OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Monday **19 MAY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Section A (30 marks)

1 Find $\int 6x^3\,dx$. [2]

2 Find the remainder when $x^3 - 2x + 10$ is divided by $(x + 2)$. [2]

3 Use the binomial theorem to find the coefficient of x^3 in the expansion of $(1 + 3x)^8$. [3]

4 Solve the inequality $x^2 + 2x - 3 > 0$. [3]

5 A curve has gradient given by $\frac{dy}{dx} = 4x - 3$. The curve passes through $(1, 5)$. Find the equation of the curve. [4]

6 Sketch the graph of $y = \cos x$ for values of x from 0° to 360°.

Calculate, correct to 1 decimal place, the values of x in this range for which $\cos x = -0.6$. [4]

7 Find the x-coordinates of the points of intersection of the curves with equations $y = x^2 - 4x + 1$ and $y = 4x^2 + 5x - 7$. Give your answers correct to 2 decimal places. [4]

8 Use calculus to find the values of x at the turning points of the curve $y = x^3 - 6x^2 + 9x$. [4]

9

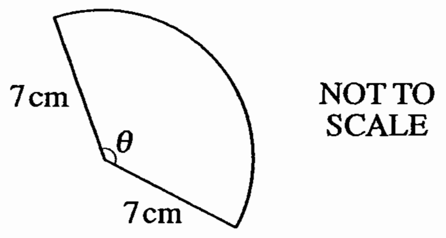

Fig. 9

Fig. 9 shows a sector of a circle with radius 7 cm. The sector angle, θ, is 1.4 radians.

Calculate the total perimeter of the sector.

Calculate also the area of the sector. [4]

Section B (30 marks)

10 A circle has centre $(3, -1)$ and radius $\sqrt{18}$.

(i) Write down the greatest and least values of the y-coordinates for points on the circle. [2]

(ii) Show that the origin lies inside the circle. [2]

(iii) Show that the line $y = x - 4$ is a diameter of the circle.

Show that the diameter which is perpendicular to $y = x - 4$ has equation $y + x = 2$. [4]

(iv) Show that the equation of the circle can be written as

$$x^2 + y^2 - 6x + 2y = 8.$$ [3]

(v) Find the coordinates of the two points of interseection of $y + x = 2$ with the circle.

(vi) Find the equation of one of the lines parallel to $y = x - 4$ which is a tangent to the circle. [5]

[Total 16]

Turn over for question 11.

11

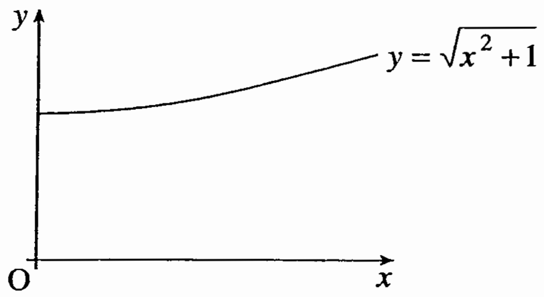

Fig. 11

Fig. 11 shows a sketch of part of the curve with equation $y = \sqrt{x^2 + 1}$.

(i) Copy and complete the table of values for the curve with equation $y = \sqrt{x^2 + 1}$. Give the missing y values to 5 decimal places.

x	0	0.25	0.5	0.75	1
y	1			1.25	1.41421

[2]

(ii) Use the trapezium rule with four strips to find an estimate of the area between the graph of $y = \sqrt{x^2 + 1}$ and the x-axis from $x = 0$ to $x = 1$. Give your answer to 4 decimal places.

State with a reason whether the trapezium rule estimate gives an upper or a lower bound in this instance. Use rectangles to find an opposite bound. [6]

(iii) The true value of the area between the graph of $y = \sqrt{x^2 + 1}$ and the x-axis from $x = 0$ to $x = 1$ is 1.1478 to 4 decimal places.

Find, to 2 significant figures, the relative error in the estimate you found using the trapezium rule in part **(ii)**. [2]

(iv) Find the volume generated when the region between the curve $y = \sqrt{x^2 + 1}$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 360° about the x-axis. [4]

[Total 14]

Mark Scheme

	Section A			
1	$\frac{3}{2}x^4 + c$	B1+B1	B1 for $6x^4/4$ i.s.w.	2
2.	f(−2)attempted or division as far as $x^2 - 2x$ 6	M1 A1	 B2 for 6 www	2
3.	1512	B3	B1 for $\begin{pmatrix}8\\3\end{pmatrix}$ or $\begin{pmatrix}8\\5\end{pmatrix}$ or 56 seen, and B1 for 3^3 or $(3x)^3$ seen, but not $3x^3$	3
4.	$x < -3$ or $x > 1$	B3	B2 for $x < -3$ or for identifying both −3 and 1 or M1 for $(x-1)(x+3)$ or B1 for $x > 1$ or just one of $x \geq 1$ or $x \leq -3$	3
5.	[$y =$] $2x^2 - 3x$ [$+ c$] subst of $y = 5, x = 1$ $c = 6$	B1+B1 M1 A1	or B4 for $y = 2x^2 - 3x + 6$ subst in their integral with $+ c$	4
6.	sketch of correct shape both axes scaled – at least 1 and 180 or 360 126.9 and 233.1	G1 G1 B1+B1	condone more than one period for G2 must be correct period B1 for both solns given with extras in range or B1 for other 126 to 127 <u>and</u> 233 to 234	4
7.	$4x^2 + 5x - 7 = x^2 - 4x + 1$ o.e. $3x^2 + 9x - 8 = 0$ $[x =]\dfrac{-9 \pm \sqrt{81+96}}{6}$ 0.72 and −3.72	M1 M1 M1 A1	valid method for eliminating y rearranging to = 0, condone one error on LHS attempt at quadratic formula, ft their quadratic, or completing square allow 0.717... and −3.717... rot to 2 or more sf	4
8.	$3x^2 - 12x + 9$ $dy/dx = 0$ seen or used $x = 3$ or 1	M1 M1 A1+A1	condone one error	4
9.	23.8 [cm] 34.3 [cm^2]	B2 B2	accept 23.77 to 23.8, M1 for 1.4 × 7 o.e. or 9.77 to 9.8 accept 34.2 to34.3, M1 for 0.5 × 7^2 × 1.4 o.e.	4
			Total section A	30

Section B					
10	(i)	$-1 \pm \sqrt{18}$ isw	B2	accept 3.24.. and –5.24.. rot to 2 sf or more, B1 for one correct	2
	(ii)	e.g. $3^2 + 1^2$ [=10] < 18 [so inside] or circle crosses y axis at (0, 2) and (0, –4) and (0,0) is between them	B2	B1 for partially complete clear method eg attempt at sketch of circle [may not be labelled (ii)] or $(0-3)^2 + (0+1)^2$ [= 10] B0 for finding x max and min and saying (0,0) between x and y max and min	2
	(iii)	subst of (3 , –1) into $y = x - 4$ 'passes thro' centre' <u>or</u> 'so is diameter' grad of normal = –1 $(y+1) = -1(x-3)$ or subst. of (3, –1) in $y = -x + c$ or checking (3, –1) fits $y + x = 2$	B1 B1 M1 M1	accept accurate graphs drawn explicit statement of at least one of these ft their gradient of normal. NB answer $y + x = 2$ given	4
	(iv)	$(x-3)^2 + (y+1)^2 = 18$ $x^2 - 6x + 9$ and $y^2 + 2y + 1$	B2 B1	B1 for LHS or RHS or quote of $(x-a)^2 + (y-b)^2 = r^2$ NB simplified answer given	3
	(v)	(0, 2) and (6, –4)	B3	B2 for one of these or for both x coords or both y coords correct. If B0, then M1 for start of geometric method e.g. $\sqrt{18}\cos 45°$ seen or for subst for x or y from $y + x = 2$ into eqn of circle.	3
	(vi)	$y = x + 2$ or $y = x - 10$ o.e.	B2	for one of these; M1 for eqn is $y = x + k$ o.e. [k may be numerical, except –4]	2
11.	(i)	1.03078, 1.11803	B2	B1 for one correct, or for both given but not to 5dp	2
	(ii)	1.1515 overestimate – curve below tops of traps [between 0 and 1] 1.0997, to 2 dp or more	B3 B1 B2	M2 for ½×0.25×[1+1.41421+ 2(1.03078+1.11803+1.25)] o.e. correct or ft their (i). M1 if one error; condone 1.1514(...) for B3 M1 for 1+1.03078+1.11803+1.25 or B1 for 1.2032(...) [the upper bound]	6
	(iii)	(trap ans – 1.1478)/1.1478 0.0032 or 0.32%	M1 A1	o.e., ft from (ii) NB B0	2
	(iv)	$\int_0^1 \pi(x^2+1)dx$ $[\pi]\left[\frac{x^3}{3} + x\right]$ 4π/3 or 4.1 – 4.2	M2 A1 A1	M1 for π omitted [but condone inserted in final answer] or other constant or for $\int \pi y^2 dx$; condone wrong / no limits allow A1 following M1	4
				Total Section B	30
				Total for paper	60

Examiner’s Report

2601 Pure Mathematics 1

General Comments

The full spread of marks was seen on this paper. Some candidates demonstrated excellent understanding and competence and many seemed to be well-prepared for the paper. Sadly, at the other end of the spectrum, there were again some very weak candidates who could attempt little successfully.

In section A, candidates often lost marks on questions 2, 3 and 4 but went on to earn good marks on the rest of the questions.

Time did not appear to be a problem, although occasionally candidates who used long methods in question 10 may have had been under time pressure during question 11.

Comments on individual questions

Q.1 Some weaker candidates confused differentiation and integration but nearly all gained at least one mark here. Omitting the constant of integration was more prevalent in some centres than others.

$\frac{3}{2}x^4 + c$.

Q.2 Many candidates attempted long division instead of using the remainder theorem, with weaker candidates often making errors in the division. Those who used the remainder theorem were usually successful in reaching the correct answer.

6.

Q.3 Most managed to find the binomial coefficient 56, but many spoilt their answer by using $3x^3$ instead of $(3x)^3$ or by wrongly working out the latter as $9x^3$.

1512.

Q.4 The factorising was usually correct, and candidates could often find the boundaries as –3 and 1, but handling of the inequalities was poor. The most successful were those who recognised a quadratic curve and used a sketch as well as the factors.

$x < -3$ or $x > 1$.

Q.5 Many candidates treated the curve as a straight line and attempted to use $y = mx + c$ with $m = 4x - 3$ or $m = 1$. Of those who realised that integration was required, most successfully went on to find the constant.

$y = 2x^2 - 3x + 6$.

Q.6 The sketch graphs varied tremendously in quality – many plotted points for $x = 0$, 90, 180 etc and some joined them in a V shape rather than the shape of a cosine curve. Some failed to indicate the scale on the y-axis by labelling 1. Adding 90 to the first answer was a common error in solving the equation.

sketch graph of cosine curve; 126.9 and 233.1.

Q.7 Many candidates coped well with this question, eliminating y successfully and solving the resulting quadratic equation. Weaker candidates, however, often made errors in rearranging the equation and in using the quadratic formula. Some wasted time by going on to find the y coordinates of the intersections as well as the required x coordinates.

0.72 and –3.72.

Q.8 Finding the x coordinates of the turning points was well done by the majority – each step was well known and usually correctly carried out.

1, 3.

Q.9 The response to this type of question showed a marked improvement in handling radians to that in some previous papers, with fewer thinking 1.4 radians is 1.4π radians, for example. A considerable number lost a mark for failing to add 14 to the arc length to find the perimeter.

23.8 cm; 34.3 cm^2.

Q.10 Most solutions to this question relied on algebra rather than geometry or graphical methods. Some candidates were unsure how to proceed in parts (i) to (iii) but recovered for parts (iv) to (v). Candidates did best on this question who thought clearly about what they were doing at each stage.

(i) Usually the greatest and least values of y were found correctly, but many candidates involved the equation of the circle unnecessarily at this stage. Some candidates did not appear to appreciate that the instruction 'Write down...' implies that it is possible to do just that.

(ii) Some found the greatest and least values of x and thought that this was sufficient, with part (i), to show that the origin is inside the circle. The most common valid method used was to show that the distance of the origin from the centre is $\sqrt{10}$, which is less than the radius so the origin is inside the circle. A few found the intersections of the circle with the axes and correctly deduced that the origin is inside since it lies between these points, although some candidates started this longer method and tailed off halfway. An accurate graph of the circle was also deemed sufficient, with partial credit given for a sketch graph.

(iii) Many candidates showed that $y = x - 4$ passes through the point $(3, -1)$ but stating that this meant the line passes through the centre and hence is a diameter was often omitted. Alternate methods included finding the intersections of the line with the circle and showing that the distance between them is double the radius or, rarely, showing that $(3, -1)$ was the midpoint of the line segment joining these intersections. Either of these two latter methods entailed a great deal more work for the candidate.

Many then showed correctly that the line passing through the centre perpendicular to $y = x - 4$ has the given equation. A good number of candidates started from the given equation and showed that $y = x - 4$ and $y + x = 2$ are perpendicular, but not all realised that they needed to show that the centre lies on $y = x + 2$.

(iv) Using the given centre and radius, most candidates wrote down $(x - 3)^2 + (y + 1)^2 = 18$ and then went on to expand the brackets correctly, even if they made errors in expanding brackets elsewhere in the paper such as part (v).

(v) Many used a correct method to find the intersection of the line and the circle, but mistakes in algebraic manipulation were common so that many failed to find $(6, -4)$ although $(0, 2)$ was more common.

(vi) Few candidates used the hint given in part (v), although many realised the equation was $y = x + c$ for some c, and received partial credit. Those who had a good picture of the circle in their minds, or even constructed on graph paper, were at an advantage here, as elsewhere in this question.

(i) $-1 \pm \sqrt{18}$ (v) $(0, 2)$ and $(6, -4)$ (vi) $y = x + 2$ or $y = x - 10$.

Q.11 Most candidates attempted all parts of this question. A few omitted the last part, but this appeared usually to be because of lack of knowledge of the topic rather than a time problem.

(i) Most candidates calculated correctly the missing values in the table. A minority lost a mark by not giving the answers to the 5 decimal places requested.

(ii) Many candidates used the trapezium rule or calculated separate trapezia successfully. However there were disappointingly still a considerable number of misquotes of the formula in spite of its being given in the formula booklet. Some candidates were confused as to whether the rule gave an upper or a lower bound in this case. The most convincing reasoning often involved a diagram showing the extra area lying above the curve. A considerable number of candidates omitted the calculation of an opposite bound. Of those who did calculate it, incorporating an extra rectangle was a common error.

(iii) Many did not understand the phrase 'relative error'. They calculated the absolute error and left this as their answer. A considerable number who used the correct method did not give their answer to 2 significant figures.

(iv) Candidates appeared to be more familiar with this topic than they have been in the past, and many good solutions were seen. Of those who could quote the correct formula, some had problems in squaring the square root to obtain y^2. Some had no idea how to proceed; others omitted π or lost it in the course of their calculation; others confused volume and area.

(i) 1.03078, 1.11803 (ii) 1.1515, overestimate – curve below tops of trapezia, 1.0997;
(iii) 0.0032 or 0.32% (iv) $4\pi/3$.