OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2602/1**

Pure Mathematics 2

Monday **19 MAY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.
- An **INSERT** is provided for Question **4** parts **(ii)** and **(iv)**.

This question paper consists of 5 printed pages, 3 blank pages and an insert.

1 (a) Fig. 1 shows the graph of $y = a + b \sin cx$, where a, b and c are constants.

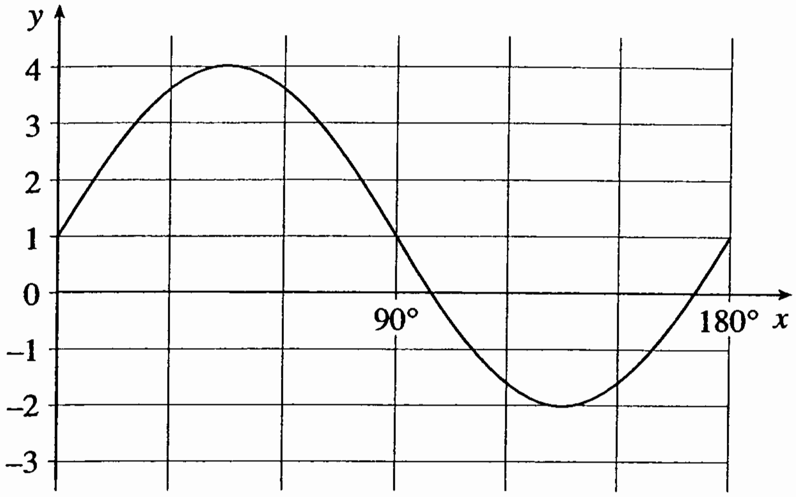

Fig. 1

Find the values of a, b and c. [3]

(b) Differentiate $\dfrac{\ln x}{1+\ln x}$, simplifying your answer. [3]

(c) Expand $(e^x + e^{-x})^2$. Hence find $\int (e^x + e^{-x})^2 \, dx$. [4]

(d) The function $g(x)$ is defined by $g(x) = a^x$, where a is a positive constant.

(i) Given that, for all values of x, $g(x + 2) = 3g(x)$, show that $a = \sqrt{3}$. [2]

(ii) Given also that $g(b) = 5$, find b to 3 significant figures. [3]

2 Fig. 2 shows the graph of $y = x\sqrt{1+x}$. The point P on the curve is on the x-axis.

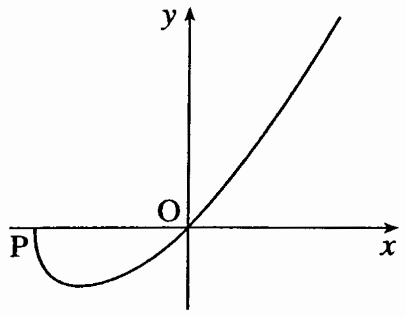

Fig. 2

(i) Write down the coordinates of P. [1]

(ii) Show that $\dfrac{dy}{dx} = \dfrac{3x+2}{2\sqrt{1+x}}$. [3]

(iii) Hence find the coordinates of the turning point on the curve. What can you say about the gradient of the curve at P? [4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1} (u^{\frac{3}{2}} - u^{\frac{1}{2}})\,du.$$

Evaluate this integral, giving your answer as an exact fraction.

What does this value represent? [7]

3 **(i)** For each of the following sequences, state whether it is arithmetic, geometric or neither of these. For those that are arithmetic or geometric, find the sum of the first 20 terms of the corresponding series.

(A) $50, 52, 54, 56, \ldots$

(B) $u_n = 2 \times 0.8^n$, $\quad n = 1, 2, 3, \ldots$

(C) $u_n = 2n + 3$, $\quad n = 1, 2, 3, \ldots$

(D) $u_n = n^2$, $\quad n = 1, 2, 3, \ldots$

(E) $u_{n+1} = -u_n, u_1 = 2$, $\quad n = 1, 2, 3, \ldots$

(F) $u_{n+1} = 2u_n + 1, u_1 = 1$, $\quad n = 1, 2, 3, \ldots$ [13]

(ii) In the case of one of these sequences, the corresponding series has a sum to infinity. Calculate the sum to infinity of this series. [2]

4 **[Use the insert provided to answer parts (ii) and (iv) of this question.]**

In a scientific study, a researcher measures the mass of a cockroach at regular intervals. Let w milligrams be the mass of the cockroach t days after hatching. Fig. 4 shows the values of $\ln w$ and $\ln t$ plotted on graph paper. This is repeated on the insert.

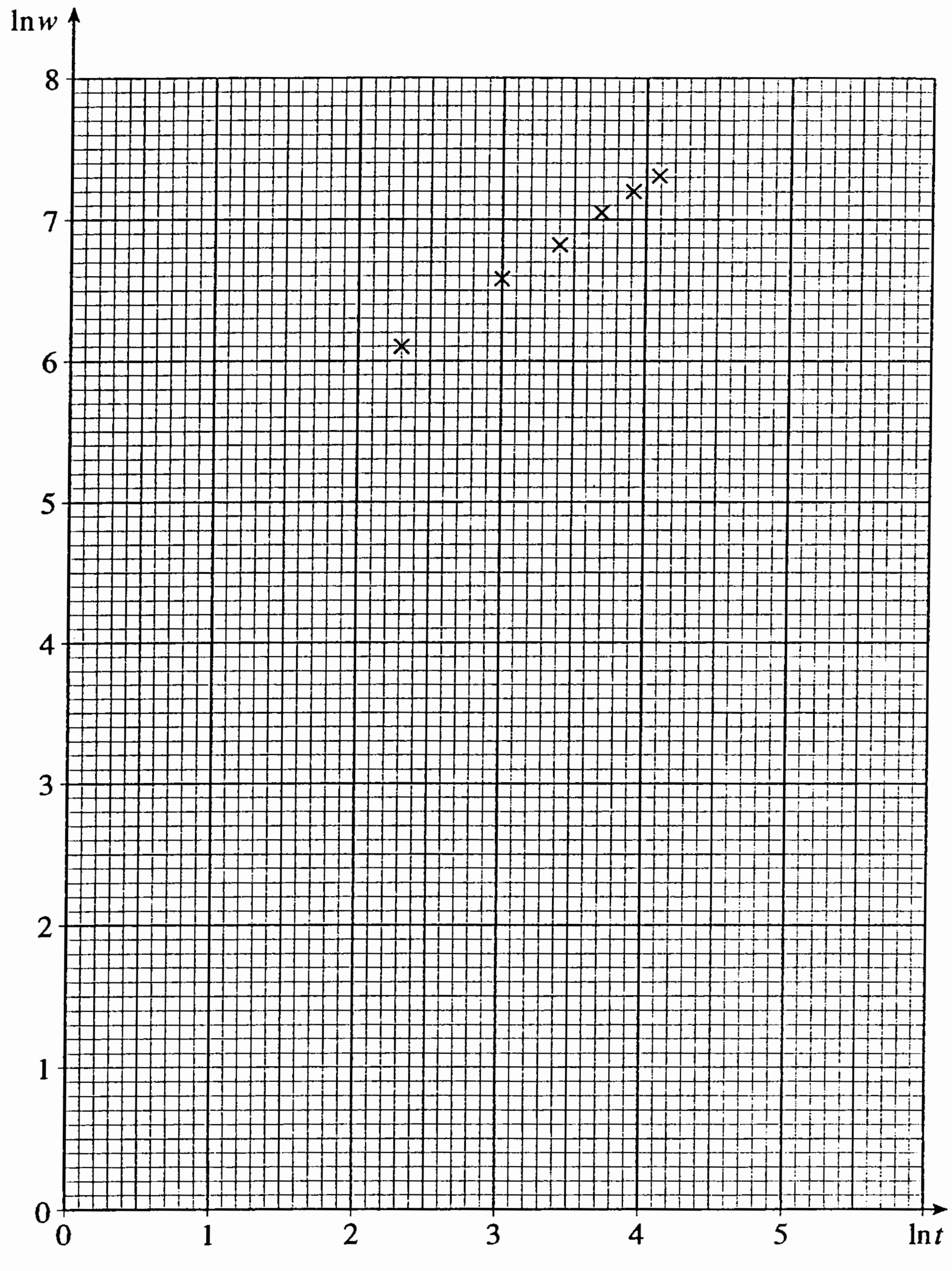

Fig. 4

(i) Show that a model of the form $w = at^b$ is appropriate for the data. [2]

(ii) Use the graph **on the insert** to estimate the values of a and b, giving your answers to 2 significant figures. [4]

(iii) (A) Find the mass predicted by the model when $t = 100$.

(B) Comment on the suitability of the model for predicting the mass of the cockroach immediately after hatching. [4]

(iv) The mass w' milligrams of a beetle is modelled by the equation $w' = 150t^{0.5}$, for $t \geqslant 0$.

By plotting the graph of $\ln w'$ against $\ln t$ **on the insert**, or otherwise, estimate the value of t when the mass of the beetle is the same as the mass of the cockroach. [5]

Candidate Name	Centre Number	Candidate Number

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Pure Mathematics 2

INSERT

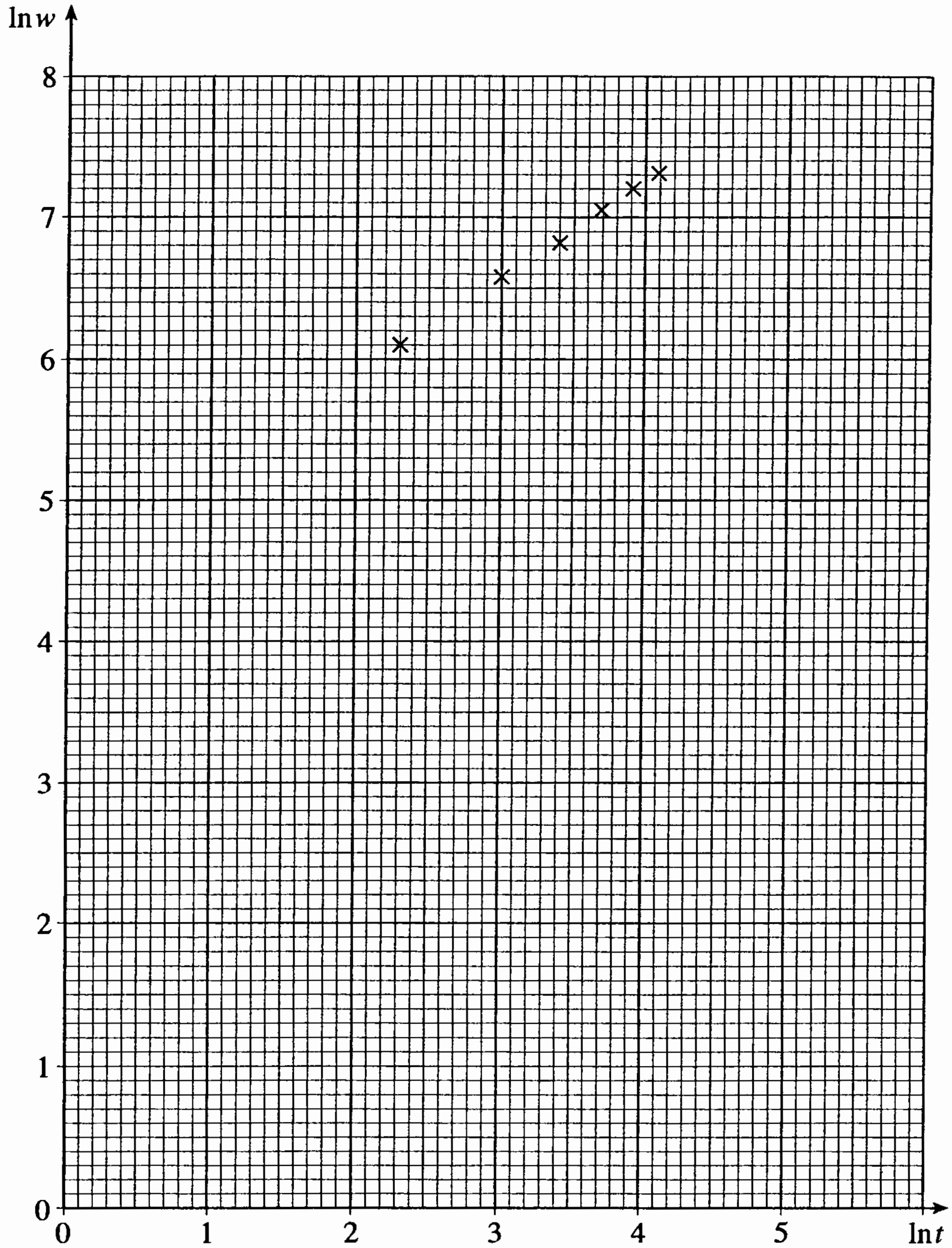
Monday **19 MAY 2003** Morning 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **4** parts **(ii)** and **(iv)**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Attach the insert securely to your answer booklet.

This insert consists of 2 printed pages.

4 (ii), (iv)



Mark Scheme

1(a) $a = 1$ $b = 3$ $c = 2$	B1 B1 B1 [3]	
(b) $y = \dfrac{\ln x}{1+\ln x}$ $\dfrac{dy}{dx} = \dfrac{(1+\ln x)\frac{1}{x} - \ln x.\frac{1}{x}}{(1+\ln x)^2}$ $= \dfrac{1}{x(1+\ln x)^2}$	M1 M1 A1 [3]	$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$ quotient rule or product rule (see below) consistent with their derivatives must cancel terms in numerator, but need not bring down the x
product rule: $\dfrac{dy}{dx} = \ln x(-1)(1+\ln x)^{-2}.\dfrac{1}{x} + \dfrac{1}{x}.(1+\ln x)^{-1}$ $= (1+\ln x)^{-2}.\dfrac{1}{x}(-\ln x + 1 + \ln x)$ $= \dfrac{1}{x(1+\ln x)^2}$	[3]	
(c) $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$ $\int (e^x + e^{-x})^2\, dx = \int (e^{2x} + 2 + e^{-2x} dx$ $= ½ e^{2x} + 2x - ½ e^{-2x} + c$	M1 A1 M1 A1 cao [4]	Correct expansion – condone e^{x^2} simplified – must have 2, allow $(e^x)^2$ and $(e^{-x})^2$ one or other of $\int e^{2x} dx = \dfrac{1}{2}e^{2x}$ or $\int e^{-2x} dx = -\dfrac{1}{2}e^{-2x}$ condone no c
(d) (i) $g(x + 2) = a^{x+2} = 3\, a^x$ $\Rightarrow a^x . a^2 = 3\, a^x$ $\Rightarrow a^2 = 3$ $\Rightarrow a = \sqrt{3}$	M1 E1 [2]	$a^{x+2} = 3\, a^x$ or equivalent equation with a particular value of x, e.g. $a^2 = 3$ or $a^3 = 3a$, etc… Allow verifications, but must be exact.
(ii) $(\sqrt{3})^b = 5$ $\Rightarrow \sqrt{3}^{\,b} = 5$ $\Rightarrow b \ln \sqrt{3} = \ln 5$ $\Rightarrow b = \ln 5 / \ln \sqrt{3}$ $\Rightarrow b = 2.93$	M1 M1 A1 [3]	 Taking lns and bringing the power down cao – must be 3 s.f.

2(i) (−1, 0)	B1 [1]	or $x = -1$, or P = −1. Not (0, -1) or $y = -1$.
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}.1$ $= \frac{1}{2}(1+x)^{-1/2}(x+2+2x)$ $= \frac{1}{2}(1+x)^{-1/2}(3x+2)$ $= \frac{3x+2}{2\sqrt{1+x}}$ *	M1 M1 E1 [3]	$x.\frac{1}{2}(1+x)^{-1/2}$ $+ (1+x)^{1/2}.1$
(iii) $\frac{dy}{dx} = 0$ when $3x+2=0$ $\Rightarrow x = -2/3$ or −0.67 or better $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}} = -\frac{2}{3\sqrt{3}} = -0.385$ Gradient is infinite or undefined at P	M1 B1 cao B1 B1 [4]	$3x + 2 = 0$ soi condone rounding errors, e.g. 0.666 Any correct expression for y. For numerical answers, -0.38 or better, but isw after correct surd expressions.
(iv) Let $u = 1 + x$, du = dx, when $x = -1$, $u = 0$; when $x = 0$, $u = 1$ $\Rightarrow \int_{-1}^{0} x\sqrt{1+x}\,dx = \int_0^1 (u-1)u^{1/2}\,du$ $= \int_0^1 (u^{3/2} - u^{1/2})\,du$ * $= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \frac{2}{5} - \frac{2}{3} = -\frac{4}{15}$ Area (or −area) between curve and x axis	M1 M1 E1 B1 M1 A1 cao B1 [7]	Changing limits – must show evidence. Substituting $(u-1)\,u^{1/2}$ for $x\sqrt{(1+x)}$ and du = dx or dx/du = 1 or du/dx = 1 www $\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}$ or equivalent substituting limits, but must have integrated (not differentiated) must mention x-axis or points O and P, or −1 and 0, ignore negatives.

3(i) (*A*) AP sum = (20/2)(100 + 19×2) = 1380	B1 M1 A1cao [3]	Correct expression Allow unsupported answers. Strictly 'could be arithmetic' or 'not possible to determine' is more correct. If no sum calculated thereafter, allow SCB3.
(*B*) GP sum = $\dfrac{1.6(1-0.8^{20})}{1-0.8}$ = 7.9	B1 M1 A1 cao [3]	 Condone 2 for 1.6 7.9 or better. Condone rounding errors.
(*C*) AP sum = (20/2)(10 + 19×2) = 480	B1 M1 A1cao [3]	 Correct expression. Condone $a = 3$ for M1 Allow unsupported answers.
(*D*) Neither	B1 [1]	
(*E*) GP sum = 0	B1 B1 [2]	
(*F*) Neither	B1 [1]	
(ii) (*B*) has a sum to infinity $S_\infty = \dfrac{1.6}{1-0.8}$ = 8	 M1 A1 [2]	 Condone $a = 2$ f.t. on a only from **(i)** (***B***), giving 10

4 (i) $w = a\,t^b$ $\Rightarrow \ln w = \ln a + b \ln t$ of form $y = c + m\,x$ so plotting $\ln w$ against $\ln t$ produces a straight line graph	M1 A1 [2]	or logs Comparing with $y = c + m\,x$. Need not explicitly say that $c = \ln a$ and $m = b$, but www
(ii) $\ln a = 4.5 \Rightarrow a = e^{4.5} = 90$ $b = \dfrac{y - step}{x - step}$ $= 0.70$	M1 A1 M1 A1 [4]	$\ln a$ = their intercept 81 to 99 but must be 2 s.f. any reasonable values from graph 0.65 to 0.75 By simultaneous equations, award M1 for each equation with correct coordinates substituted. Then A1 A1 for a and b, same range as above. Penalise s.f. once only
(iii) (A) $t = 100 \Rightarrow \ln t = 4.61$ $\Rightarrow \ln w = 4.5 + 0.7 \times 4.61$ $= 7.73$ $\Rightarrow w = 2270$ mg *or* $w = 90 \times 100^{0.70}$ $= 2260$ mg (B) $t = 0 \Rightarrow w = 0$ not appropriate	M1 M1 A1ft M2 A1ft B1 [4]	ln 100 = 4.61 soi Calculating their value for $\ln w$ or reading (correctly) off from graph ft only on values of a and b within ranges specified in (ii). ft on their 90, 0.70 ft on values of a and b as\before Reasonable comment on $w = 0$. Allow, for example, 'mass of egg is negligible, so model is appropriate'.
(iv) $w' = 150\,t^{0.5}$ $\Rightarrow \ln w' = 5.01 + 0.5 \ln t$ Graphs meet at $\ln t = 2.6$, $\Rightarrow t = 13$	M1 A1 A1 A1 A1ft	$\ln w' = 5.01 + 0.5 \ln t$ soi Graph crosses vertical axis at 5(.01) Correct gradient $2.3 \le \ln a \le 2.8$ dep on last A1 art range 10 to 17
or $150\,t^{0.5} = 90\,t^{0.70}$ $\Rightarrow 150 / 90 = t^{0.70-0.5}$ $\Rightarrow t = \left(\dfrac{150}{90}\right)^{\frac{1}{0.2}}$ $= 13$ *or* $\ln 150 + 0.5 \ln t = \ln 90 + 0.70 \ln t$ Þ $\ln 150 - \ln 90 = 0.20 \ln t$ Þ $t = 13$	M1ft M1 M1 A2 ft M1 M1 A2 ft [5]	Equating with their a and b collecting t's (no need to move the '90') solving for t as shown or taking lns correctly taking lns and bringing powers down collecting $\ln t$ s on one side art range 10 to 17

Examiner’s Report

2602 Pure Mathematics 2

General Comments

The paper attracted the full range of marks, from 0 to 60. There were some excellent centres, with the majority of scores in the 50s; even very weak candidates managed a score in double figures. There appeared to be ample time to complete all four questions – very few candidates ran out of time. Fragile algebra was, as usual, the major reason for lost marks – an inability to do the basics being evident from questions 1(b), 1(c) and 2(ii). However, the calculus questions were generally soundly done. The standard of presentation was, as ever, variable

In the report on the January paper, we commented on the necessity to attach insert sheets firmly to booklets. Sadly, in this examination, there were still too many instances of missing insert sheets, and some centres with all the insert sheets left unattached. While in marking such scripts we try to err on the side of generosity, the insert sheet for this examination included essential working – we would strongly urge centres to ensure that they are attached, and that their candidates marks are therefore not placed in jeopardy.

Comments on Individual Questions

Q.1 Various Topics

This question was the least well done on the paper.

(a) Most candidates scored 1 for a, but b and c were less frequently correct, with $c = ½$ a common error.

(b) Most candidates used the quotient rule correctly and knew the derivative of ln x, but even quite good candidates failed to gain the mark for simplifying the numerator.

(c) Expanding the brackets was very poorly done. Many missed out the inner and outer products. Any correct expression, even with the ambiguous e^{x^2}, gained M1. After that, candidates needed to simplify this to e^{2x}, and use $e^x.e^{-x} = 1$. The integration invariably falters unless they have done this simplification, but a substantial minority at this stage muddled their integration and differentiation of e^{kx} and integrated e^{2x} to $2e^{2x}$, etc.

(d) Function notation was often poorly understood, with $g(x + 2) = g(x) + 2$ a very common error in (i). Verifications were allowed, provided they were exact, and it is perfectly correct in this case to choose your value for x. Part (ii) was answered better, generally.

(a) $a = 1, b = 3, c = 2$; (b) $\dfrac{1}{x(1+\ln x)^2}$; (c) $½ e^{2x} + 2x - ½ e^{-2x} + c$; (d)(ii) $b = 2.93$.

Q.2 Calculus, curve sketching

This question was generally quite well answered.

Part (i) was correct in most instances, although some threw the mark away with (0, –1).

In part (ii), most used the product rule with the correct derivatives to get $x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}.1$, but few completed the algebraic simplification to arrive at the stated answer.

Part (iii) was successfully done by most sound candidates. Weaker ones set the denominator to zero as well, or set the numerator equal to the denominator. It is worth noting that ‘asymptote’ was not an acceptable answer for the gradient at P.

Part (iv) was a fairly simple substitution, with the answer given. For this reason, we insisted on correct usage of notation; in particular wanting to see some evidence of limits change and also du = dx or du/dx = 1. Weaker candidates then failed to integrate before substituting limits, or got the formula mixed up. The interpretation of area was generally well done, although 'area under the graph' was insufficient to gain the mark.

(i) $(-1, 0)$, (iii) $(-2/3, -2/3\sqrt{3})$, gradient infinite or undefined, (iv) $-\frac{4}{15}$.

Q.3 Sequences and series

This question was the highest scoring on the paper, and many candidates scored full marks. Weaker candidates made errors in their sum formulae.

Part (i) (*A*) was usually correct. Teachers may well prefer the answer 'we cannot tell' as there is no definition of the sequence; this was allowed full marks, but no bonus mark, I fear. Needless to say, I saw no scripts with this answer, but it would be a valuable teaching point nevertheless.

In part (ii) (*B*) the most common error was to take $a = 2$ instead of 1.6, and a similar error in (*C*) was to take $a =$ [illegible](*D*) and (*F*) was usually correctly identified as 'neither', but (*E*) foxed even good candidates, who saw it as oscillating but not geometric. Part (ii) was usually correct – we allowed full marks if they followed through with $a = 2$.

(i) (*A*) AP, 1380, (*B*) GP, 7.91, (*C*) AP, sum 480, (*D*) Neither, (*E*) GP, sum = 0, (*F*) Neither,
(ii) (*B*) has sum to infinity 8.

Q.4 Logarithms, reduction to linear form

This question gained quite a good response on the whole.

Part (i) was well done – candidates needed to compare with $y = mx + c$ to gain the second 'A' mark. In part (ii), some candidates got their lns muddled up, for example giving $a = 4.5$ or, for example, $b = \frac{\ln 7.2 - \ln 6.1}{\ln 7.2 - \ln 6.1}$. One mark was deducted for failure to round to 2 s.f. Candidates gained generous follow through in part (iii), and there were some erudite, but unrewarded, theses on the life cycle of the cockroach to explain the appropriateness of the model when $t = 0$.

Part (iv) was done in numerous ways. The graphical method of plotting $\ln w = 5.01 + .5 \ln t$ required quite careful plotting to achieve an answer in range, and there was some evidence of plotting errors at the intercept with the $\ln w$ axis (5.1 instead of 5.01). A common mistake was to plot t (instead of $\ln t$) against $\ln w$'. Algebraic methods yielded good answers provided they could handle the manipulation of the lns. Another aberrance worth mentioning was calculating logs to base 10 rather than e.

(ii) $a = 90$. $b = 0.70$, (iii) $w = 2260$ (ft), (iv) $t = 13$ (with generous follow through).