RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2606

Pure Mathematics 6

Tuesday **10 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Option 1: Vectors and Matrices

1 Four points have coordinates A(10, 11, 12), B(15, 15, 15), C(4, 1, 2) and D(k, 9, 8).

In the case $k \neq 14$, find the following, giving your answers as simply as possible:

(i) the volume of the tetrahedron ABCD, [5]

(ii) the shortest distance between the lines AB and CD. [6]

In the case $k = 14$,

(iii) show that the lines AB and CD are parallel, [1]

(iv) find the shortest distance between the lines AB and CD, [4]

(v) find the shortest distance from the origin (0, 0, 0) to the plane containing the lines AB and CD. [4]

Option 2: Limiting Processes

2 **(a)** Given that a is a positive constant, find $\lim\limits_{x \to a} \dfrac{e^x - e^a}{x^e - a^e}$. [3]

(b) Prove from first principles that if $y = \sqrt{x}$ then $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$. [5]

(c) **(i)** Show that

$$\int_2^{n+1} \frac{1}{\sqrt{x}}\,dx < \sum_{r=2}^{n} \frac{1}{\sqrt{r}} < \int_1^{n} \frac{1}{\sqrt{x}}\,dx.$$ [4]

(ii) Hence prove that the infinite series $\sum\limits_{r=1}^{\infty} \dfrac{1}{\sqrt{r}}$ is not convergent. [3]

(iii) Given that N is a positive integer, show that

$$2N < \sum_{r=1}^{(N+1)^2} \frac{1}{\sqrt{r}} < 2N + 1.$$ [5]

Option 3: Multi-Variable Calculus

3 A surface has equation $z = e^x(12 + 4xy + y^2)$.

(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [3]

(ii) Show that $(1, -2, 8e)$ is a stationary point on the surface, and find the coordinates of the other stationary point. [6]

(iii) Sketch the section of the surface given by $x = 1$, and, on a separate diagram, sketch the section of the surface given by $y = -2$.

What can you deduce about the nature of the stationary point $(1, -2, 8e)$? [6]

(iv) Find, in the form $ax + by + cz + d = 0$, the equation of the tangent plane to the surface at the point P$(0, 3, 21)$. [3]

(v) The point $(h, 3 - h, 21 + k)$, where h and k are small, is a point on the surface close to P. Find an approximate expression for k in terms of h. [2]

Option 4: Differential Geometry

4 **(a)** Find the arc length of the polar curve $r = e^{k\theta}$ for $0 \leqslant \theta \leqslant 2\pi$, where k is a positive constant. [5]

(b) A curve C has parametric equations

$$x = 3at, \quad y = at^3,$$

where a is a positive constant.

(i) Find the equation of the normal to C at the general point $(3at, at^3)$. [3]

(ii) Hence or otherwise find parametric equations for the evolute of C. [6]

(iii) Find the curved surface area formed when the arc of C for which $0 \leqslant t \leqslant 1$ is rotated through 2π radians about the x-axis. [6]

Option 5: Abstract Algebra

5 The real vector space V consists of all vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, where a, b and c are real numbers.

Three particular vectors are $\mathbf{e}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$ and $\mathbf{e}_3 = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$.

(i) Express $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ as a linear combination of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$.

Deduce that $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is a basis for the vector space V. [6]

A linear mapping $\mathrm{T}: V \to V$ is defined by $\mathrm{T}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} b - 2c \\ 2a + 3c \\ 4a + 3b \end{pmatrix}$.

(ii) Find the matrix of T with respect to the basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. [5]

The subspace K of V consists of all vectors $\mathbf{x}$ for which $\mathrm{T}\mathbf{x} = \mathbf{0}$.

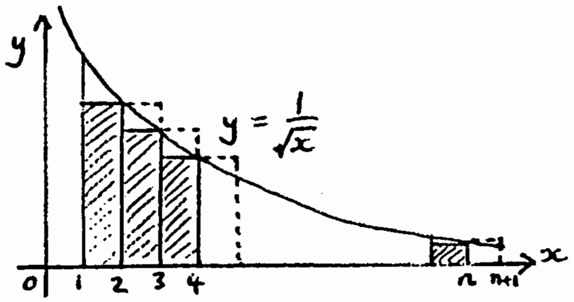
(iii) Prove that $\{\mathbf{e}_3\}$ is a basis for K. [3]

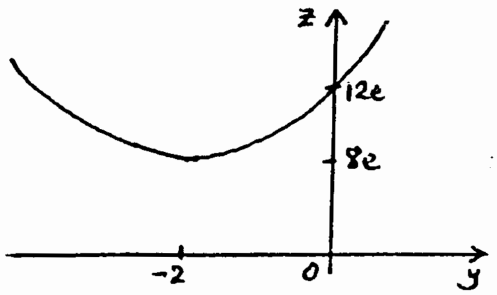
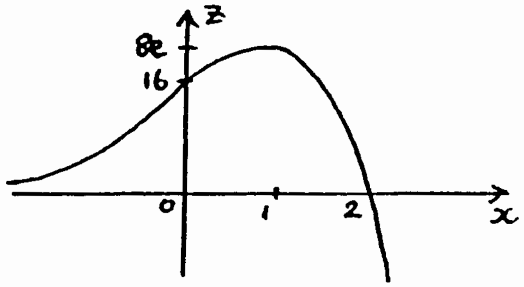
(iv) Simplify $\mathrm{T}(\mathbf{e}_1 + \lambda\mathbf{e}_3)$, where λ is a real number. [2]

(v) Hence find the vector $\mathbf{v}$ such that $\mathrm{T}\mathbf{v} = \mathbf{e}_2$ and $\mathbf{v}$ is perpendicular to $\mathbf{e}_2$. [4]

Mark Scheme

1 (i)	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}5\\4\\3\end{pmatrix}\times\begin{pmatrix}-6\\-10\\-10\end{pmatrix}=\begin{pmatrix}-10\\32\\-26\end{pmatrix}$	M1A1	Vector product of two sides
	$V=\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}=\frac{1}{6}\begin{pmatrix}-10\\32\\-26\end{pmatrix}\cdot\begin{pmatrix}k-10\\-2\\-4\end{pmatrix}$	M1 M1	Scalar triple product Fully correct method (including $\frac{1}{6}$)
	$=\frac{5}{3}\lvert(14-k)\rvert$	A1 5	
(ii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}5\\4\\3\end{pmatrix}\times\begin{pmatrix}k-4\\8\\6\end{pmatrix}=\begin{pmatrix}0\\3k-42\\-4k+56\end{pmatrix}$	M1A1	
	$=(k-14)\begin{pmatrix}0\\3\\-4\end{pmatrix}$	M1	Finding unit vector
	Distance is $\frac{1}{5}\begin{pmatrix}0\\3\\-4\end{pmatrix}\cdot\begin{pmatrix}-6\\-10\\-10\end{pmatrix}=2$	M1 A1 ft A1 cao 6	Fully correct method Correct answer in any form Simplified answer
(iii)	$\overrightarrow{AB}=\begin{pmatrix}5\\4\\3\end{pmatrix}$, $\overrightarrow{CD}=\begin{pmatrix}10\\8\\6\end{pmatrix}=2\overrightarrow{AB}$, so lines are parallel	B1 1	
(iv)	$\overrightarrow{AC}\times\overrightarrow{AB}=\begin{pmatrix}-6\\-10\\-10\end{pmatrix}\times\begin{pmatrix}5\\4\\3\end{pmatrix}=\begin{pmatrix}10\\-32\\26\end{pmatrix}$	M1A1	
	Distance is $\dfrac{\lvert\overrightarrow{AC}\times\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}=\dfrac{\sqrt{10^2+32^2+26^2}}{\sqrt{5^2+4^2+3^2}}$	M1	Fully correct method
	$=\dfrac{\sqrt{1800}}{\sqrt{50}}=6$	A1 4	
	OR Take $P(10+5\lambda,\ 11+4\lambda,\ 12+3\lambda)$ on AB		
	$\overrightarrow{CP}\cdot\overrightarrow{AB}=\begin{pmatrix}6+5\lambda\\10+4\lambda\\10+3\lambda\end{pmatrix}\cdot\begin{pmatrix}5\\4\\3\end{pmatrix}=0$ M1		
	$5(6+5\lambda)+4(10+4\lambda)+3(10+3\lambda)=0$ A1 $\lambda=-2$		
	$\overrightarrow{CP}=\begin{pmatrix}-4\\2\\4\end{pmatrix}$ and $CP=\sqrt{16+4+16}$ M1		
	$=6$ A1		
(v)	Equation of plane ABCD is $5x-16y+13z=30$	M1A1	
	Distance is $\dfrac{30}{\sqrt{5^2+16^2+13^2}}=\dfrac{30}{\sqrt{450}}=\sqrt{2}$	M1A1 4	

2 (a)	Limit is $\lim_{x\to a}\dfrac{e^x}{ex^{e-1}}$ $=\dfrac{e^a}{ea^{e-1}}=\dfrac{e^{a-1}}{a^{e-1}}$	M1A1 A1 3	
(b)	$\dfrac{\delta y}{\delta x}=\dfrac{\sqrt{x+\delta x}-\sqrt{x}}{\delta x}$ $=\dfrac{(\sqrt{x+\delta x}-\sqrt{x})(\sqrt{x+\delta x}+\sqrt{x})}{\delta x(\sqrt{x+\delta x}+\sqrt{x})}$ $=\dfrac{x+\delta x-x}{\delta x(\sqrt{x+\delta x}+\sqrt{x})}=\dfrac{1}{\sqrt{x+\delta x}+\sqrt{x}}$ Letting $\delta x\to 0$, $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{x}+\sqrt{x}}=\dfrac{1}{2\sqrt{x}}$	M1A1 M1 M1 A1 5	Use of $(\sqrt{x+\delta x}+\sqrt{x})$ *SR:* Allow M1M1A0 for use of $(x+h)^{\frac{1}{2}}=x^{\frac{1}{2}}\left(1+\dfrac{h}{2x}+\dots\right)$ Considering $\delta x\to 0$ Correct completion
(c)(i)	 $\sum_{r=2}^{n}\dfrac{1}{\sqrt{r}}$ is the area of rectangles of width 1 below the curve $y=\dfrac{1}{\sqrt{x}}$ for $1\le x\le n$ and above the curve for $2\le x\le n+1$ Hence $\int_2^{n+1}\dfrac{1}{\sqrt{x}}dx<\sum_{r=2}^{n}\dfrac{1}{\sqrt{r}}<\int_1^{n}\dfrac{1}{\sqrt{x}}dx$	B1 B1 B1 B1 4	Completion
(ii)	$\sum_{r=2}^{n}\dfrac{1}{\sqrt{r}}>\left[2\sqrt{x}\right]_2^{n+1}=2\sqrt{n+1}-2\sqrt{2}$ $\to\infty$ as $n\to\infty$ Hence $\sum_{r=1}^{\infty}\dfrac{1}{\sqrt{r}}$ is not convergent	M1A1 A1 3	Evaluation of a relevant definite integral (possibly infinite) For a fully correct argument
(iii)	$\sum_{r=2}^{(N+1)^2}\dfrac{1}{\sqrt{r}}<\left[2\sqrt{x}\right]_1^{(N+1)^2}=2(N+1)-2=2N$ Hence $\sum_{r=1}^{(N+1)^2}\dfrac{1}{\sqrt{r}}=1+\sum_{r=2}^{(N+1)^2}\dfrac{1}{\sqrt{r}}<2N+1$ $\sum_{r=2}^{(N+1)^2}\dfrac{1}{\sqrt{r}}>\left[2\sqrt{x}\right]_2^{(N+1)^2+1}>2(N+1)-2\sqrt{2}$ $\sum_{r=1}^{(N+1)^2}\dfrac{1}{\sqrt{r}}>1+2(N+1)-2\sqrt{2}=2N+3-2\sqrt{2}>2N$	M1 A1 M1 M1A1 5	OR $\sum_{r=1}^{(N+1)^2}\dfrac{1}{\sqrt{r}}>\int_1^{(N+1)^2+1}\dfrac{1}{\sqrt{x}}dx$ M1 $=2\sqrt{(N+1)^2+1}-2$ $>2(N+1)-2=2N$ M1A1

3 (i)	$\dfrac{\partial z}{\partial x} = e^x(12 + 4xy + y^2 + 4y)$	M1A1	
	$\dfrac{\partial z}{\partial y} = e^x(4x + 2y)$	B1 3	
(ii)	For stationary points, $\dfrac{\partial z}{\partial x} = \dfrac{\partial z}{\partial y} = 0$	M1	
	$y = -2x$ and $12 - 2y^2 + y^2 + 4y = 0$ $y^2 - 4y - 12 = 0$ $y = -2,\ 6$ $x = 1,\ -3$	M1A1	Quadratic in x or y
	One stationary point is $(1,\ -2,\ 8e)$	B1	Can be verified by substitution
	Other stationary point is $(-3,\ 6,\ -24e^{-3})$	A2 6	Give A1 for one coordinate correct
(iii)	When $x = 1$, $z = e(12 + 4y + y^2)$		
		B1 B1	Parabola with minimum Minimum in 2nd quadrant
	When $y = -2$, $z = e^x(16 - 8x)$		
		B1 B1 B1	Curve approaching x-axis from above as $x \to -\infty$ Curve approaching $-\infty$ as $x \to +\infty$ Maximum in 1st quadrant
	Stationary point is neither a maximum nor a minimum (saddle point)	B1 6	Only if the graphs show this
(iv)	At P, $\dfrac{\partial z}{\partial x} = 33,\ \dfrac{\partial z}{\partial y} = 6$	B1	
	Tangent plane is $33x + 6y - z = 0 + 18 - 21$ $33x + 6y - z + 3 = 0$	M1A1 3	
(v)	$33h + 6(3 - h) - (21 + k) + 3 \approx 0$ $27h - k \approx 0$	M1	*Or* $\delta z \approx 33\delta x + 6\delta y$ $k \approx 33h + 6(-h)$
	$k \approx 27h$	A1 2	

4 (a)	$\int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} \sqrt{\left(e^{k\theta}\right)^2 + \left(ke^{k\theta}\right)^2}\, d\theta$	M1A1	
	$= \sqrt{1+k^2} \int_0^{2\pi} e^{k\theta}\, d\theta$	M1	
	$= \frac{\sqrt{1+k^2}}{k}\left(e^{2\pi k} - 1\right)$	M1A1 5	
(b)(i)	$\frac{dy}{dx} = \frac{3at^2}{3a} = t^2$	M1	
	Equation of normal is $y - at^3 = -\frac{1}{t^2}(x - 3at)$	M1A1 3	
	$y = -\frac{x}{t^2} + at^3 + \frac{3a}{t}$		
(ii)	Differentiating partially with respect to t		
	$0 = \frac{2x}{t^3} + 3at^2 - \frac{3a}{t^2}$	M1A1	
	$x = \frac{3at}{2} - \frac{3at^5}{2}$	M1A1	
	$y = -\frac{1}{t^2}\left(\frac{3at}{2} - \frac{3at^5}{2}\right) + at^3 + \frac{3a}{t}$	M1	
	$y = \frac{3a}{2t} + \frac{5at^3}{2}$	A1 6	
	OR $\rho = \frac{\left(\dot{x}^2 + \dot{y}^2\right)^{\frac{3}{2}}}{\dot{x}\ddot{y} - \ddot{x}\dot{y}} = \frac{3a\left(1+t^4\right)^{\frac{3}{2}}}{2t}$ M1A1		
	$\hat{\mathbf{n}} = \frac{1}{\sqrt{1+t^4}}\begin{pmatrix} -t^2 \\ 1 \end{pmatrix}$ M1		
	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3at \\ at^3 \end{pmatrix} + \frac{3a(1+t^4)}{2t}\begin{pmatrix} -t^2 \\ 1 \end{pmatrix}$ M1		
	$x = \frac{3at}{2} - \frac{3at^5}{2},\quad y = \frac{3a}{2t} + \frac{5at^3}{2}$ A1A1		
(iii)	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (3a)^2 + (3at^2)^2 = 9a^2(1+t^4)$	M1	
	$\int 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 2\pi(at^3)(3a)\sqrt{1+t^4}\, dt$	M1A1	
	$= \pi a^2 \int_0^1 6t^3\sqrt{1+t^4}\, dt = \pi a^2\left[\ (1+t^4)^{\frac{3}{2}}\ \right]_0^1$	M1A1	
	$= \pi a^2(2\sqrt{2} - 1)$	A1 6	

5 (i)	Let $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda\mathbf{e}_1 + \mu\mathbf{e}_2 + \nu\mathbf{e}_3$ then		
	$a = -\mu - 3\nu,\ b = \lambda + 3\mu + 4\nu,\ c = \lambda + 3\mu + 2\nu$	M1A1	
	$\lambda = \frac{1}{2}(6a + 7b - 5c),\ \mu = \frac{1}{2}(-2a - 3b + 3c)$	M1	Solving to obtain one of λ, μ, ν
	$\nu = \frac{1}{2}(b - c)$	A2	Give A1 for two correct
	The set is also linearly independent	B1 **6**	*Or* V has dimension 3
(ii)	$\mathrm{T}\mathbf{e}_1 = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} = \mathbf{e}_2,\ \mathrm{T}\mathbf{e}_2 = \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix} = 3\mathbf{e}_1 + \mathbf{e}_3,\ \mathrm{T}\mathbf{e}_3 = \mathbf{0}$	M1 A1A1A1	
	Matrix is $\begin{pmatrix} 0 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	A1 **5**	
(iii)	If $\mathrm{T}\mathbf{x} = \mathbf{0},\ b - 2c = 2a + 3c = 4a + 3b = 0$	M1	
	$b = 2c,\ a = -\frac{3}{2}c$, so $\mathbf{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{1}{2}c\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$	A1A1 **3**	
	$\mathrm{T}\mathbf{x} = \mathbf{0} \Leftrightarrow \mathbf{x} = \lambda\mathbf{e}_3$ so $\{\mathbf{e}_3\}$ is a basis for K		
	OR If $\mathbf{x} = \lambda\mathbf{e}_1 + \mu\mathbf{e}_2 + \nu\mathbf{e}_3$ then		
	$\mathrm{T}\mathbf{x} = 3\mu\mathbf{e}_1 + \lambda\mathbf{e}_2 + \mu\mathbf{e}_3$ M1A1		
	$\mathrm{T}\mathbf{x} = \mathbf{0} \Leftrightarrow \lambda = \mu = 0 \Leftrightarrow \mathbf{x} = \nu\mathbf{e}_3$ A1		
(iv)	$\mathrm{T}(\mathbf{e}_1 + \lambda\mathbf{e}_3) = \mathrm{T}\mathbf{e}_1 + \lambda\mathrm{T}\mathbf{e}_3$	M1	
	$= \mathbf{e}_2 + \mathbf{0}$		
	$= \mathbf{e}_2$	A1 **2**	
(v)	Let $\mathbf{v} = \mathbf{e}_1 + \lambda\mathbf{e}_3 = \begin{pmatrix} -3\lambda \\ 1 + 4\lambda \\ 1 + 2\lambda \end{pmatrix}$	M1	
	$\mathbf{v}.\mathbf{e}_2 = 0 \Rightarrow -(-3\lambda) + 3(1 + 4\lambda) + 3(1 + 2\lambda) = 0$	M1A1	
	$21\lambda + 6 = 0$		
	$\lambda = -\frac{2}{7}$		
	$\mathbf{v} = \begin{pmatrix} \frac{6}{7} \\ -\frac{1}{7} \\ \frac{3}{7} \end{pmatrix}$	A1 **4**	

Examiner’s Report

2606 Pure Mathematics 6

General Comments

This was found to be considerably more straightforward than last year's paper. There were many excellent scripts, with one quarter of the candidates scoring 50 marks or more (out of 60), and there was a wide range of ability, with a quarter scoring less than 30 marks. About 45% of candidates chose questions 1, 3 and 4; other popular selections were 1, 2 and 3 (24% of candidates) and 1, 3 and 5 (16% of candidates). Some candidates appeared to have difficulty completing three questions in the time allowed.

Comments on Individual Questions

Q.1 Vectors

This question was attempted by most candidates, and half the attempts scored 13 marks or more (out of 20). There were four standard calculations, and three of these were generally well understood and accurately applied. The exception was part (iv), the shortest distance between two parallel lines; many candidates just used their formula from part (ii). Part (iii) was intended to help the candidates, but some did manage to lose this mark, usually for arguing that since the volume of the tetrahedron is zero (and so A, B, C, D are coplanar) AB must be parallel to CD. Solutions to this question were often very long. Few candidates seemed to appreciate that the same vector product AB × AC could be used in parts (i), (iv) and (v), or that taking out the factor $(k-14)$ from the vector product AB × CD greatly simplifies finding the unit common perpendicular in part (ii).

(i) $\frac{5}{3}|(14-k)|$; (ii) 2; (iv) 6; (v) $\sqrt{2}$.

Q.2 Limiting Processes

This question was attempted by about one third of the candidates.

L'Hôpital's rule in part (a) was well understood and the limit was usually found correctly.

In part (b) there was some misunderstanding of the phrase 'from first principles'. A few candidates quoted the formula nx^{n-1}, which earned no marks. Several used the binomial series to approximate $(x+h)^{\frac{1}{2}}-x$; the infinite binomial series is a more sophisticated concept than differentiation so it is not really a 'first principle', but its correct use was awarded 4 marks out of the 5.

In part (c)(i) almost all candidates explained the inequalities by reference to rectangles and the area under the curve $y=\frac{1}{\sqrt{x}}$, although not all these explanations were good enough to earn full marks; for each part of the inequality candidates were expected to identify precisely which rectangles were being considered. Shading on a diagram is perfectly acceptable provided it is unambiguous. In part (ii) the divergence of the series was usually established by evaluating a definite integral and showing it to be unbounded; sometimes only the upper bound was considered and this lost the final mark. In part (iii) most candidates earned some marks, although full marks was not very common. Adapting the inequalities involving $\sum_{r=2}^{n}\frac{1}{\sqrt{r}}$ to inequalities involving $\sum_{r=1}^{n}\frac{1}{\sqrt{r}}$ can be done by adding 1 throughout; this gives the upper bound easily, but some work is necessary to show that the lower bound obtained as $1+2\sqrt{(N+1)^2+1}-2\sqrt{2}$ is greater than $2N$. Many candidates used the alternative method of subtracting 1 from the lower limits of the integrals; this works well for the lower bound, but unfortunately gives an upper bound of $2(N+1)$ instead of the required $2N+1$.

(a) $\dfrac{e^{a-1}}{a^{e-1}}$.

Q.3 Multi-Variable Calculus

This question was attempted by most candidates, and half the attempts scored 15 marks or more. The partial differentiation in part (i), and finding the stationary points in part (ii), were very often done correctly. However, many candidates were unable to sketch the graphs in part (iii). In part (iv) the equation of the tangent plane was usually found correctly. Part (v) was also answered well.

(i) $\dfrac{\partial z}{\partial x} = e^x(12 + 4xy + y^2 + 4y)$, $\dfrac{\partial z}{\partial y} = e^x(4x + 2y)$; (ii) $(-3,\ 6,\ -24e^{-3})$;

(iii) saddle point; (iv) $33x + 6y - z + 3 = 0$; (v) $k \approx 27h$.

Q.4 Differential Geometry

This question was attempted by just over half the candidates. It was the best answered question, with half the attempts scoring 16 marks or more and one fifth of attempts scoring full marks. All parts were well understood. In part (b)(ii) the great majority used the expected method of differentiating the equation of the normal to find the evolute, with just a few candidates preferring to start afresh and find the radius of curvature. In part (b)(iii) many candidates were unable to integrate $t^3\sqrt{1+t^4}$. Quite a few attempted to use the parametric equations they had just found in part (ii) instead of those for the original curve *C*.

(a) $\dfrac{\sqrt{1+k^2}}{k}\left(e^{2\pi k} - 1\right)$;

(b)(i) $y = -\dfrac{x}{t^2} + at^3 + \dfrac{3a}{t}$; (ii) $x = \dfrac{3at}{2} - \dfrac{3at^5}{2}$, $y = \dfrac{3a}{2t} + \dfrac{5at^3}{2}$; (iii) $\pi a^2(2\sqrt{2} - 1)$.

Q.5 Vector Spaces

This was the least popular question, attempted by about one fifth of the candidates. It was by far the worst answered question with half the attempts scoring 7 marks or less, and no candidate scored full marks. In part (i) many candidates did not seem to realise that explicit expressions for the three coefficients were required. In part (ii) the concept of the matrix corresponding to a linear transformation and given bases was very poorly understood; most candidates gave the matrix corresponding to the standard basis. In part (iii) only a handful of candidates even attempted to show that every vector in *K* is a multiple of $\mathbf{e}_3$; most considered it sufficient to show that $\mathbf{e}_3$ is in *K*. Parts (iv) and (v) were quite often answered correctly.

(i) $\frac{1}{2}(6a + 7b - 5c)\mathbf{e}_1 + \frac{1}{2}(-2a - 3b + 3c)\mathbf{e}_2 + \frac{1}{2}(b - c)\mathbf{e}_3$;

(ii) $\begin{pmatrix} 0 & 3 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$; (iv) $\mathbf{e}_2$; (v) $\begin{pmatrix} \frac{6}{7} \\ -\frac{1}{7} \\ \frac{3}{7} \end{pmatrix}$.