OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2608/1**

Mechanics 2

Wednesday **15 JANUARY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8$ m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 **In this question take $g = 10\,\mathrm{m\,s^{-2}}$** and neglect the effect of air resistance.

A smooth ball of mass 0.1 kg is projected from ground level over smooth horizontal ground with an initial speed of 65 m s^{-1} at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of restitution between the ball and the ground is 0.4.

(i) Show that the ball leaves the ground after the first bounce with speeds of 52 m s^{-1} in the horizontal direction and 15.6 m s^{-1} in the vertical direction. Explain your reasoning carefully. [5]

(ii) Calculate the impulse on the ball at its first bounce. [3]

The ball is in the air for T_1 seconds between projection and bouncing the first time, T_2 seconds between the first and the second bounces and T_n seconds between the $(n-1)$th and the nth bounces.

(iii) Show that $T_1 = \frac{39}{5}$ and $T_n = \frac{39}{5} \times (0.4)^{n-1}$. [3]

(iv) Calculate the total time after projection until the ball stops bouncing.

Calculate also the total horizontal distance travelled by the ball in this time.

According to the assumptions in this question, what happens after the ball stops bouncing? Give a brief reason for your answer. [4]

[Total 15]

2

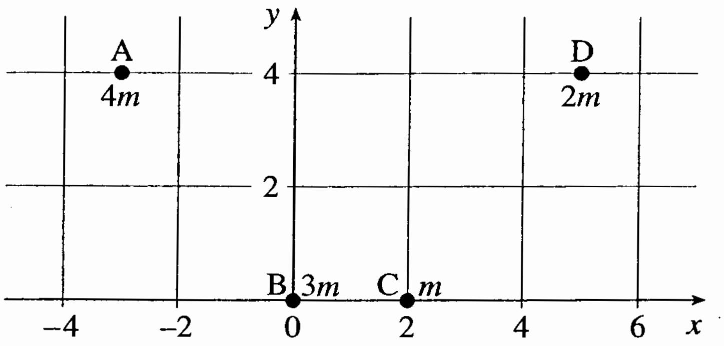

Fig. 2.1

In this question, coordinates are referred to the axes shown in Fig. 2.1.

Four particles at A, B, C and D with masses $4m$, $3m$, m and $2m$ lie in a plane with positions (–3, 4), (0, 0), (2, 0) and (5, 4), respectively.

(i) Calculate the coordinates of the centre of mass of the four particles. [5]

A thin, uniform, rigid wire of mass $12m$ connects A to B, B to C and C to D, as shown in Fig. 2.2.

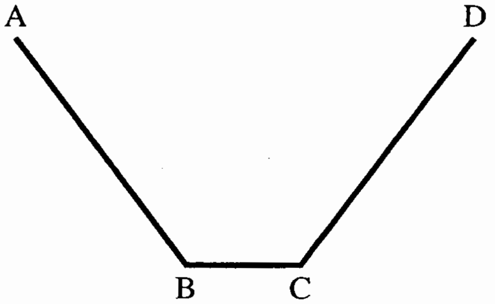

Fig. 2.2

(ii) Calculate the coordinates of the centre of mass of the wire. [5]

(iii) Calculate the coordinates of the combined centre of mass of the wire and the particles at A, B, C and D. [3]

A is connected to D by means of a further straight rigid wire of negligible mass. The combined system of wires and the particles at A, B, C and D is suspended freely from the mid-point of AD.

(iv) What extra mass must be added at C if the system is to hang in equilibrium with AD horizontal? [3]

[Total 16]

3 A car of mass 800 kg is travelling along a straight, level road.

The car is travelling at a steady speed of 25 m s^{-1} when the engine is developing 32 kW of power.

(i) Calculate the resistance to the motion of the car.

Assuming that the resistance to the car's motion is constant, how much work is done against it as the car travels 100 m? [4]

The power developed by the engine is increased so that it averages 45 kW over the next 10 seconds. At the end of this time the car is travelling at a speed of $v \text{ m s}^{-1}$. The total work done against the resistance to motion during the 10 seconds is 340 kJ.

(ii) Show that $v = 30$. [6]

With the engine developing no power, the speed of the car decreases from 35 m s^{-1} to 15 m s^{-1} as it travels 200 m *up* a slope at an angle of θ to the horizontal, where $\sin\theta = \frac{1}{14}$, as shown in Fig. 3.

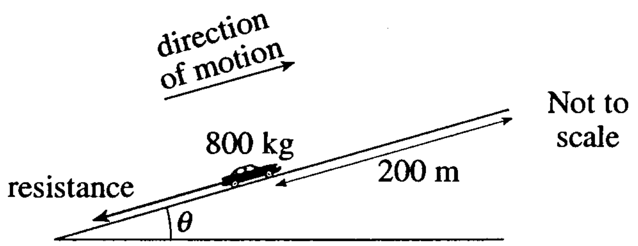

Fig. 3

(iii) How much work is done against the resistance to motion over this 200 m? [5]

[Total 15]

4

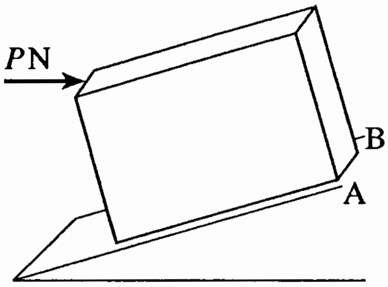

Fig. 4.1

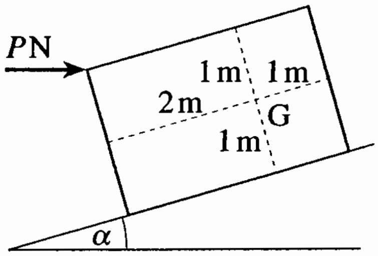

Fig. 4.2

A packing case in the shape of a cuboid is on a rough plane inclined at an angle α to the horizontal. The packing case is being pushed by a **horizontal** force of P N applied perpendicular to and in the centre of an edge of the case, as shown in Fig. 4.1. Fig. 4.2 is a side elevation showing the dimensions of the packing case and the position of G, the centre of mass of the packing case and its contents.

The weight of the packing case and contents is 840 N, $\sin\alpha = \frac{7}{25}$, $\cos\alpha = \frac{24}{25}$ and the coefficient of friction between the packing case and the plane is μ.

(i) Initially $P = 0$ and the packing case is in equilibrium. Show that $\mu \geqslant \frac{7}{24}$. [4]

(ii) Subsequently $P > 0$. Write down the components of P parallel to and perpendicular to the plane. Show that the moment of the pushing force about the edge AB, shown in Fig. 4.1, is $\frac{27}{25}P$ N m clockwise. [4]

(iii) The value of P is such that the packing case is in equilibrium but about to turn about the edge AB.

Draw a diagram showing all of the forces acting on the packing case.

Show that $P = 964$, correct to three significant figures. [6]

[Total 14]

Mark Scheme

Q 1		Mark				
(i)	At projection					
	$\uparrow\ 65\sin\alpha = 39$ $\rightarrow\ 65\cos\alpha = 52$	M1 A1	Attempt to find components Both correct			
	(Since no air resistance,) before first bounce $\downarrow 39, \quad \rightarrow 52$					
	After bounce LM conserved horiz (smooth plane) so $\rightarrow 52$	E1	Must be explicit. Condone no reference to smooth. Accept 'NEL doesn't apply' 'Horiz motion not affected by impact'			
	NEL vert so $\uparrow\ 39\times 0.4 = 15.6$	M1 E1	Explicit ref to NEL or 'bouncing principle' applied perpendicular to plane [Award B1 for $39\times 0.4 = 15.6$ seen without explanation]	5		
(ii)	$\uparrow\ 0.1(15.6-(-39)) = 5.46$ N s	M1 A1 A1	Attempt at $m\mathbf{v} - m\mathbf{u}$ for the ball Signs correct cao with direction explicit or implied by a diagram	3		
(iii)	$0 = UT - 5T^2 \Rightarrow T = 0, \frac{U}{5}$	M1	Use of *uvast* to find time in the air. Condone no ref to $T = 0$.			
	$U_1 = 39$ gives $T_1 = \frac{39}{5}$,	E1	Clearly established			
	$U_n = 39\times 0.4^{n-1}$ gives $T_n = \frac{39}{5}\times 0.4^{n-1}$	E1	Clearly established (e.g. at least 2 members of sequence)	3		
(iv)	GP with $a = \frac{39}{5}$ and $r = 0.4$	M1	Use of S_∞ for a GP or attempt at limit			
	so $T_\infty = \frac{\frac{39}{5}}{1-0.4} = 13$ so 13 s	A1	cao			
	Total horiz dist is $52\times 13 = 676$ m	F1	FT their time			
	Carries on after bouncing phase as constant horiz momentum.	B1	Accept any indication that they understand that the motion continues.	4		
		Tot 15				

Q 2		Mark				
(i)	$10m\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 4m\begin{pmatrix}-3\\ 4\end{pmatrix} + 3m\begin{pmatrix}0\\ 0\end{pmatrix} + m\begin{pmatrix}2\\ 0\end{pmatrix} + 2m\begin{pmatrix}5\\ 4\end{pmatrix}$	M1	Appropriate method for at least one cpt			
		A1	**Either** at least two non-zero RHS terms correct **or** all masses and at least 3 cpts correct			
	giving (0, 2.4)	A1	All correct			
		A1	Each cpt			
		A1		5		
(ii)	By symmetry $\bar{x} = 1$	B1	Accept no reason given			
	$12m\bar{y} = 5m \times 2 + 2m \times 0 + 5m \times 2 \Rightarrow \bar{y} = \frac{5}{3}$	M1	Lengths of rods and their mass			
		A1				
		M1	Award for method using appropriate masses and c.m. at mid-points of rods			
		A1		5		
(iii)	$22m\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10m\begin{pmatrix}0\\ 2.4\end{pmatrix} + 12m\begin{pmatrix}1\\ \frac{5}{3}\end{pmatrix}$	M1	Any correct method			
	giving $\left(\frac{6}{11}, 2\right)$	F1	Each component			
		F1		3		
(iv)	Take moments about line of symmetry **either** for composite body	M1	Award for any clear attempt			
	$22mg\left(1 - \frac{6}{11}\right) = Mg \times 1$	B1	Distance on LHS Award for use of **their** values. Condone use of mass not weight.			
	Hence $M = 10m$	A1	cao	3		
	or omitting rods					
	$(10m + M) \times 1 \times g = 10m \times 0 + M \times 2 \times g$ giving $M = 10m$	M1	Any clear attempt using original values or quoting c.m.			
		A1	Use of **their** values if c.m. quoted. All terms present. Accept mass used.			
		A1	cao			
		Tot 16				

Q 3		Mark				
(i)	Driving force F and resistance R					
	$32000 = 25F$ giving $F = 1280$	M1	Use of $P = Fv$			
	also $F - R = 0$ so $R = 1280$ and resistance is 1280 N	A1	Need some reference to $F = R$			
	$1280 \times 100 = 128000$ J	M1 F1	Use of WD = Fd	4		
(ii)	Use of work-energy	M1	Must have KE term + WD by engine			
	$\frac{1}{2} \times 800v^2 - \frac{1}{2} \times 800 \times 25^2$	M1 A1	One correct KE term Both KE terms correct			
	$= 45000 \times 10$	B1	Work done by engine			
	-340000	A1	All correct			
	hence $v = 30$	E1		6		
(iii)	**either** Use of work-energy	M1	Use of work-energy with KE and GPE			
	$\frac{1}{2} \times 800 \times 15^2 - \frac{1}{2} \times 800 \times 35^2$	M1 A1	Attempt at GPE Correct GPE including sign			
	$= -200 \times \frac{1}{14} \times 9.8 \times 800 - J$ $J = 288\ 000$ so 288 000 J	A1 F1	All signs correct	5		
	or *uvast* up the plane					
	$15^2 = 35^2 + 2a \times 200$	M1	Use of appropriate *uvast*			
	so $a = -2.5$	A1	Accept sign not clearly defined			
	N2L up the plane					
	$-F - 800g \times \frac{1}{14} = 800 \times -2.5$	M1	Use of N2L with all forces present. Condone sign errors.			
	$F = 1440$ WD is $1440 \times 200 = 288$ kJ	A1 F1	FT error in F.			
		Tot 15				

Q 4		Mark				
			[If sin $\leftrightarrow$ cos lose mark at first occurrence and then FT. All E marks lost]			
(i)	Resolving down the plane					
	$840\sin\alpha \le F_{max}$ Perp to plane	B1	Accept $= F$			
	$R = 840\cos\alpha$	B1				
	$F_{max} = \mu R$ Hence $840\sin\alpha \le 840\mu\cos\alpha$	M1	Allow for $F = \mu R$ used.			
	and $\mu \ge \tan\alpha = \dfrac{7}{24}$	E1	Award only if inequality clearly established and value of tan demonstrated [for $\mu = \tan\alpha$ WW, award SC1 with or without evaluation of tan]	4		
(ii)	Parallel $P\cos\alpha$ Perp to plane downwards $P\sin\alpha$	B1 B1	Accept all forces resolved Accept all forces resolved			
	Moment c.w. is $2P\cos\alpha - 3P\sin\alpha$	M1	FT components and lengths			
	$= \dfrac{P}{25}(2\times 24 - 3\times 7) = \dfrac{27P}{25}$	E1	Clearly shown	4		
(iii)	Must have P, weight through G (approx), F plane parallel and thro' base and R perp to plane thro' upper face	B1	All correct. Accept F and R combined at top edge			
	Moment of weight about AB is	M1	Attempt to find moment of weight about AB			
	c.w. $-840\cos\alpha\times 1 - 840\sin\alpha\times 1 = \dfrac{-5208}{5}$ N m	A1				
	Sum of moments about AB is zero so $\dfrac{27P}{25} = \dfrac{5208}{5}$	M1 A1	Equating moments about AB. Dependent on previous M1			
	and $P = 964.444....$ so 964 (3 s. f.)	E1		6		
		Tot 14				

Examiner’s Report

2608 Mechanics 2

General Comments

Although some candidates scored very well on the paper, others found it much more difficult and there was a wide range of scores; in particular, some parts of Q1 and Q4 that were intended to be straightforward caused many candidates considerable problems. There were a number of excellent responses to all of the questions and relatively few candidates scored poorly on every question. When diagrams were drawn they were on the whole of a good standard but many candidates failed to illustrate their answers with a diagram or give an indication of a sign convention and so could not gain full credit. The parts of questions requiring modelling or explanations were, as usual, often not done well because of a poor use of technical language. Candidates seemed to be able to finish all they could do in the time allocated.

Comments on Individual Questions

Question 1 (Impulse and Momentum)

The numerical aspect of part (i) was done well by the majority of candidates who were able to resolve correctly and produce the required answers. However, a significant minority of the candidates did not explain why the final velocity in the vertical direction before the first bounce should be multiplied by the coefficient of restitution to produce the velocity in this direction after the bounce. Also, many candidates failed to state explicitly that the component of linear momentum in the horizontal direction is conserved.

There were few completely correct answers to part (ii) because many candidates did not make the direction of the impulse explicit or obvious from a supporting diagram.

Part (iii) was well done on the whole with most candidates able to establish the given result. Candidates were often less successful with part (iv) because many of them tried to 'guess' the number of bounces instead of using the sum to infinity of a GP. Most of the candidates correctly stated what happens after the ball stops bouncing although a small minority argued that the ball continued to bounce even though this could not be seen.

(ii) 5.46 Ns vertically upwards (iv) 13 s; 676 m.

Question 2 (Centre of Mass)

This question was the highest scoring for the vast majority of candidates. Parts (i), (ii) and (iii) were usually done very well by clear methods. Part (iv) caused some problems for the small minority of candidates who attempted to take moments about a point not on the line of symmetry.

(i) (0, 2.4); (ii) (1, 5/3); (iii) (6/11, 2); (iv) $10m$.

Question 3 (Work and Energy)

There were many good answers to this question and the majority of candidates were able to score quite well.

The calculation posed few problems in part i) but many candidates did not explicitly relate the resistance to the driving force.

In part (ii), most candidates used the work-energy principle and went on to establish the given result.

In part (iii), most candidates who continued to use work and energy gained full marks but, as usual, there were some who used the longer method based on constant acceleration formulae and Newton's second law and these attempts were typically not as successful.

(i) 128 kJ; (iii) 288 kJ.

Question 4 (Friction and Moments)

Many candidates found this question difficult and failed to achieve half marks. In part (i), many could resolve the weight correctly and use $F = \mu R$ but could not establish the required inequality.

In part (ii), the majority of candidates could write down the components of P but were then unable to establish the correct moment; it was quite common to see the expression $3P \sin\alpha = 2P\cos\alpha$ given as the moment.

In part (iii) many candidates failed to draw a correct diagram showing all of the forces acting on the case; a very common mistake was to think that the normal reaction acted through the centre of mass. Some candidates wasted time and effort by re-establishing the result for part (ii) and others forgot one component of the weight in their attempt to find its moment.