OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2613

Statistics 1

Friday **17 JANUARY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 A motoring magazine carried out a survey of the value of petrol-driven cars that were five years old. In the survey, the value of each car was expressed as a percentage of its value when new. The results of the survey are summarised in the following table.

Percentage of original value (x)	Number of cars
$15 \leqslant x < 20$	4
$20 \leqslant x < 25$	12
$25 \leqslant x < 30$	18
$30 \leqslant x < 35$	13
$35 \leqslant x < 40$	6
$40 \leqslant x < 45$	5
$45 \leqslant x < 55$	2

(i) Draw a histogram on graph paper to illustrate the data. [4]

(ii) Calculate an estimate of the median of the data. [2]

(iii) Calculate estimates of the mean and standard deviation of the data, giving your answers correct to 2 decimal places. Hence identify any outliers, explaining your method. [7]

A similar survey of 60 diesel-driven cars produced a mean of 34.2% and a standard deviation of 11.7%.

(iv) Use these statistics to compare the values of petrol and diesel cars five years after they were purchased as new. [2]

2 A practical music examination can be taken once or twice. Those candidates who fail it on the first occasion take it a second time.

For those having their first attempt, 25% pass with distinction and 45% gain an ordinary pass. For those taking the examination for a second time, the corresponding figures are 5% and 70% respectively. The tree diagram in Fig. 2 illustrates the situation.

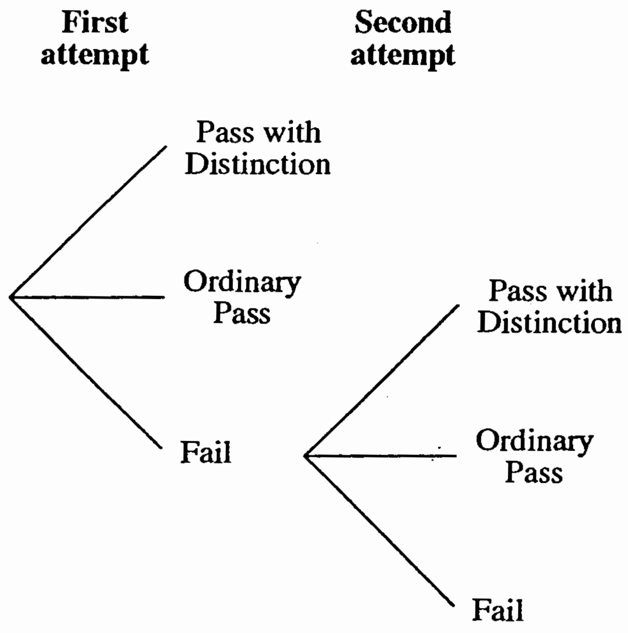


Fig. 2

(i) Find the probability that a randomly chosen candidate

(A) fails the examination,

(B) passes the examination (with or without distinction). [4]

(ii) Given that a randomly chosen candidate passes the examination, find the probability that he or she passes with distinction. [3]

(iii) Jill and Jo are two randomly chosen entrants for the examination. Find the probabilities that

(A) both pass (with or without distinction), but just one of them needs a second attempt,

(B) Jill gets a better result than Jo. [8]

[Turn over

3 In a certain region of the country there are 10000 blood donors. The percentages of donors in each of four different blood groups are as follows.

Group	O	A	B	AB
Percentage	45%	43%	9%	3%

(i) A stratified sample of size 400, in which each group is represented proportionately, is to be taken. Find the sample sizes in each of the strata, and state the sampling method that should be used within each stratum. [3]

(ii) Name another method of sampling in which each group is represented proportionately. Describe this method briefly. [2]

In a group of 36 donors, 16 are male and 20 are female. Four of these 36 people are chosen at random for an interview.

(iii) In how many ways can they be chosen? [2]

(iv) Find the probability that exactly two of the four people chosen are male. [3]

Two people are chosen at random from the population in the region. You may assume that the percentages of each blood group are the same as for donors.

(v) Find the probability that

(*A*) at least one has blood group O,

(*B*) both have the same blood group. [5]

4 During busy periods at a call centre, callers either get through to an operator immediately or are put on hold. A large survey revealed that 80% of callers were put on hold.

(i) Write down the probability that a caller gets through immediately. [1]

(ii) For a random sample of 25 callers, find the probability that

(*A*) exactly 6 callers get through immediately,

(*B*) at least 2 callers get through immediately. [6]

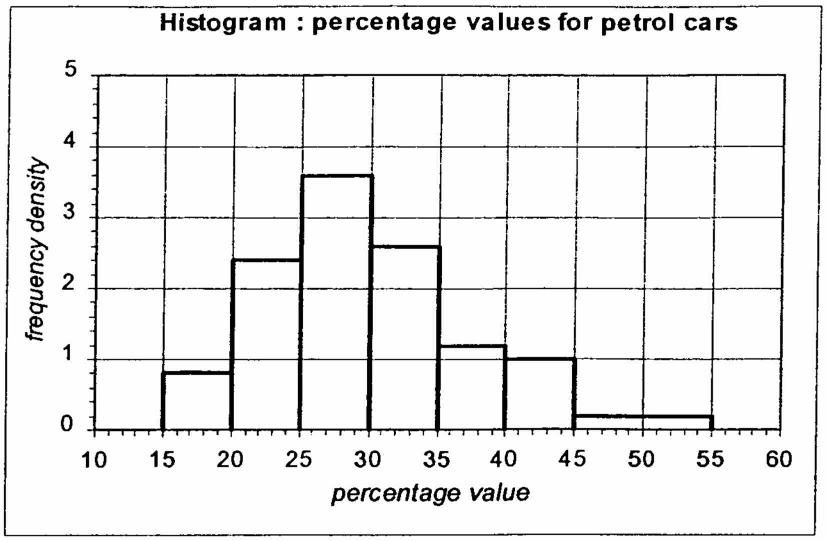
The call centre increases the number of operators with the intention of reducing the proportion of callers who are put on hold. After the change, exactly half of a random sample of 20 callers get through immediately.

(iii) Carry out a suitable hypothesis test to examine whether the centre has been successful. Use a 5% significance level and state your hypotheses and conclusions carefully. Determine the critical region for the test. [8]

Mark Scheme

January 2003 2613 MEI Statistics 1

Question 1

(i)		G1 for linear scaled axes G1 for frequency density *or* equivalent *or* key G1 for heights of first 6 bars (joined); all correctly positioned G1 for size of 7th bar	**4**
(ii)	Find median by simple interpolation $= 25 + \frac{14.5}{18} \times 5 = 29$ *or* $25 + \frac{14}{18} \times 5 = 28.9$	M1 for identifying interval containing median [SOI] A1 for precise value	**2**
(iii)	Mid-interval points: 17.5, 22.5, 27.5, 32.5, 37.5, 42.5, 50 Mean $= \frac{1795}{60} = 29.9$ (to 1 d.p.) *or* 29.92 (to 2 d.p.) s.d. $= \sqrt{\frac{57112.5}{60} - 29.92^2} = 7.53$ *or* 7.54 (to 2 d.p.) [*Allow n*–1 *divisor giving* s.d. = 7.60] Mean – 2 s.d. = 29.92 – 2 x 7.54 = 14.84 (to 2 d.p.) Mean + 2 s.d. = 29.92 + 2 x 7.54 = 45.00 (to 2 d.p.) Hence the 2 percentages in range 45 to 55 are outliers since they lie more than 2 s.d. from the mean	B1 for mid-interval points [SOI] B1 ft M1 for variance A1 ft their mean B1 for mean – 2 s.d. B1 for mean + 2 s.d. E1 for "2 outliers" *or equivalent*	**7**
(iv)	On average cars with diesel engines held their value better than cars with petrol engines Greater variation in the percentage values for cars with diesel engines compared to cars with petrol engines.	E1 E1	**2**
			15

Question 2

(i)	(*A*) P(failing examination) = $0.3 \times 0.25 = 0.075$ *or* $\frac{3}{10}\times\frac{1}{4}=\frac{3}{40}$ (*B*) P(passing, with or without distinction) $= 1 - 0.075 = 0.925$ [*or* $0.7 + 0.3 \times 0.75 = 0.925$]	M1 A1 M1 for "1 – their (*A*)" A1	**4**
(ii)	P(passing with distinction | passed the examination) $= \dfrac{\text{P(passing with distinction)}}{\text{P(passing the examination)}}$ $= \dfrac{0.25 + 0.3\times 0.05}{0.925} = \dfrac{0.265}{0.925} = 0.286$ (3 s.f.)	B1 for numerator M1 for fraction A1 **cao**	**3**
(iii)	(A) P(both pass, but just one needs a second attempt) $= 2\,(0.25 + 0.45) \times (0.3 \times 0.75) = 2 \times 0.7 \times 0.225$ $= 0.315$ (B) P(Jill gets a better result than Jo): *Method 1:* P(Jill passes with distinction *and* Jo does not *or* Jill passes without distinction *and* Jo fails) $= [0.25(0.45 + 0.21 + 0.075) + 0.015(0.45 + 0.21 + 0.075)] + [0.45\times 0.075 + 0.21\times 0.075]$ $= 0.265 \times 0.735 + 0.66 \times 0.075$ $= 0.194775 + 0.0495 = 0.244275 = 0.24$ (to 2 s.f.) *Method 2:* P(Jill passes with dist. *and* Jo passes without dist. *or* Jill passes with distinction *and* Jo fails *or* Jill passes without distinction *and* Jo fails) $= [0.25(0.45 + 0.21) + 0.015(0.45 + 0.21)] + [0.25\times 0.075 + 0.015\times 0.075] + [0.45\times 0.075 + 0.21\times 0.075]$ $= 0.265 \times 0.66 + 0.265 \times 0.075 + 0.66 \times 0.075$ $= 0.1749 + 0.019875 + 0.0495 = 0.24$ (to 2 s.f.) *Method 3:* [1 – P(both get same result)]/2 $= [1 - (0.265^2 + 0.66^2 + 0.075^2)]/2 = 0.24$ (to 2 s.f.)	B1 for "0.7" M1 for "… × 0.225" A1 **cao** M1 for attempt to enumerate 2, 3 or 8 cases A1 evaluating 1 case M1 for complete enumeration M1 for sum of 2, 3 or 8 products A1 **cao**	**8**
			15

Question 3

(i)	Divide the population into strata and choose samples proportionately: *Group* **O** **A** **B** **AB** [Pop. Size 4500 4300 900 300] Sample size 180 172 36 12 Within each stratum use random *or* systematic sampling.	M1 for finding sample sizes A1 E1 for sampling method	**3**
(ii)	Quota sampling. Search the population until you have found the required number (quota) for each sample. *Allow opportunity sampling and/or alternative description*	B1 for method E1 for description	**2**
(iii)	Number of ways 4 may be chosen from 36 $= \binom{36}{4} = 58905$	M1 for $\binom{36}{4}$ term A1	**2**
(iv)	P(two out of the four chosen are male) $= 6 \times \frac{20}{36} \times \frac{19}{35} \times \frac{16}{34} \times \frac{15}{33} = 0.39 \text{ (2 s.f.)} = 0.387 \text{ (3 s.f.)}$ *or* $= \frac{\binom{20}{2} \times \binom{16}{2}}{\binom{36}{4}} = \frac{190 \times 120}{58905} = \frac{1520}{3927} = 0.39 \text{ (to 2 s.f.)}$	M1 for "$20 \times 19 \times 16 \times 15$" M1 for "$36 \times 35 \times 34 \times 33$" A1 **cao** M1 for numerator M1 for quotient A1 **cao**	**3**
(v)	(*A*) P(at least one has blood group O) $= 1 - 0.55^2 = 0.698$ (to 3 s.f.) *or* 0.70 (to 2 s.f.) [*or* $0.45^2 + 2 \times 0.45 \times 0.55$] (*B*) P(both have same blood group) $= 0.45^2 + 0.43^2 + 0.09^2 + 0.03^2$ $= 0.396$ (to 3.s.f.) *or* 0.40 (to 2 s.f.)	M1 for calculation A1 M1 for at least 2 squares M1 for sum of 4 squares A1	**5**
			15

Question 4

(i)	$p = 1 - 0.8 = 0.2$	B1 for probability	1
(ii)	P(exactly 6 callers get through immediately) $= {}^{25}C_6 \times 0.2^6 \times 0.8^{19}$ $= 0.16$ (to 2 s.f.) $= 0.163$ (to 3 s.f.) P(at least 2 callers get through immediately) $= 1 - $P(0 or 1 callers get through immediately) $= 1 - [0.8^{25} + {}^{25}C_1 \times 0.2 \times 0.8^{24}]$ $= 1 - [0.00378 + 0.02361]$ $= 0.973$ (to 3 s.f.) *or* 0.97 (to 2 s.f.)	M1 for $0.2^6 \times 0.8^{19}$ M1 for ${}^{25}C_6 \times$ A1 **cao** M1 for ${}^{25}C_1 \times 0.2 \times 0.8^{24}$ M1 for $1 - [.... + ...]$ A1 **cao**	6
(iii)	*Either* H_0: $p = 0.2$ and H_1: $p > 0.2$ Consider $X \sim B(20, 0.2)$: $P(X \geq 10) = 1 - P(X < 10) = 1 - 0.9974 = 0.0026$ Since $0.0026 < 0.05$, do not accept H_0 There is enough evidence to accept the hypothesis that the proportion of callers getting through first time has increased. The critical region for the test is $\{8, 9, ..., 19, 20\}$. *or* H_0: $p = 0.8$ and H_1: $p < 0.8$ Consider $X \sim B(20, 0.8)$: $P(X \leq 10) = 0.0026$ Since $0.0026 < 0.05$, do not accept H_0 There is enough evidence to accept the hypothesis that the proportion of callers getting through first time has increased. The critical region for the test is $\{0, 1, ..., 11, 12\}$. *If critical region found first, all marks are possible, with probability marks implied. Explicit conclusion to test still required, using position in critical region.*	B1 for H_0, B1 for H_1 M1 for probability A1 M1 for comparison A1 for conclusion in words (dependent) B1 for 8 (at start of upper tail) B1 for $\{9, 10, .., 19, 20\}$ B1 for H_0, B1 for H_1 M1 for probability A1 M1 for comparison A1 for conclusion in words (dependant) B1 for 12 (at end of lower tail) B1 for $\{0, 1, .., 10, 11\}$	8
			15

Examiner’s Report

2613 Statistics 1

General Comments

Overall candidates generally found this paper more demanding than previous papers. There were fewer higher marks than usual, with results more densely grouped around the mean. However, the majority of candidates showed a fair degree of understanding of many of the topics.

In particular, the hypothesis test answers revealed an improvement in this examination. The work on sampling techniques still caused problems, as did the probability question, which was found difficult by many. The correct construction of histograms is misunderstood by many candidates.

Candidates in general do not understand that premature rounding causes loss of accuracy. They will happily round a result to 1 d.p. another to 2 d.p., another to 3 d.p. then add them up and give the answer to 3 d.p. – totally oblivious that they are doing something silly.

Increasing numbers of candidates now prefer to work with percentages rather than true probabilities but this causes great difficulty for them when faced with $(20\%)^{19}$ instead of 0.2^{19}. Centres should encourage the correct probability use of decimals or fractions.

Comments on Individual Questions

Question 1 (Data analysis; histogram, measures of central tendency and standard deviation; depreciation of second-hand cars)

A good starting question for most candidates. Most parts were well answered, but many failed to construct the histogram properly or find a reasonable estimate of the median. Candidates who scored close to full marks were few and far between. It was noticeable, for a small minority of students, that meeting the standard deviation for the first time at AS Level caused problems in its correct calculation.

(i) For the histogram, almost all drew linear scaled axes on graph paper. A few labelled the x-axis with $15 \leq x < 20$ etc, which is not recommended at this level. Many candidates labelled the vertical axis correctly as 'frequency density' but plotted frequencies with the final bar of height 2 instead of 1. A significant minority displaced the boundaries: 14.5 – 19.5; 19.5 – 24.5; etc. An extremely small number of all correct solutions were seen.

(ii) The calculation of the median was very poorly answered. Most left their answer as $25 \leq x < 30$ or the mid-point 27.5. Very few candidates attempted interpolation within the interval. Even worse, some thought the frequency of the interval (i.e. 18) was the median.

(iii) The mean calculation was attempted by most with success. Most obtained $\frac{1795}{60}$ or their equivalent with mid-point errors. The commonest errors were the mid–interval values. The standard deviation was less well answered. Often the examiners saw evidence of $\sum (fx)^2$ and $\sum f \times \sum x^2$ instead of $\sum fx^2$. A significant number (even otherwise strong candidates) did not seem perturbed by a huge standard deviation of 90+.

Outliers: most candidates used the ± 2 s.d. rule, but due to the above errors and/or premature rounding very few had the correct limits of 14.84 and 45.00. Those with sensible limits often did not know how to conclude, making statements like '$45 \leq x < 55$ is an outlier' or outliers = '$45 \leq x < 55$' and similar. It is important that candidates specify how many outliers there are; not just their location. Some candidates looked only above the mean and so failed to get full marks, even with a correct conclusion.

(iv) Most candidates understood that petrol cars had depreciated more than diesels and were able to write comments to that effect. The larger s.d. for diesels led many to say that they must have a larger range, which is not necessarily true. A significant few were not specific about which type of car held its value best or failed to comment in context at all.

(i) histogram; (ii) median = 28.9 or 29.0 (to 3 s.f.); (iii) mean = 29.92, s.d. = 7.54 (to 2 d.p.); outliers: 2 values in interval $45 \le x < 55$; (iv) comments

Question 2 (Probability; combining probabilities by multiplication and addition; Jill and Jo taking one or two attempts to pass a music examination)

A large range of marks was seen. Overall candidates found this question very challenging. Weaker candidates failed to appreciate that overall passing or failing was required early on. The final part was found difficult by even the strongest candidates, with hardly anyone managing the finish satisfactorily.

(i) Part (A): Many candidates failed from the start, being unable to answer part (i) correctly. They either considered 1st attempt only p (fail) =0.3 *or* considered the 1st and 2nd attempts as mutually exclusive with P(fail) = 0.3 + 0.25 or even 0.3 + 0.075. Some stated that they did not understand if the question meant 'failed once' or 'failed twice' and gave alternative answers.

Part (B): Many did not understand that this was "1 – answer (A)" and worked it out another way. It was not unusual to see the correct answer here after getting part (A) wrong. The most common mistake seen was 0.7 from 0.25 + 0.45.

(ii) The majority of candidates did not recognise or attempt a conditional probability at all. Many simply worked out the 'pass with distinction' and left it at that. Of those who did realise that a conditional probability was involved, too many then offered a product of probabilities in the numerator, one of which tended to cancel with the probability in the denominator.

(iii) Part (A): Quite poorly answered with 0.7 + 0.225 instead of 0.7×0.225 being the most common error. Omission of the factor of 2 was also prevalent, even by strong candidates.

(iv) Very poorly answered. The vast majority scored two marks here for a sensible set up and showing evidence of at least one correct calculation. Very few got all 8 cases in the $\sum 8$ method, listing only 3 or 4 possibilities. The error by stronger candidates using this method was to have 7 correct cases and omit "$0.3 \times 0.05 \times 0.45$". The more able candidates using the $\sum 2$ or $\sum 3$ methods fared better. Very few candidates got the final correct answer.

(i) (*A*) 0.075, (*B*) 0.925; (ii) 0.286; (iii) (*A*) 0.315, (*B*) 0.244 (to 3 s.f.)

Question 3 (Sampling and probability; blood donors)

This was once again a poorly answered question. Many candidates managed to calculate the number in each strata but seemed unable to distinguish between sampling methods for the next part. The probability parts met mixed responses, with most candidates confusing sampling *with* and sampling *without* replacement.

(i) This was usually well answered with sample sizes nearly always correct. Mostly, if a sampling method was stated, it was either random or systematic. Those who opted for systematic often said 'take every 10th on the list'; not realising that it should be every 25 in this case.

(ii) Very few stated 'Quota sampling', most opting for Cluster sampling. Fewer still could describe quota sampling. Curiously a significant minority stated 'stratified'. This was the method of part (i) and the question clearly asks for '... another method'.

(iii) Most candidates obtained 58905. ${}^{36}P_4$ was seen only rarely. Some very small numbers, namely 5,12,16 also appeared, showing little understanding of what was being asked.

(iv) There were surprisingly few correct responses here. The ${}^{n}C_r$ method, when used, usually gave the correct answer, but a few used ${}^{20}C_2 + {}^{16}C_2$ rather than ${}^{20}C_2 \times {}^{16}C_2$. The probability method was most often wrong: either by not 'counting down' or using the wrong ${}^{n}C_r$ term. ${}^{4}C_2$ was often replaced by ${}^{36}C_{16}$, ${}^{20}C_{16}$ or similar. Most candidates chose to sample with replacement and gave a binomial answer.

(v) Part (A) proved more difficult than part (B), where most understood that the sum of 4 squares was required. In part (A) many did not think to say P(not O) = 0.55 and instead considered separate cases: P(OO) + P(OA) + P(OB) + P(OAB), 'forgetting' to multiply by 2 in the last 3 cases.

(i) sample sizes: 180, 172, 36, 12; (ii) quota sampling and description; (iii) 58905;
(iv) 0.385 (to 3 s.f.); (v) (*A*) 0.698 (to 3 s.f.), (*B*) 0.396 (to 3 s.f.).

Question 4 (Binomial distribution and hypothesis testing; call centre, being put on hold or getting through first time)

A full spectrum of marks was seen. This remains a challenging topic for some candidates, where many misconceptions abound.

(i) The vast majority of candidates deduced the probability correctly.

(ii) Part (*A*): This was well answered by the majority of candidates. Reversing powers i.e. $0.2^{19}0.8^{6}$ or omitting ${}^{25}C_6$ were the most common errors.

Part (B): In attempting to find P(at least 2), the most common errors were $1 - P(X = 1)$ or $1 - P(X \leq 2)$. Those with the correct method often got 0.972 instead of 0.973 due to rounding errors.

(iii) The hypothesis test was well answered by many, with about equal numbers using the null hypothesis "$p = 0.2$" and "$p = 0.8$". However, the following faults are still too prevalent:

- Surprisingly, hypotheses are often poorly stated.
- Tail probabilities are not explicitly compared with the significance level.
- Point probabilities are used instead of tail probabilities.
- Conclusions are not given in context.
- Critical regions are not clearly identified and are often given as a single value.
- The given observation, 10 in this case, is not explicitly compared with the critical region.

Many candidates lose a significant number of marks through poor or non-existent explanation as detailed above.

(i) 0.2; (ii) 0.163, 0.973 (to 3 s.f.);
(iii) *Either* H_0: $p = 0.2$, H_1: $p > 0.2$; $P(X \geq 10) = 0.0026 < 0.05$, hence do not accept H_0; critical region = {8, 9, ..., 19, 20}.
Or H_0: $p = 0.8$, H_1: $p < 0.8$; $P(X \leq 10) = 0.0026 < 0.05$, hence do not accept H_0; critical region = {0, 1, ..., 11, 12}.