{1}{2}\frac{1}{u} du$ ½ ln(1 + 2x) or B2 by inspection $+ c$
(c)  	B1 B1	Parabola – any u shape translated $\begin{pmatrix}3\\2\end{pmatrix}$ - must have coordinates of turning point clear.
g(x) $= (x - 3)^2 + 2$ $= x^2 - 6x + 11$ $\Rightarrow a = -6, b = 11$	M1 A1cao A1cao [5]	$(x - 3)^2 + 2$ $a = -6$ $b = 11$
(d) $y = \ln(1 + x^2)$ let $u = 1 + x^2$, $\Rightarrow \frac{du}{dx} = 2x, y = \ln u, \frac{dy}{du} = \frac{1}{u}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{2x}{1+x^2}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1+x^2).2 - 2x.2x}{(1+x^2)^2}$ $= \frac{2+2x^2-4x^2}{(1+x^2)^2}$ $= \frac{2-2x^2}{(1+x^2)^2}$ * *or* $\frac{d^2y}{dx^2} = -2x(1+x^2)^{-2}.2x + 2(1+x^2)^{-1}$ $= \frac{2-2x^2}{(1+x^2)^2}$ *	B1 B1 M1ft M1ft E1 M1ft M1ft E1 [5]	$\frac{1}{1+x^2}$ $\times 2x$ their $v\mathrm{d}u/\mathrm{d}x - u\mathrm{d}v/\mathrm{d}x$ (but $u \neq 1$) their $(1 + x^2)^2$ [SCB1 $\frac{-2x}{(1+x^2)^2}$ from $\frac{1}{1+x^2}$] www $-2x(1+x^2)^{-2}.2x + \ldots$ $\ldots + 2(1+x^2)^{-1}$ www

2(i) t | 0 5 10 15 20 25 log V 4 **4.26** 4.52 **4.79** 4.95 **5.30** Plotting points (see below for completed graph)	B2,1,0 B2,1,0 [4]	Must be to 3 s.f., but penalise once only. Condone 5.3 allow ½ square tolerance
(ii) $t = 20$, $\log V = 5.04$, $\Rightarrow V = 10^{5.04}$ $= 110\,000$	B1 M1 A1 [3]	allow $5.0 \le \log V \le 5.1$ $V = 10^{\text{their}5.04}$ (soi) $100\,000 \le V \le 130\,000$
(iii) $\log V = \log a + t \log b$ $\log a = 4 \Rightarrow a = 10^4 = 10\,000$ gradient $= \dfrac{5.30-4}{25-0} = 0.052$ $\Rightarrow \log b = 0.052$ $\Rightarrow b = 1.127$	M1 A1 B1 M1 A1 [5]	$\log a$ = their intercept or $10\,000 = a\,b^0$ $a = 10\,000 \pm 500$ gradient = 0.052 ± 0.002 $\log b$ = their gradient (soi) art $1.12 \le b \le$ art 1.13
(iv) $1\,000\,000 = 10\,000 \times 1.127^t$ $\Rightarrow 1.127^t = 100$ $\Rightarrow t \log 1.127 = 2$ $\Rightarrow t = 2/(\log 1.127)$ $= 39$ [so model predicts 2014]	M1 M1 A1ft [3]	Substituting $t = 1\,000\,000$ and their values of a and b into $V = ab^t$ or $\log V = \log a + t \log b$ equation (e.g. $\log 1\,000\,000 = 4 + t \log 1.127$) Taking logs (if needed) and simplifying to make t subject, but www for a and b $36 \le t \le 41$ Do not accept unsupported answers



3(i) $u_1 = 0.368,\ u_2 = 0.135$ $u_3 = 0.04978\ldots = 0.05$ to 2 d.p. $r = e^{-1}$ or 1/e	B1 E1 B1 cao [3]	Both correct (or 0.37, 0.14) Must show either • u_3 to more than 2 d.p or • $e^{-3} = 0.05$ or • $u_3 = e^{-1} \times 0.135$ or $0.14 = 0.05$ Must be exact . Not e^{-2}/e^{-1}
(ii) $e^{-i} < 10^{-12}$ $\Rightarrow\ -i < \ln 10^{-12} = -27.6$ *or* $-i \log e < -12$ $\Rightarrow\ -i < -27.63\ldots$ $\Rightarrow\ i = 28$	M1 M1 A1 A1	Equating or in-equating taking lns or logs (dep 1st M1) −27.63…(soi) 28 cao
or $e^{-27} = 1.88 \times 10^{-12}$ $e^{-28} = 6.91 \times 10^{-13}$ $\Rightarrow i = 28$	M1 M1 A2 [4]	Dep both M1s
(iii) $-1 < \frac{1}{e} < 1$ so convergent $S_\infty = \frac{a}{1-r} = \frac{e^{-1}}{1-e^{-1}}$ $= \frac{1}{e-1}$ *	E1 M1 E1 [3]	Or $1/e^n \to 0$ in S_n formula. Condone $\frac{1}{e} < 1$ S_∞ formula with $a = e^{-1}$ and $r = e^{-1}$ (allow approximations) www – must be exact working
(iv) $\ln u_i = \ln e^{-i} = -i$ So −1, −2, −3, … This is an A.P. with $a = -1,\ d = -1$ $S_{100} = \frac{100}{2}[2\times(-1) + 99\times(-1)]$ $= -5050$	M1 A1 M1 A1 A1 [5]	$\ln e^{-i} = -i$ stated for any value of i without approximation $a = -1,\ d = -1$ sum of AP formula (correct) (soi) used with $a = (-)1,\ d = (-)1$ and $n = 100$ cao

4(i) P is (½ , 0)	B1 [1]	Allow P = ½ or x = ½ but not (0, ½)
(ii) Let $u = x$ and $v = (1-2x)^{1/2}$ $du/dx = 1$, $dv/dx = \frac{1}{2}.(-2).(1-2x)^{-1/2}$ $= -(1-2x)^{-1/2}$ $\frac{dy}{dx} = -x(1-2x)^{-1/2} + (1-2x)^{1/2}.1$ $= (1-2x)^{-1/2}[-x+1-2x]$ $= \frac{1-3x}{\sqrt{1-2x}}$ *	 B1 B1 M1 E1 (4)	 $\frac{1}{2}(1-2x)^{-1/2}$ × (−2) Product rule consistent with their derivatives www
$\frac{dy}{dx} = 0$ when $1 - 3x = 0$ ⇒ $x = 1/3$, $y = \frac{1}{3}\sqrt{1-\frac{2}{3}} = \frac{1}{3\sqrt{3}}$	M1 A1 B1 (3) [7]	$1 - 3x = 0$ (only) $x = 1/3$ $y = \frac{1}{3\sqrt{3}}$ or equivalent, but must be exact , with 1 − 2/3 simplified – mark final irrational answer
(iii) $A = \int_0^{\frac{1}{2}} x\sqrt{1-2x}\,dx$ let $u = 1 - 2x$, $du = -2dx$ ⇒ $x = (1-u)/2$ when $x = 0$, $u = 1$ when $x = ½$, $u = 0$ ⇒ $A = \int_1^0 \frac{1-u}{2}.u^{1/2}.(-\frac{1}{2})du$ $= \frac{1}{4}\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}})du$ *	 M1 M1 M1 E1 (4)	 $\frac{1}{2}(1-u).u^{1/2}$ × − ½ du $x = 0$, $u = 1$, $x = ½$, $u = 0$ [even if wrongly entered into integral] www
$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$ $= \frac{1}{4}(\frac{2}{3} - \frac{2}{5})$ $= \frac{1}{2} \times \frac{2}{15} = \frac{1}{15}$	B1 M1 A1 (3) [7]	$u^{3/2}$ and $u^{1/2}$ correctly integrated substituting (correct) limits (but must have clearly attempted to integrate) cao – must be exact fraction

Examiner’s Report

2602/01 Pure Mathematics 2

General Comments

This proved to be a reasonably straightforward paper. Question 2 scored well, with some easy marks available for parts (i) and (ii), and this question ensured that very few candidates scored in single figures. However, Question 3 placed series work in a somewhat unfamiliar context, and this redressed the overall mark for many candidates. Nevertheless, there were plenty of strong scripts scoring over 50 out of 60.

Virtually all candidates seemed to have enough time to answer all the questions, although if they were very inefficient in their methods of solution (for example plotting a graph for the function g(x) in question 1(c), they did on occasions run out of time in question 4. The insert in question 2 was still not securely tied into the paper from some centres – and very occasionally was missing, with a consequent loss of marks.

The effect of giving answers on the paper, as in questions 1(d), 3(iii), 4(ii) and 4(iii), is to encourage 'fiddling' which is often counter-productive – students should be advised that they could score more marks by leaving their attempts unsullied by unsound efforts to derive the printed result.

Comments on Individual Questions

Question 1 (Various topics)

This question proved to contain relatively straightforward tests of syllabus topics, and many candidates scored well.

(a) Most candidates successfully identified an even function, with $y = x^2$ by far the most popular, although some candidates hazarded more interesting functions. Answers such as f(x^2) were penalised. Most candidates also successfully quoted symmetry about the y-axis for the second mark.

(b) This part was less successfully answered, with the ½ commonly omitted from the integral in solutions by inspections. The arbitrary constant, when present, gained an easy mark.

(c) Any parabola shape gained an easy mark for all and sundry; placing the minimum was also quite well done. Stating the quadratic equation for the function was either right or wrong, and tested candidates' understanding in a straightforward context.

(d) Quite a lot of candidates scored full marks here, but they needed to get the first derivative correct to make much progress. Omitting the $2x$ from this led to fudged second derivatives. The quotient rule was generally well done.

(b) ½ ln(1 + $2x$) + c;   (c) $a = -6$, $b = 11$

Question 2 (Reduction to linear form)

This proved to be the easiest question on the paper, with plenty of accessible marks.

Part (i) was usually correct, with answers given to 3 significant figures and accurate plotting of the points.

Part (ii) was also very well done, with virtually all candidates successfully anti-logging their log V value. The mark scheme erred on the side of generosity in its range of permissible values.

Part (iii) was usually done by calculating the gradient and the intercept, although a can also be found easily by substituting $t = 0$ in the equation $V = a\ b^t$. Some candidates took the gradient to be b rather than $\log b$, or even got the gradient upside down.

Part (iv) required a little more understanding, but there were still plenty of good answers. Extrapolations of the graph scored zero.

(ii) $100\,000 < V < 130\,000$, (iii) $a = 10\,000$, $1.12 < b < 1.13$ (iv) $36 \le t \le 41$

Question 3 (Sequences and series)

The unfamiliar context here lowered the marks for this question, although many candidates recovered to gain full marks for the last part.

Part (i) gave candidates a gentle introduction, with u_3 giving the intended re-assurance. However, the common ratio was often approximate or not simplified, e.g. e^{-2}/e^{-1}.

In part (ii), there was some uncertain use of negatives in solving the inequality. Most used logarithms, but there was the occasional successful use of trial and improvement.

Part (iii) was poorly done. Many candidates took partial sums to 'show' that the sum to infinity existed, rather than noting that $-1 < e^{-1} < 1$. The sum to infinity formula was often quoted wrongly as $\frac{1}{r-1}$, the answer in the paper perhaps implying this.

In part (iv), approximate answers to –1, –2, –3, etc. indicated ignorance that $\ln e^{-i} = -i$. However, many recovered to score the final three marks. Some refused to believe that the sum could be negative and gave the final answer as 5050.

(i) 0.368, 0.135, $0.04978.. \approx 0.05$, e^{-1}, (ii) $i = 28$, (iii) $-1 < e^{-1} < 1$, (iv) –5050

Question 4 (Calculus)

Part (i) was usually correct.

In part (ii), the factor –2 was frequently omitted from the derivative of $(1-2x)^{\frac{1}{2}}$, and the algebra required to derive the given result for $x(1-2x)^{\frac{1}{2}}$ eluded all but the more competent candidates. The turning point was generally found correctly, albeit with an approximate rather than exact surd value for the y-coordinate.

The integration by substitution in part (iii) was quite tricky, with the reversal of the limits required to achieve the printed result prompting a great deal of fiddling which proved counter-productive. However, if candidates stuck to their guns and substituted correctly for dx, x and the limits change, they scored 3 out of the 4 marks. There were many recoveries to evaluate the integral for the final three marks. Weak candidates substituted limits without integrating, or differentiated instead.

(i) $\left(\frac{1}{2}, 0\right)$, (ii) $\left(\frac{1}{3}, \frac{1}{3\sqrt{3}}\right)$, (iii) $\frac{1}{15}$