OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS — 2615

Statistics 3

Thursday **30 MAY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 The weekly takings at three cinemas are modelled as independent Normally distributed random variables with means and standard deviations as shown in the table, in £.

	mean	standard deviation
Cinema A	6000	400
Cinema B	9000	800
Cinema C	5100	180

(i) Find the probability that the weekly takings at cinema A will be less than those at cinema C. [4]

(ii) Find the probability that the weekly takings at cinema B will be at least twice those at cinema C. [4]

(iii) The parent company receives a weekly levy consisting of 12% of the weekly takings at cinema A, 20% of those at cinema B and 8% of those at cinema C. Find the probability that this levy exceeds £3000 in any given week. Hence find the probability that in a 4-week period the weekly levy exceeds £3000 at least twice. [7]

2 At a bottling plant, wine bottles are filled automatically by a machine. The bottles are meant to hold 75 cl. Under-filling leads to contravention of regulations and complaints from customers. Over-filling prevents the bottles being sealed securely.

The contents of 10 bottles are carefully measured and found to be as follows, in cl.

75.6	76.2	74.3	74.8	75.3	76.3	75.9	74.2	75.6	76.7

(i) State appropriate null and alternative hypotheses for the usual t test for examining whether the bottles are being filled correctly. [2]

(ii) State the conditions necessary for correct application of this test. [2]

(iii) Carry out the test, using a 5% significance level. [7]

(iv) Provide a two-sided 99% confidence interval for the true mean amount of wine delivered into the bottles. [4]

3 The length of metal rods used in an engineering structure is specified as being 40 cm. It does not matter if the rods are slightly longer, but they should not be any shorter. These rods are made by a machine in such a way that their lengths are Normally distributed with standard deviation 0.2 cm. An operator sets the machine so that the mean length, μ cm, is slightly greater than 40.

The operator takes a random sample of 12 rods. Their lengths, in cm, are as follows.

40.43 40.49 40.19 40.36 40.81 40.47 40.46 40.63 40.41 40.27 40.34 40.54

The operator wishes to examine whether μ may be assumed to be 40.5, as experience shows that a smaller mean would not give an adequate margin for error.

(i) State suitable null and alternative hypotheses for the test. [2]

(ii) Carry out the test at the 5% level of significance. [7]

(iii) Write down the probability of a Type I error for the test. [1]

(iv) Calculate the probability of a Type II error for the test if in fact $\mu = 40.3$. [5]

4 The manager of a large supermarket has recently moved from one store to another. At the previous store, it was known from surveys that 42% of the customers lived within 5 miles of the store, 35% lived between 5 and 10 miles from the store and the remaining 23% lived more than 10 miles from the store. The manager wishes to test whether the same proportions apply at the new store.

One Saturday morning, the first 100 customers to enter the store after 11 00 am were asked how far from the store they lived. The results, grouped into the same categories as for the previous store, were as follows.

Distance (miles)	Number of customers
0 - 5	34
5 - 10	48
more than 10	18

(i) Assuming that this is a random sample of all the customers at the store, test at the 5% level of significance whether the proportions at the manager's new store may be taken as the same as those at the previous store. Discuss your conclusions briefly. [8]

(ii) Discuss whether this is likely to be a random sample. [3]

The marketing manager believes that the actual distance from the store for all customers in the 5 - 10 miles category is uniformly distributed over that range.

(iii) Assuming that this is correct, write down the probability density function for the random variable giving the distances for these customers. State its mean and use calculus to find its variance. [4]

Mark Scheme

Q1	$A \sim N(6000, \sigma = 400)$ $B \sim N(9000, \sigma = 800)$ $C \sim N(5100, \sigma = 180)$ Do not allow negative variance in any part of the question.		
(i)	Want $P(A - C < 0)$ (or $P(C - A > 0)$) $A - C \sim N(900,$ $400^2 + 180^2 = 192400$ [$\sigma = 438.63$]) $\therefore$ want $P(N(0, 1) < \frac{-900}{438.63} = -2.05)$ $= 1 - 0.9798$ $= 0.0202$	B1 M1 for $400^2 + 180^2$. M1 for standardising using c's parameters. Inequality and sign to be consistent; possibly implied later. A1 c.a.o. Min 3 dp required. Accept awrt 0.020	4
(ii)	Want $P(B - 2C > 0)$ (or $P(2C - B < 0)$) $B - 2C \sim N(-1200,$ $800^2 + 4\times180^2 = 769600$ [$\sigma = 877.27$] $\therefore$ want $P(N(0, 1) > \frac{1200}{877.27} = 1.368)$ $= 1 - 0.9144$ $= 0.0856$	Or equivalent. B1 M1 for $800^2 + 4\times180^2$. M1 for standardising using c's parameters. Inequality and sign to be consistent; possibly implied later. A1 c.a.o. Min 3 dp required. Accept anything between 0.085 and 0.086.	4
(iii)	Levy $L = 0.12A + 0.2B + 0.08C$ $L \sim N(\mu = 720 + 1800 + 408 = 2928$ $\sigma^2 = (0.12)^2(400)^2 + (0.2)^2(800)^2 + (0.08)^2(180)^2$ $= 2304 + 25600 + 207.36 = 28111.36$ [$\sigma = 167.66$]) $P(L > 3000) = P(N(0, 1) > \frac{72}{167.66} = 0.4294)$ $= 1 - 0.6662$ $= 0.3338$	B1 M1 Attempt to combine variances correctly. M1 for standardising using c's parameters. A1 c.a.o. Min 3dp required. Accept anything between 0.3325 and 0.3345	4
	Now have $X \sim B(4, 0.3338)$ $P(X \geq 2) = 1 - \{(0.6662)^4 + 4(0.3338)(0.6662)^3\}$ $= 1 - 0.5918$ $= 0.4082$	M1 Accept any evidence that candidate realises to use $B(4, p)$ or $B(4, 1 - p)$, where p = c's 0.3338. M1 Correct binomial terms present and no extras. Could be the other three terms. A1F ft c's value of p provided $0 < p < 1$. Min 3dp required.	3
			15

Q2 (i)	$H_0 : \mu = 75$ $H_1 : \mu \neq 75$ (Do **not** allow $\bar{x} = \ldots$ or similar) Where μ is the (population) mean amount of wine delivered into a bottle.	B1 Both must be correct. Allow statements in words (see below). B1 Must indicate "mean"; condone "average". If the symbol used is not μ, or no symbol is used, then insist on "population".	2
(ii)	Underlying distribution is Normal. Sample is random.	B1 Must be describing the population and not the sample or sample means. There may be other supporting evidence e.g. $X \sim N(.. , ..)$ earlier. B1 Condone "independent" and ignore all else.	2
(iii)	$\bar{x} = 75.49,\ s_{n-1} = 0.8439\ \ (s_{n-1}^2 = 0.7121)$ Test statistics is $\dfrac{75.49 - 75}{\frac{0.8439}{\sqrt{10}}}$ $= 1.836$	B1 Allow $s_n = 0.8006,\ (s_n^2 = 0.6409)$ only if correctly used in sequel. M1 Allow alternative: $75 + (\text{c's } 2.262) \times \frac{0.8439}{\sqrt{10}}$ (= 75.60) for subsequent comparison with $\bar{x}$. A1 c.a.o. (but ft from here if this is wrong.) Use of $75 - \bar{x}$ or $\bar{x} - 2.262 \times \frac{0.8439}{\sqrt{10}}$ scores M1A0, but next 4 marks still available.	3
	Refer to t_9 Double-tailed 5% point is 2.262. Not significant Seems we may accept that on average 75cl is being delivered into each bottle	M1 A1 A1F ft only c's test statistic. A1F ft only c's test statistic.	4
(iv)	CI is given by 75.49 ± 3.250 $\times \dfrac{0.8439}{\sqrt{10}}$ $= 75.49 \pm 0.86(73)$ $= (74.62(27), 76.35(73))$	M1 c's $\bar{x}$ ± … B1 M1 c's $s_{n-1}/\sqrt{10}$ A1 c.a.o. Min 2 dp required. Must be given as an interval. N.B. The *same* wrong distribution used can score max M1B0M1A0. ZERO out of 4 if candidate *changes* to a wrong distribution. Recovery to t_9 can score full marks.	4
			15

Q3 (i)	$H_0 : \mu = 40.5$ $H_1 : \mu < 40.5$ (Do **not** allow $\bar{x} = \ldots$ or similar)	B1 B1 No need for candidates to define μ verbally – it is given in the question.	2
(ii)	$n = 12$ $\Sigma x = 485.4, \quad \bar{x} = 40.45$ Test statistic is $\dfrac{40.45 - 40.5}{\frac{0.2}{\sqrt{12}}}$ $= -0.866$ Refer to $N(0, 1)$ Lower 5% point is -1.645 Not significant Reasonable to accept that $\mu = 40.5$	B1 M1 Use of any alternative to $\sigma = 0.2$, e.g. $s_{n-1} = 0.1636$ or $s_n = 0.1566$, scores M0A0. Allow alternative: $40.5 +$ (c's -1.645) $\times \frac{0.2}{\sqrt{12}}$ ($= 40.405$) for subsequent comparison with $\bar{x}$. A1 c.a.o. (but ft from here if this is wrong.) Use of $40.5 - \bar{x}$ or $\bar{x} + 1.645 \times \frac{0.2}{\sqrt{12}}$ scores M1A0, but next 4 marks still available. M1 A1 or $P(\bar{X} < 40.45) = 0.1932$ A1F ft only c's test statistic. A1F ft only c's test statistic.	3 4
(iii)	P(Type I error) $= 0.05$	B1 accept "5%".	1
(iv)	If $\mu = 40.3$, $\bar{X} \sim N\left(40.3, \frac{(0.2)^2}{12}\right)$ H_0 is accepted if $\bar{X} > 40.5 - 1.645\frac{0.2}{\sqrt{12}} = 40.405(03)$ So P(Type II error) $= P\left(N\left(40.3, \frac{(0.2)^2}{12}\right) > 40.405\right)$ $= P(N(0, 1) > 1.818(65))$ $= 0.0345$	M1 Evidence of the use of this distribution is sufficient. M1 Finding the critical value. M1 For showing a correct definition of a Type II error. Minimum acceptable is P(accept $H_0 \mid \mu = 40.3$). Need not include 40.405, but must not include anything other than it. A1F Depends on all 3 M's. ft c's σ. A1F Depends on all 3 M's. ft c's σ. If $\sigma = 0.1636$ then cv=40.422, $z = 2.59$, $p = 0.0048$. If $\sigma = 0.1566$ then cv=40.426, $z = 2.78$, $p = 0.0027$.	5
			15

Q4 (i)	o_i: 34, 48, 18 e_i: 42, 35, 23		
	$X^2 = \frac{(34-42)^2}{42} + \ldots = 1.524 + 4.829 + 1.087$ $= 7.44$	M1 A1	2
	Refer to χ_2^2 Upper 5% point 5.991 Significant Seems proportions do not apply.	M1 A1 A1F ft only c's test statistic. A1F ft only c's test statistic.	4
	We find more than expected in the 5-10 miles category, fewer than expected in the other categories.	E2 Should discuss these details of the "quality of fit". Do not accept discussion of the size of the test statistic or a restatement of the conclusions of the test. A less detailed comment such as "This is not surprising because the supermarkets are in different locations" may score 1 out of 2.	2
(ii)	Sample is very unlikely to be random … … e.g. all at about the same time, customers arriving in groups, etc.	B1 E2 Allow any reasonable discussion of why **not** random. As a guide look for two points (e.g. timing and clustering) or one point + some justification (e.g. reference to the definition of a random sample or linking time of arrival to distance travelled).	3
(iii)	Uniform (5, 10): $f(x) = \frac{1}{5}$ (for $5 \le x \le 10$)	B1 Domain not required.	
	Mean is $7\frac{1}{2}$	B1 c.a.o.	
	$E[X^2] = \int_5^{10} \frac{1}{5}x^2 dx = \left[\frac{x^3}{15}\right]_5^{10} = \frac{1000-125}{15} = 58.\dot{3}$	M1 for setting up the integral, including limits.	
	$\therefore$ variance $= 58.\dot{3} - (7.5)^2 = 2.08\dot{3}$ $\left(\text{or } \frac{25}{12}\right)$	A1 c.a.o.	4
			15

Examiner’s Report

2615 Statistics 3

General Comments

The general standard of the candidates' work for this module was pleasing: many were clearly well prepared for it. There continues to be improvement in their ability to tackle questions on the main topics of the syllabus, and throughout the paper, with the exception of Question 3 (iv), the parts calling for calculation were done well. However, as in the past, candidates' explanations were often woolly, indicating that they had not considered carefully enough what they were being asked.

Invariably all four questions were attempted. Marks for Question 2 were consistently high, with Questions 1 and 4 only slightly behind. Candidates were least successful in Question 3, particularly in the last part, to find the probability of the Type II error.

The quality of presentation of scripts – the handwriting and attention to accuracy of answers – still leaves room for improvement.

Comments on Individual Questions

Q.1 This question was well answered by many candidates. Not surprisingly it was the inability of candidates to combine variances correctly which proved to be the main cause of lost marks.

(i) For most candidates this turned out to be a reasonably easy starter. Weaker candidates experienced difficulty in formulating the problem – a common error was to find $P(A < 5100)$.

(ii) This time the formulation of the problem caused more difficulty. There was also the added problem of the variances which were often combined incorrectly, usually because the candidate used twice the variance of C instead of 2^2 times.

(iii) This part was often done better than part (ii); some fairly weak candidates were seen to recover here. Problems over the rules for combining variances persisted, but it was clear that their understanding of what the question required them to do was better.

The second half of this part required candidates to recognise that the binomial distribution was appropriate and apply it. There was relatively little trouble with doing so and the last 3 marks were often earned in full. Any marks lost could usually be attributed to a misunderstanding (or oversight) of the words "at least twice".

(i) 0·0202; (ii) 0·0856; (iii) 0·3338, 0·4082.

Q.2 (i) Most candidates stated their hypotheses correctly, but few defined μ.

(ii) As in the past the necessary conditions were not as well remembered as perhaps they ought to be. The statements offered were frequently vague: e.g. "it is random", "it is Normal", and "the data/sample is Normal".

(iii) There were many fully correct answers to this part. The work seen indicated a continuation of the improvement noted in last January's report. There were fewer instances of candidates using s_n instead of s_{n-1} to calculate the test statistic, and candidates were more likely to identify the correct t distribution and critical value than in the past.

(iv) Apart from a number of candidates who reverted to a Normal distribution for their confidence interval, this part was generally well done.

(iii) test statistic 1·836, critical value 2·262; (iv) (74·62, 76·35).

Q.3 (i) Once again the hypotheses were often given correctly, though this time there was more uncertainty about the alternative hypothesis – a significant number of candidates took it as a two-tailed test.

(ii) Too many candidates spoiled their attempts to answer this part by using the sample standard deviation (the population value was given in the question) and/or using the t distribution. In addition many candidates appeared to be too casual in their treatment of the negative sign of the test statistic.

(iii) Most candidates knew the answer to this part.

(iv) This part of the question was not answered at all well, though it is clear that candidates' performance with the relatively new topic "Type I and Type II errors" is gradually improving. Many candidates displayed a flawed understanding of what they needed to do, and it still seems that the level of preparation varies from centre to centre. It was necessary for candidates to find the critical value of the original test in part (ii), to recognise that there was a new distribution, with mean 40·3, for the sample means, and to put these together to find the required probability. It was noticeable that many of the more successful candidates made good use of sketches.

(ii) Test statistic –0·866, critical value –1·645; (iii) 0·05; (iv) critical value 40·405, 0·0345.

Q.4 (i) Most candidates scored the first few marks of this part of the question fairly easily. By asking candidates to "Discuss your conclusions briefly" it was intended that they would highlight some aspects of the available evidence which might account for the conclusion to the hypothesis test. The vast majority gave no explanation whatsoever and of those who did attempt to explain only a few managed to say anything appropriate.

(ii) Answers to this part were also disappointing, with very many candidates confusing random with representative. Explanations tended to be rambling and contained a lot of padding, and said little about candidates' understanding of sampling methods. Even so, many scored at least some credit for saying that the sample was unlikely to be random and making a plausible attempt to justify it.

(iii) This part was often done well, but there was a significant minority who quoted from memory the general formula for the variance of a uniform distribution, instead of using calculus, and so lost the final marks.

(i) test statistic 7·44, critical value 5·991; (iii) $f(x) = 1/5$ for $5 \leqslant x \leqslant 10$, mean 7·5, variance 2·083.