OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2604

Pure Mathematics 4

Monday **20 MAY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

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1 A curve has equation $y = \dfrac{(x+4)^2}{x-3}$.

(i) Write down the equation of the asymptote parallel to the y-axis.

Express the equation of the curve in the form

$$y = Ax + B + \frac{C}{x-3}.$$

Hence write down the equation of the oblique asymptote. [4]

(ii) Find $\dfrac{dy}{dx}$, and hence find the coordinates of the stationary points. [5]

(iii) Sketch the curve. [3]

(iv) Solve the inequality $\dfrac{(x+4)^2}{x-3} > -36$. [4]

(v) On a separate diagram, sketch the curve with equation $y = \left|\dfrac{(x+4)^2}{x-3}\right|$.

Give the equations of the three asymptotes of this curve. [4]

2 (a) Express $\dfrac{2r}{(r+1)(r+2)(r+3)}$ in partial fractions, and hence find the sum of the first n terms of the series

$$\frac{2}{2\times3\times4} + \frac{4}{3\times4\times5} + \frac{6}{4\times5\times6} + \ldots.$$ [8]

(b) Two straight lines have equations

$$L: \frac{x-21}{9} = \frac{y+1}{7} = \frac{z+14}{2} \quad \text{and} \quad M: \frac{x+30}{-6} = \frac{y-6}{7} = \frac{z-12}{8}.$$

(i) Show that L and M intersect, and find the coordinates of their point of intersection. [7]

(ii) Find, in the form $ax + by + cz + d = 0$, the equation of the plane which contains both the lines L and M. [5]

3 (a) A sequence of integers $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 7$ and $u_{n+1} = 2u_n + 3 - n^2$ for $n \geqslant 1$.

Prove by induction that $u_n = n(n+2) + 2^{n+1}$. [7]

(b) The equation $z^2 + 6z + 16 = 0$ has complex roots α and β, where α is the root with positive imaginary part.

(i) Find α and β in the form $a + b\mathrm{j}$. Give the exact values of a and b, using surds if necessary. [3]

(ii) Find the modulus and argument of each of the complex numbers α, β, $\frac{\alpha}{\beta}$, $\frac{\beta}{\alpha}$, giving the arguments in radians between $-\pi$ and π, correct to 2 decimal places.

Illustrate these four complex numbers on an Argand diagram. [8]

(iii) Write down the exact value of $\arg\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)$. [2]

4 (a) Find an expression for $\sum_{r=1}^{n} (r+2)(3r+1)$, giving your answer in a fully factorised form. [5]

(b) **M** is the transformation of the plane defined by the matrix $\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$.

(i) Show that $y = 3x$ is an invariant line of the transformation **M**, and find the equation of the other invariant line. [5]

P is the transformation defined by the matrix $\begin{pmatrix} 5 & -1 \\ -8 & 2 \end{pmatrix}$.

The line L is the image of the line $y = 3x$ under the transformation **P**.

(ii) Find the equation of L. [2]

The transformation **Q** is the inverse of **P**.

(iii) State the image of the line L under the transformation **Q**. [1]

(iv) Find the matrix corresponding to **Q**. [2]

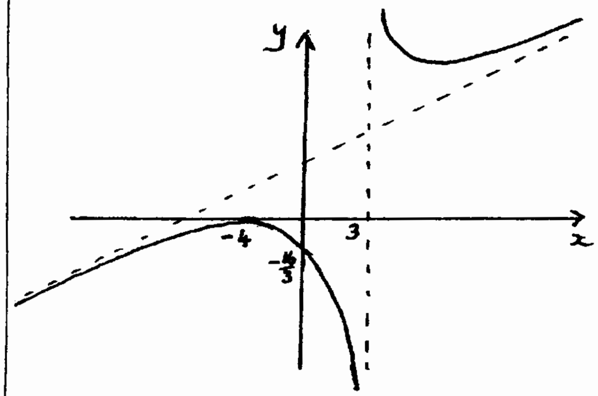
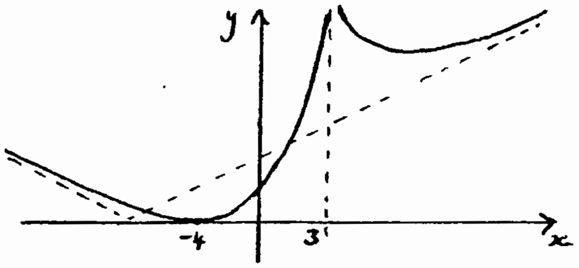
The transformation **R** is **Q** followed by **M** followed by **P**.

(v) Show that L is an invariant line of the transformation **R**. [2]

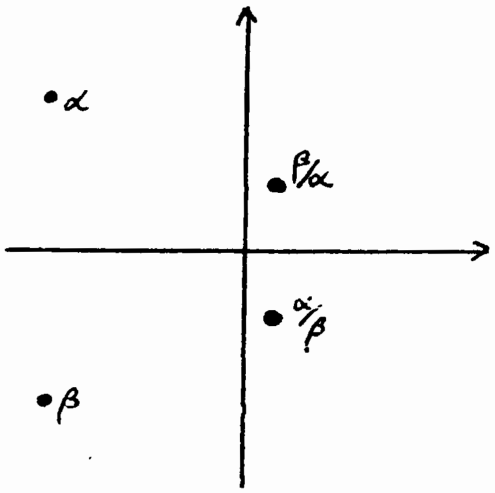
(vi) Find the matrix corresponding to **R**. [3]

Mark Scheme

MARKING SCHEME

1 (i)	Vertical asymptote is $x = 3$ $y = \dfrac{(x-3)(x+11)+49}{x-3}$ $= x + 11 + \dfrac{49}{x-3}$ Oblique asymptote is $y = x + 11$	B1 M1 A1 A1 ft **4**	Method for finding *A*, *B*, *C*
(ii)	$\dfrac{dy}{dx} = 1 - \dfrac{49}{(x-3)^2}$ $= 0$ when $(x-3)^2 = 49$ $x = -4,\ 10$ Stationary points are $(-4, 0)$ and $(10, 28)$	M1A1 M1 A2 cao **5**	Or $\dfrac{(x-3)(2x+8)-(x+4)^2}{(x-3)^2}$ Or $x^2 - 6x - 40 = 0$ Give A1 for one point correct, or for $x = -4,\ 10$
(iii)		B1 B1 B1 **3**	Max on LH branch, min on RH branch Cutting *y*-axis at $\left(0, -\frac{16}{3}\right)$ Touching *x*-axis and approaching asymptotes correctly
(iv)	$\dfrac{(x+4)^2}{x-3} = -36$ when $x^2 + 44x - 92 = 0$ $x = -46,\ 2$ $\dfrac{(x+4)^2}{x-3} > -36 \Leftrightarrow -46 < x < 2,\quad x > 3$	M1 A1 M1 A1 cao **4**	Or $(x+4)^2(x-3) + 36(x-3)^2 > 0$ Or $\dfrac{(x+4)^2 + 36(x-3)}{x-3} > 0$ Or factors $(x+46)(x-2)$ If M0 then B1B1 for $-46, 2$ or factors Considering intervals defined by $-46, 2, 3$ (ft)
(v)	 Asymptotes $x = 3,\ y = x + 11$ $y = -x - 11$	B2 cao B1 ft B1 ft **4**	Give B1 (ft) for negative section reflected in *x*-axis

2 (a)	$\frac{2r}{(r+1)(r+2)(r+3)} = -\frac{1}{r+1} + \frac{4}{r+2} - \frac{3}{r+3}$	M1 A1	Finding partial fractions
	Sum $= \left(-\frac{1}{2}+\frac{4}{3}-\frac{3}{4}\right)$ $+\left(-\frac{1}{3}+\frac{4}{4}-\frac{3}{5}\right)$	M1	Using partial fractions in at least two terms
	$+\ldots$	A1 ft	At least three terms correct
	$+\left(-\frac{1}{n}+\frac{4}{n+1}-\frac{3}{n+2}\right)$ $+\left(-\frac{1}{n+1}+\frac{4}{n+2}-\frac{3}{n+3}\right)$	M1	Cancelling to leave 3 fractions at beginning and / or 3 fractions at end
	$= -\frac{1}{2}+\frac{4}{3}-\frac{1}{3}-\frac{3}{n+2}+\frac{4}{n+2}-\frac{3}{n+3}$	A1 ft A1 ft	3 fractions at beginning 3 fractions at end *Condone r instead of n*
	$= \frac{1}{2}+\frac{1}{n+2}-\frac{3}{n+3} \quad \left(= \frac{n(n+1)}{2(n+2)(n+3)}\right)$	A1 cao **8**	
(b)(i)	For a point of intersection, $21+9\lambda = -30-6\mu$ (1) $-1+7\lambda = 6+7\mu$ (2) $-14+2\lambda = 12+8\mu$ (3)	M1 A1	Equating two components, *using different parameters* Two equations correct
	Solving (1) and (2), $\lambda = -3,\ \mu = -4$	M1A1	
	Check in (3), LHS $= -14-6 = -20$ RHS $= 12-32 = -20$	M1 A1	Checking consistency
	Hence the lines intersect Point of intersection is $(-6, -22, -20)$	A1 **7**	*Dependent on first M1A1M1*
(ii)	Normal is $\begin{pmatrix}9\\7\\2\end{pmatrix} \times \begin{pmatrix}-6\\7\\8\end{pmatrix} = \begin{pmatrix}42\\-84\\105\end{pmatrix} = 21\begin{pmatrix}2\\-4\\5\end{pmatrix}$	M1 A2	Vector product, or other method for finding normal vector Give A1 if just one error
	Equation of *P* is $2x-4y+5z = -12+88-100$ i.e. $2x-4y+5z+24=0$	M1 A1 **5**	Finding constant Accept $42x-84y+105z=-504$ etc

3 (a)	When $n=1$, $n(n+2)+2^{n+1}=1\times 3+2^2=7$ So formula is true when $n=1$ Assume true for $n=k$, then $u_{k+1}=2\left\{k(k+2)+2^{k+1}\right\}+3-k^2$ $=k^2+4k+3+2^{k+2}$ $=(k+1)(k+3)+2^{k+2}$ True for $n=k \Rightarrow$ True for $n=k+1$ (So formula is true for all positive integers n)	B1 M1A2 M1 A1 A1 7	 Use of $2\times 2^{k+1}=2^{k+2}$ Stated or clearly implied. *Dependent on previous M1A2M1A1*
(b)(i)	$z=\dfrac{-6\pm\sqrt{-28}}{2}$ $\alpha=-3+\sqrt{7}\,\mathrm{j}$, $\beta=-3-\sqrt{7}\,\mathrm{j}$	M1 M1 A1 3	Quadratic formula (or equivalent) Use of $\sqrt{-n}=k\,\mathrm{j}$ Accept $\dfrac{-6+\sqrt{28}\,\mathrm{j}}{2}$ etc
(ii)	$\|\alpha\|=\|\beta\|=4$ $\arg\alpha=2.42$ $\arg\beta=-2.42$ $\left\|\dfrac{\alpha}{\beta}\right\|=\left\|\dfrac{\beta}{\alpha}\right\|=1$ $\arg\dfrac{\alpha}{\beta}=-1.45$, $\arg\dfrac{\beta}{\alpha}=1.45$	B1 ft B1 ft B1 ft B1 M1 A1 cao	 Must be in radians and 2 dp *Accept* 0.77π $\arg\beta=-\arg\alpha$ Using $\arg\alpha-\arg\beta$ (or other valid method) *Accept* -0.46π, 0.46π *Allow anything in range* 1.44 *to* 1.45
		B2 cao 8	Give B1 for two points correct Give B1 for all correct but not labelled
(iii)	$\arg\left(\dfrac{\alpha}{\beta}-\dfrac{\beta}{\alpha}\right)=-\tfrac{1}{2}\pi$	B2 2	Accept $-90°$, $\tfrac{3}{2}\pi$, $270°$ Give B1 for $\tfrac{1}{2}\pi$, $90°$, -1.57

4 (a)	$\sum_{r=1}^{n}(3r^2+7r+2)$ $=\frac{1}{2}n(n+1)(2n+1)+\frac{7}{2}n(n+1)+2n$ $=\frac{1}{2}n(2n^2+3n+1+7n+7+4)=\frac{1}{2}n(2n^2+10n+12)$ $=n(n+2)(n+3)$	B1B1B1 M1 A1 **5**	Expressing as a simple cubic *Accept* $\frac{1}{2}n(2n+4)(n+3)$ *etc*
(b)(i)	$\begin{pmatrix}-1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}t\\mt\end{pmatrix}=\begin{pmatrix}-t+2mt\\3t+4mt\end{pmatrix}$ The line $y=mx$ is invariant if $3t+4mt=m(-t+2mt)$ $2m^2-5m-3=0$ $(m-3)(2m+1)=0$ $m=3,\ -\frac{1}{2}$ The other invariant line is $y=-\frac{1}{2}x$	M1 A1 M1 B1 A1 **5**	For LHS (or equivalent) For $m=3$. Can be given for $\begin{pmatrix}-1 & 2\\3 & 4\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}5\\15\end{pmatrix}$, also on $y=3x$
(ii)	$\begin{pmatrix}5 & -1\\-8 & 2\end{pmatrix}\begin{pmatrix}t\\3t\end{pmatrix}=\begin{pmatrix}2t\\-2t\end{pmatrix}$ Equation of L is $y=-x$	M1 A1 **2**	Any one non-zero point on $y=3x$ is sufficient
(iii)	The line $y=3x$	B1 **1**	
(iv)	$\mathbf{Q}=\mathbf{P}^{-1}=\frac{1}{2}\begin{pmatrix}2 & 1\\8 & 5\end{pmatrix}$	B1B1 **2**	For $\frac{1}{2}$ and $\begin{pmatrix}2 & 1\\8 & 5\end{pmatrix}$
(v)	**Q** transforms L to $y=3x$ **M** transforms $y=3x$ to $y=3x$ **P** transforms $y=3x$ to L Hence **R** transforms L to L	M1 A1 **2**	Considering at least two successive transformations of L
	OR, if done after **(vi)**, $\begin{pmatrix}16 & 11\\-18 & -13\end{pmatrix}\begin{pmatrix}t\\-t\end{pmatrix}=\begin{pmatrix}5t\\-5t\end{pmatrix}$ M1 which is also on L A1		
(vi)	$\mathbf{R}=\mathbf{PMQ}=\begin{pmatrix}5 & -1\\-8 & 2\end{pmatrix}\begin{pmatrix}-1 & 2\\3 & 4\end{pmatrix}\frac{1}{2}\begin{pmatrix}2 & 1\\8 & 5\end{pmatrix}$ $=\begin{pmatrix}16 & 11\\-18 & -13\end{pmatrix}$	M1A1 ft A1 cao **3**	Give M1A0 if order wrong

Examiner’s Report

2604 Pure Mathematics 4

General Comments

This paper was found to be slightly harder than those in recent sessions. However, there was a great deal of good work, with 30% of candidates scoring 50 marks or more (out of 60) and only about 10% of candidates scoring les than 20 marks. The great majority of candidates demonstrated some competence in all the topics examined, although there were many errors resulting from careless algebra and arithmetic (often involving signs). Some candidates appeared to have insufficient time to complete three questions. Q.4 was by far the least popular question.

Comments on Individual Questions

Q.1 This was the most popular question and it was very well answered, with half the attempts scoring 16 marks or more (out of 20). In part (i) most candidates wrote down the vertical asymptote and carried out the long division correctly, although there were some sign errors. The weaker candidates just multiplied out $(x+4)^2 = x^2 + 8x + 16$ then wrote $A = 1$, $B = 8$, $C = 16$. In part (ii) only a few candidates differentiated the result of their long division, with most preferring to use the quotient rule, which was generally applied confidently and with care. Most went on to find the stationary points correctly. In part (iii) the curve was usually well drawn, although some did not indicate the point where it crosses the y-axis, and a missing or incorrect oblique asymptote often led to a curve of the wrong shape. Some of the graphs were very scruffily drawn, and some were much too small. In part (iv) the inequality was generally well handled, with many appealing to the evidence of their graph in part (iii). However, a fair number ignored the asymptote at $x = 3$ and just solved the inequality $(x + 4)^2 > -36(x - 3)$. In part (v) most candidates reflected the negative part of their curve correctly to obtain the graph of $y = |f(x)|$, although a few attempted to draw the graph of $y^2 = f(x)$ instead. However, many failed to reflect their oblique asymptote correctly; and the equation of the reflected asymptote was often given as $y = x - 11$ instead of $y = -x - 11$.

(i) $x = 3, y = x + 11 + \frac{49}{x-3}, y = x + 11$, (ii) $\frac{dy}{dx} = 1 - \frac{49}{(x-3)^2}$, $(-4, 0)$ and $(10, 28)$,

(iv) $-46 < x < 2,\ x > 3$, (v) $x = 3, y = x + 11, y = -x - 11$.

Q.2 This question was answered well. The average mark was about 14, and more candidates scored full marks on this question than on any other. In part (a) the partial fractions were usually found correctly and although some attempts stopped at this point most knew how to proceed. The series was very often summed successfully, although there were quite a lot of errors such as having the wrong fractions left over after cancelling, having $n + 1$ terms instead of n, combining like terms incorrectly, or giving the answer in terms of r instead of n. In part (b)(i) some candidates began with $\frac{x-21}{9} = \frac{x+30}{-6}$ and so on, but the great majority introduced two different parameters and proceeded staunchly towards the point of intersection, although a great number failed to verify that the lines do indeed meet. Part (b)(ii) was very often answered correctly, with most using a vector product to find the direction of the normal. A few started with a parametric equation for the plane, but this very rarely led to the correct answer.

(a) $-\frac{1}{r+1} + \frac{4}{r+2} - \frac{3}{r+3}$, $\frac{1}{2} + \frac{1}{n+2} - \frac{3}{n+3}$.; (b)(i) $(-6, -22, -20)$, (ii) $2x - 4y + 5z + 24 = 0$.

Q.3 This was not quite so well answered as Q.1 and Q.2, the average mark being about 13. In part (a) the proof by induction was very often competently done. Common errors were starting the induction with $n = 2$ instead of $n = 1$, writing the inductive step as $u_{k+1} = 2u_k + 3 - (k + 1)^2$ or trying to 'add on the next term' with $u_{k+1} = k(k + 2) + 2^{k+1} + 2u_k + 3 - k^2$, and in the working, $2 \times 2^{k+1} = 4^{k+1}$. In part (b)(i) there were a surprising number of errors in solving the quadratic equation, the most common being $\alpha = -3 + 7j$. In part (b)(ii) almost all candidates found the four moduli correctly (for their α and β) but the arguments caused more difficulty. Having obtained $\arg\frac{\alpha}{\beta} = \arg\alpha - \arg\beta = 4.84$ very many converted this to the principal value by subtracting π instead of 2π. A fair proportion of candidates attempted to evaluate $\frac{\alpha}{\beta}\left(\text{and } \frac{\beta}{\alpha}\right)$ before finding its modulus and argument; this was sometimes successful, but errors were very common here. The Argand diagram usually followed the candidate's own version of the four complex numbers and as such frequently contained errors, usually with the plotting of $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. In part (b)(iii) only a minority of candidates gave the correct answer, with many finding it necessary to evaluate $\frac{\alpha}{\beta} - \frac{\beta}{\alpha}$ first. A common incorrect answer was $\frac{1}{2}\pi$.

(b)(i) $\alpha = -3 + \sqrt{7}\mathrm{j}$, $\beta = -3 - \sqrt{7}\mathrm{j}$, (ii) Moduli 4, 4, 1, 1, Arguments 2.42, -2.42, –1.45, 1.45, (iii) $-\frac{1}{2}\pi$.

Q.4 This question was attempted by about one third of the candidates. It was the worst answered question, with half the attempts scoring 10 marks or less. In part (a) a surprising number were unable to multiply out $(r + 2)(3r + 1)$ correctly, obtaining $3r^2 + 7r + 3$ or worse. Another common error was to take $\Sigma 2$ as 2 instead of $2n$. Further poor algebra was seen in the attempt to reduce the result to a 'fully factorised form', for example many errors were made when extracting a common factor. Only a minority produced the answer in the expected form. Part (b)(i) was often answered correctly; the most common error was using the method for finding invariant points instead of lines. Parts (b)(ii), (iii) and (iv) were usually answered correctly, although the determinant of **P** was often calculated as 18 instead of 2. In part (b)(v) correct and clear arguments were rare; a very common explanation was: 'Since **P** and **Q** are inverses, **Q** followed by **M** followed by **P** is the same as **M**'. In part (b)(vi) **R** = **M** was common (as above), and the matrix product **QMP** was seen about as often as the correct **PMQ**.

(a) $n(n+2)(n+3)$; (b)(i) $y = -\frac{1}{2}x$, (ii) $y = -x$, (iii) Line $y = 3x$, (iv) $\frac{1}{2}\begin{pmatrix} 2 & 1 \\ 8 & 5 \end{pmatrix}$, (vi) $\begin{pmatrix} 16 & 11 \\ -18 & -13 \end{pmatrix}$.