OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2602/1

Pure Mathematics 2

Monday **13 JANUARY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.
- An **INSERT** is provided for Question **2** part **(ii)**.

This question paper consists of 4 printed pages and an insert.

1 **(a)** An arithmetic progression has first term 4 and common difference 3. Find the 50th term, and the sum of the first 50 terms. [4]

(b) Differentiate $\ln(1 + x^2)$. [3]

(c) Evaluate $\int_0^{\frac{1}{3}} e^{-3x}\,dx$, leaving your answer in terms of e. [3]

(d) Given that $f(x) = x^{\frac{1}{3}}$, find the value of x for which $f'(x) = f(x)$. [4]

[Total 14]

2 **[Use the insert provided to answer part (ii) of this question.]**

(i) The number of new cases of infection from a virus in a week is modelled by a geometric progression with common ratio 1.25. The number in week 1 is 32.

(*A*) Find the number of new cases predicted by the model in each of week 2, week 3 and week 10. [3]

(*B*) Find an expression in terms of n for the *total* number of cases in the first n weeks, simplifying your answer. After how many weeks would the total number of cases first exceed 5000? [4]

(ii) After 10 weeks, a vaccination programme is introduced to control the spread of the virus. Thereafter, the actual numbers of reported cases are as follows.

Week number, x	11	12	13	14	15
Number of new cases, y	240	150	95	58	38

For $x \geqslant 11$, the number of new cases y in week x is to be modelled by an equation of the form $y = pq^x$, where p and q are constants.

Plot a graph of $\ln y$ against x **on the insert** provided, and explain why the graph confirms that the model is appropriate.

Use the graph to predict the week in which the number of new cases falls below 20 for the first time. (You are **not** required to estimate values for p and q.) [8]

[Total 15]

3 This question is about the graph of $y = \sqrt{2x - x^2}$, so that $y \geqslant 0$.

(i) Find the coordinates of the points where the graph meets the x-axis. [2]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1-x}{\sqrt{2x - x^2}}$. [2]

(iii) Hence find the coordinates of the turning point of the graph. What can you say about the gradient at $x = 0$ and $x = 2$? [4]

(iv) Use the above results to sketch the graph of $y = \sqrt{2x - x^2}$. You may assume that the turning point is a maximum point. [2]

(v) The region enclosed by the graph and the x-axis is rotated through 360° about the x-axis. Find by integration the volume of the solid of revolution produced, leaving your answer as a multiple of π. [3]

(vi) Starting from $y = \sqrt{2x - x^2}$, show that

$$(x-1)^2 + y^2 = 1.$$

What can you deduce about the shape of the graph? [3]

[Total 16]

4 Fig. 4 shows the graph of $f(x) = \dfrac{x}{x-1}$.

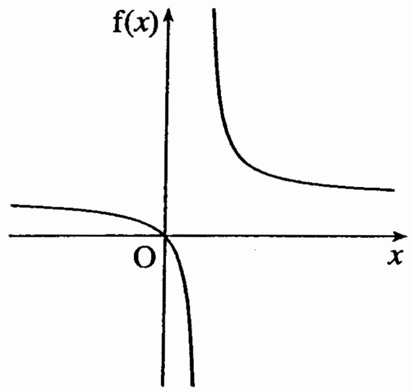

Fig. 4

(i) State the value of x for which the function $f(x)$ is undefined. [1]

(ii) Find the derivative $f'(x)$, simplifying your answer. How can you deduce that the gradient of the graph is always negative? [4]

The area enclosed by the graph, the x-axis and the lines $x = 2$ and $x = 3$ is denoted by A.

(iii) Using the substitution $u = x - 1$, show that

$$A = \int_1^2 \left(1 + \frac{1}{u}\right) du.$$

Find the exact value of A. [6]

(iv) Show algebraically that $f^{-1}(x) = f(x)$. What feature of the graph illustrates this result? [4]

[Total 15]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS **2602/1**

Pure Mathematics 2

INSERT

Monday **13 JANUARY 2003** Afternoon 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

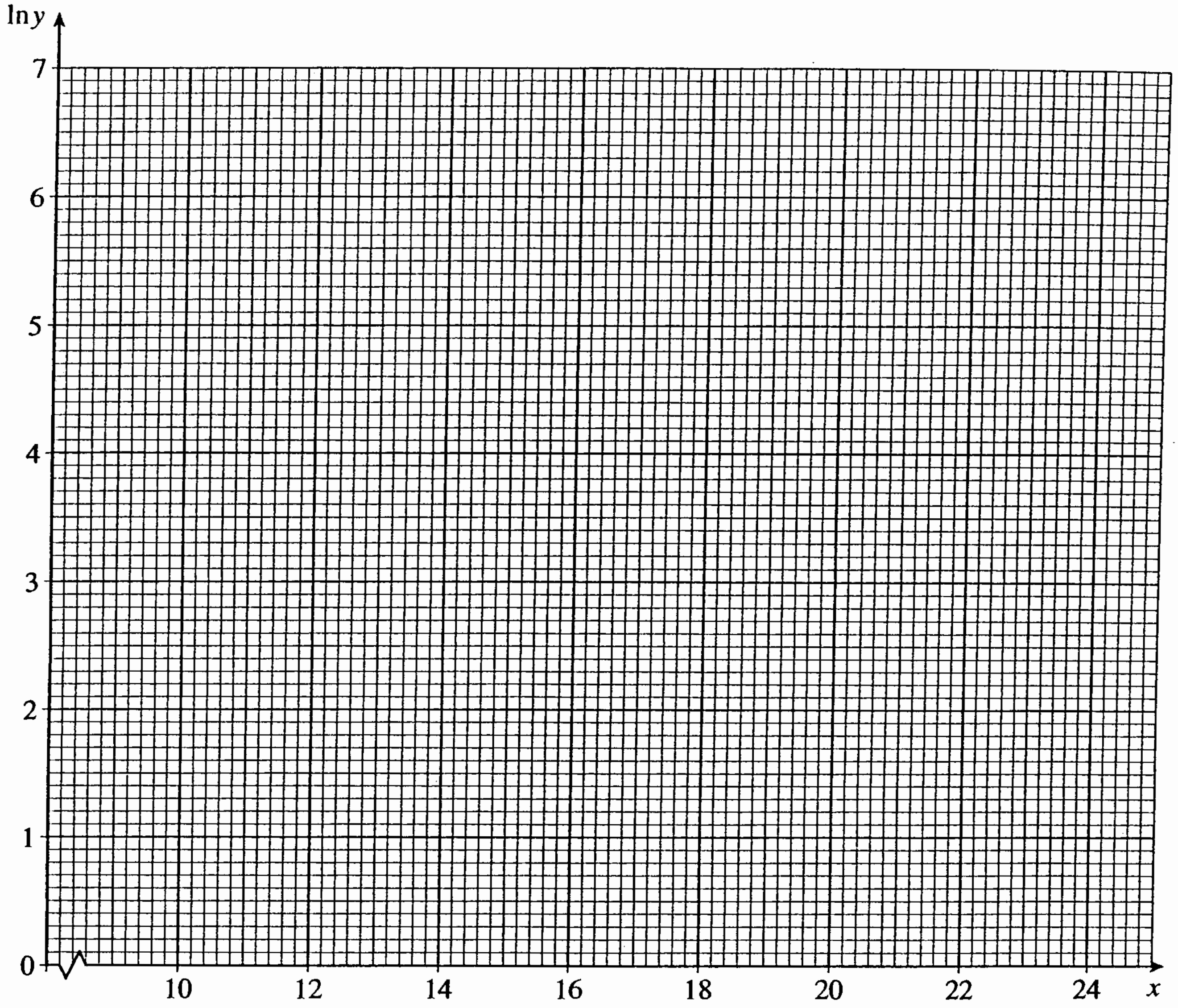
- This insert should be used in Question **2 (ii)**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Attach the insert securely to your answer booklet.

This insert consists of 2 printed pages.

HN/3

[Turn over

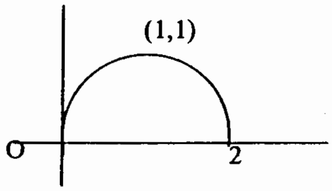
2 (ii)



Mark Scheme

1(a) 50^{th} term $= 4 + 49 \times 3$ $= 151$ $S_{50} = \frac{50}{2}(8 + 49 \times 3)$ $= 3875$	M1 A1 cao M1 A1 cao [4]	Use of $a + (n-1)d$ with $a = 4$, $n = 50$ and $d = 3$ $= 151$ For correct expression for S_{50} $= 3875$
(b) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\Rightarrow dy/dx = dy/du \times du/dx$ $= 2x/u$ $= \frac{2x}{1+x^2}$	M1 B1 A1 [3]	For using chain rule For $\frac{d}{dx}(\ln u) = \frac{1}{u}$ soi o.e.
(c) $\int_0^{\frac{1}{3}} e^{-3x} dx = \left[-\frac{1}{3}e^{-3x}\right]_0^{1/3}$ $= -\frac{1}{3}e^{-1} + \frac{1}{3}e^0$ $= \frac{1}{3}(1 - \frac{1}{e})$	B1 M1 A1cao [3]	$\left[-\frac{1}{3}e^{-3x}\right]$ for substituting correct limits into their integrand (correctly) For any equivalent expression, but must have evaluated $e^0 = 1$
(d) $f'(x) = \frac{1}{3}x^{-2/3}$ $\Rightarrow \frac{1}{3}x^{-2/3} = x^{1/3}$ $\Rightarrow \frac{1}{3} = x^{1/3} \times x^{2/3} = x$ $\Rightarrow x = 1/3$	B1 M1 M1 A1 cao [4] Total [14]	For correct derivative for forming equation with their derivative for $x^{1/3} \times x^{2/3} = x^1$

2(i) (*A*) $32 \times 1.25 = 40$ $40 \times 1.25 = 50$ $S_{10} = 32 \times 1.25^9$ $= 238(.4..)$	B1 cao M1 A1 cao [3]	For both 40, 50 For using $a \times r^{n-1}$
(*B*) $S_n = \frac{32(1.25^n - 1)}{1.25 - 1}$ $= 128(1.25^n - 1)$ $\Rightarrow 128(1.25^n - 1) > 5000$ $\Rightarrow 1.25^n > 40.0625$ $\Rightarrow n \ln 1.25 > \ln 40.0625$ Þ $n > 3.69 / 0.223 = 16.53\ldots$ *or* $n \log 1.25 > \log 40.0625$ $\Rightarrow n > 1.60/0.0969 = 16.53\ldots$ $\Rightarrow n = 17$ weeks	M1 A1cao M1 A1 cao [4]	For any correct expression using logs www *or* using trial and improvement www www
(ii) $y = p\,q^x$ $\Rightarrow \ln y = \ln p + x \ln q$ of form $y = c + x\ m$ x ln y 11 5.48 12 5.01 13 4.55 14 4.06 15 3.64 $\ln 20 = 3.00$ $x = 17$ weeks	M1 A1 M1 B2, 1, 0 M1 M1 A1 cao [8] Total 15	for taking lns or logs for comparing with $y = m\,x + c$. for calculating $\ln y$ values for plotting points – allow ± 0.5 mm for $\ln y$ values for calculating ln 20 or ln 19 = 2.9 soi for seeing when graph falls below their ln 20

3 (i) $\sqrt{2x-x^2}=0$ $\Rightarrow 2x-x^2=x(2-x)=0$ $\Rightarrow x=0, 2$ so (0, 0) and (2, 0)	B1 B1 [2]	$x=0$ or (0,0) $x=2$ or (2, 0)
(ii) $y=(2x-x^2)^{1/2}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{2}(2x-x^2)^{-1/2}(2-2x)$ $=\frac{1-x}{\sqrt{2x-x^2}}$ *	M1 E1 [2]	For a correct expression deriving result www
(iii) $\frac{dy}{dx}=0$ when $1-x=0$ $\Rightarrow x=1$ $\Rightarrow y=1$ At $x=0$ and 2, gradient is infinite or undefined	M1 A1 B1 B1 [4]	$1-x=0$ soi $x=1$ $y=1$ 'infinite', 'undefined', 'no gradient'
(iv) 	M1 A1 [2]	Through (0,0), (2,0) and max at (1,1) Gradient infinite at (0,0) and (2,0) and no y values outside (0,2) plotted
(v) $V=\int_0^2 \pi y^2 dx$ $=\pi\int_0^2(2x-x^2)dx$ $=\pi\left[x^2-\frac{1}{3}x^3\right]_0^2$ $=\pi(4-\frac{8}{3})$ $=\frac{4}{3}\pi$	M1 B1 A1 cao [3]	Correct integral and limits $x^2-\frac{1}{3}x^3$
(vi) $y^2=2x-x^2$ $\Rightarrow (x-1)^2+y^2=x^2-2x+1+2x-x^2$ $=1$* Semi-circle	M1 E1 B1 [3] Total 16	squaring verifying

4 (i) $x = 1$	B1 [1]	
(ii) $f'(x) = \frac{(x-1).1 - x.1}{(x-1)^2}$ $= -\frac{1}{(x-1)^2}$. $(x-1)^2 > 0 \Rightarrow f'(x) < 0$ for all x. $\Rightarrow$ gradient is always negative	M1 M1 A1 cao A1 [4]	Correct denominator Correct numerator 'the square is always positive'
(iii) $A = \int_2^3 \frac{x}{x-1} dx$, Let $u = x-1, \Rightarrow du = dx$ When $x = 2$, $u = 1$; when $x = 3$, $u = 2$ $\Rightarrow A = \int_1^2 \frac{u+1}{u} du$ $= \int_1^2 (1 + \frac{1}{u}) du$ * $= [u + \ln u]_1^2$ $= 2 + \ln 2 - 1 - \ln 1$ $= 1 + \ln 2$	M1 E1 E1 M1 M1 A1 cao [6]	Correct integral and limits changing limits correct derivation of transformed integral $u + \ln u$ substituting limits into correct integrand www
(iv) $y = \frac{x}{x-1}$ $x \leftrightarrow y$ $x = \frac{y}{y-1}$ $\Rightarrow xy - x = y$ $\Rightarrow xy - y = x$ $\Rightarrow y(x-1) = x$ $\Rightarrow y = \frac{x}{x-1}$ so $f^{-1}(x) = f(x)$ Symmetrical about $y = x$.	M1 M1 A1 cao B1 [4]	Attempt to reverse formula collecting terms *or* Expression for f f(x) M1 Clearing subsidiary denominators M1 Simplifying to x A1

Examiner’s Report

2602 Pure Mathematics

General Comments

The paper proved to be accessible to all but the weakest candidates. Very few failed to score at least 15 marks, and there were a pleasing number of scores over 50. Nearly all candidates answered all four questions, and there was little evidence of candidates running out of time. Some scripts lost marks through insufficient working being shown – examples being questions 2(ii) and question 4(iii) (see below). Our advice is to stress to candidates that it is safer to include more steps rather than less, especially when they are 'showing' a result given on the paper. The major area of weakness of candidates remains algebraic fragility, as displayed in, for example, 1(d), 2(i)(*B*), 3(vi) and 4(iv).

On an administrative point, it is important that centres ensure that insert sheets are firmly attached to answer booklets and correctly filled in – if they are left loose, there is an obvious danger that they will get detached from the answer booklet.

Comments on Individual Questions

Question 1 (various topics)

(a) This was a straightforward starter which was well answered. Some candidates confused their term and sum formulae.

(b) Generally well answered, either using a 'u' or directly. A product rule using u = ln and $v = 1 + x^2$ earned no marks!

(c) This was less successfully done, with a lot of errors in integrating e^{-3x} (e.g. $-3e^{-3x}$) and in substituting the limits correctly. Evaluating $e^0 = 1$ in their final answer was required for the final A mark.

(d) Most candidates did the derivative correctly, but quite a lot got in a muddle solving the equation, either by combining the indices or by logarithms.

Answers: (a) 151, 3875 , (b) $\frac{2x}{1+x^2}$, (c) $\frac{1}{3}(1-\frac{1}{e})$, (d) $x = 1/3$.

Question 2 (sequences and series, reduction to linear form)

This question was generally well answered, especially part (ii).

In part (i), all but the weakest candidates succeeded with part (i), with 32×1.25^{10} the most common error in (*A*). In part (*B*), there were plenty of good solutions – solving the equation using logarithms was well understood – but some good candidates lost a mark for not simplifying the expression to $128(1.25^n - 1)$. Poor algebra also cost marks here through errors in simplifying the equation before solving. Trial and improvement methods were allowed provided candidates explicitly tested $n = 16$ and $n = 17$ – unsupported correct answers gained no marks. Again, it is important that candidates write down their working, even if they are doing it all correctly on their calculators.

Part (ii) was on the whole very well done. The explanation of why the graph confirms the model had to include a statement that $y = p\,q^x \Rightarrow \ln y = \ln p + x \ln q$, and compare this correctly to $y = m\,x + c$. The graph was usually accurately done, but many candidates lost the final mark with $n = 16$ instead of 17. More seriously, there had to be some supporting working shown, either on the graph or on paper, to indicate their method. A clearly visible 'dot' where $\ln y = 3$ on the straight line was enough, but drawing horizontal and vertical lines was to be preferred.

Answers: (i)(*A*) 40, 50, 238 , (*B*) $n = 17$, (ii) 17 weeks.

Question 3 (calculus, curve sketching)

Although a little less successfully done than questions 1 and 2, there were still plenty of accessible marks here. In part (i), some candidates made a bit of a meal of solving the quadratic by formula, and quite a few missed the $x = 0$ solution; also (0,2) gained no marks her. Finding the maximum point in part (ii) was generally well done, but interpreting the gradient at $x = 0$ and $x = 2$ was often incorrect, with 'asymptotic' a common error.

The sketch in part (iv) usually went through (0, 0), (1, 1) and (2, 0), but often exceeded the domain 0 to 2, and few solutions showed the infinite gradients at 0 and 2 correctly. It is worth noting that sketches without an indication of the scale of both axes scored zero.

Part (v) was poorly done – candidates were not prepared for a volume of revolution question on a P2 paper. Part (vi) revealed some weaknesses in algebra; however, most candidates recognised this as a circle, and were granted a mark, even though the correct answer was, of a course, a semi-circle.

Answers: (i) (0, 0) and (2, 0) , (iii) (1, 1), gradients infinite or undefined , (v) $\frac{4}{3}\pi$, (vi) semicircle.

Question 4 (calculus, transformations of graphs)

This proved to be the hardest question of the four, albeit not by much.

Part (i) was almost universally correct. The quotient rule in part (ii) was well known and usually correct, although the terms of the numerator were sometimes the wrong way round. The product rule correctly applied was accepted, but this proved harder to simplify correctly. To gain the final mark, candidates had to mention explicitly that the square factor in the denominator is always positive.

Part (iii) was disappointingly answered. To gain the first three marks, candidates had to show or imply the correct definite integral for the area, correctly substitute for x *and* dx in terms of u and du and transform it algebraically, and show the change of limits *with some explicit working*. The integral was poorly done generally, some candidates forgetting to do any integration at all. Many good candidates also left the answer as approximate.

The final part was rarely done correctly, the algebra proving beyond the majority of candidates. There was also some confusion between $f^{-1}(x)$, $1/f(x)$ and $-f(x)$. However, identifying the symmetry in $y = x$ was more commonly done correctly.

Answers: (i) $x = 1$, (ii) $-\frac{1}{(x-1)^2}$, (iii) $1 + \ln 2$.