OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Monday **20 MAY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

Section A (30 marks)

1 Find $\int 2x^4 \, dx$. [2]

2 Express $\frac{7}{5}\pi$ radians in degrees. [2]

3 Sketch the graph of $y = |2x - 3|$, showing the values of x and y where it meets the axes. [3]

4 Expand $(1 - 2x)^4$, simplifying the coefficients. [4]

5 Solve the equation $\sin^2\theta = \frac{3}{4}$, for $0° < \theta < 360°$. [4]

6

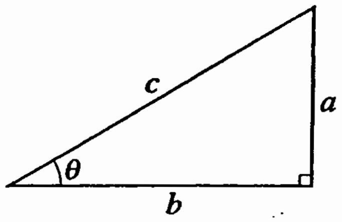


Fig. 4

Fig. 4 shows a right-angled triangle. Use Pythagoras' theorem to prove that $\sin^2\theta + \cos^2\theta = 1$. [4]

7

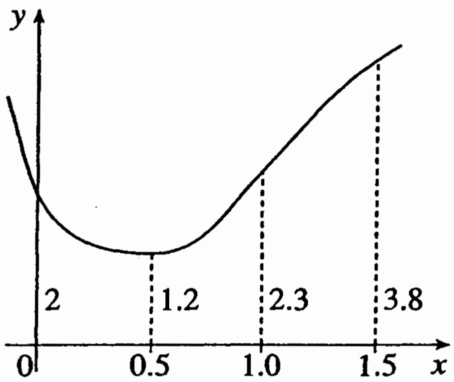


Fig. 5

Use all the information in Fig. 5 to obtain a trapezium rule estimate for the area of the region bounded by the curve, the x-axis, the y-axis and the line $x = 1.5$. [3]

8 Solve the simultaneous equations $y = 3x^2 + 2x - 1$ and $y = 1 - 3x$. [4]

9

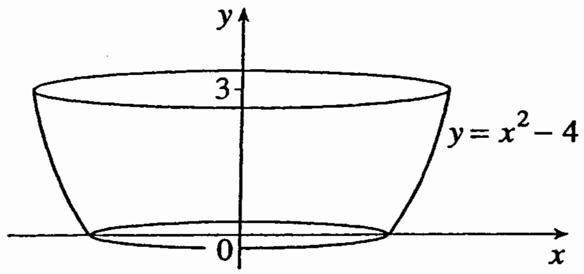


Fig. 9

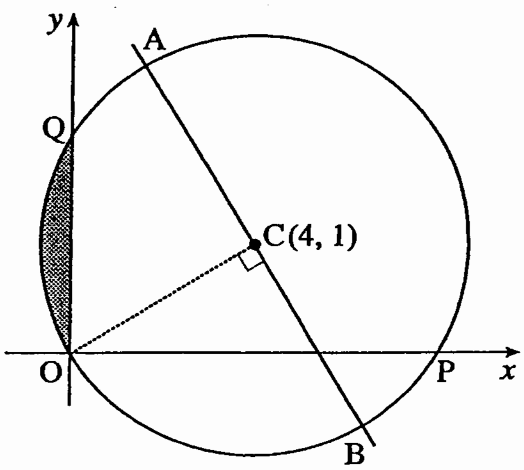
The shape of a bowl is made by rotating the curve $y = x^2 - 4$ between $y = 0$ and $y = 3$ about the y-axis as shown in Fig. 9.

Calculate $\pi \int_0^3 x^2 \, dy$ and say what this represents. [4]

Section B (30 marks)

10 (i) Show that the equation $x^2 - 8x + y^2 - 2y = 0$ represents a circle with centre (4, 1). Find the radius of this circle. [4]

(ii) This circle passes through the origin. It crosses the x- and y-axes at P and Q respectively. Find the coordinates of P and Q. [2]



Not to scale

Fig. 10

Fig. 10 shows the circle. C is the point (4, 1) and O is the origin. The diameter ACB is perpendicular to OC.

(iii) Find, in the form $y = mx + c$, the equation of the line through ACB. [4]

(iv) Show that angle OCQ is 0.49 radians to 2 significant figures and calculate the area of the segment of the circle that is shaded in Fig. 10. [5]

[Total 15]

11 In this question, $f(x) = x^3 - 3x^2 - 6x + 8$.

(i) Show that $x - 1$ is a factor of $f(x)$.

Factorise $f(x)$ completely and hence sketch the graph of $y = f(x)$. [7]

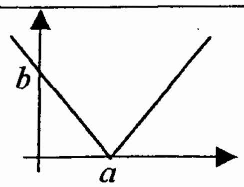
(ii) Find the equation of the tangent to the curve $y = x^3 - 3x^2 - 6x + 8$ at the point where $x = 3$. [4]

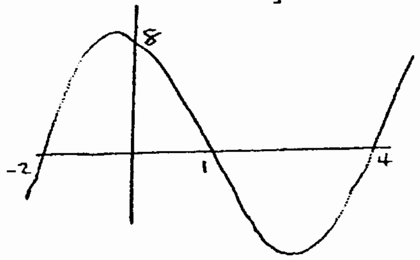
(iii) Use calculus to find the x-coordinates of the turning points on the curve

$$y = x^3 - 3x^2 - 6x + 8.$$ [4]

[Total 15]

Mark Scheme

	Section A			
1	$\frac{2}{5}x^5 + c$	B1,B1		2
2.	251-252	B2	M1 for $\frac{7\pi}{5} \times \frac{180}{\pi}$ or other ans 251-253	2
3.	sketch of line of positive or negative gradient from (a, 0), a>0 [ignore continuing for $y < 0$]; second arm of V (accept for any a) [ignore continuing below x axis; $a = 1.5$ and $b = 3$ shown, cao – V clearly chosen and reasonably symmetrical	G1 G1 G1	 condone 1 and 2 marked on x-axis and line halfway between	3
4.	$1 - 8x + 24x^2 - 32x^3 + 16x^4$ i.s.w	B4	B3 for 4 terms correct or for correct terms except for signs; B2 for 3 terms correct or B1 for 1 4 6 4 1 s.o.i.	4
5.	60, 120, 240, 300	B4	1 each;condone $\pi/3$, $2\pi/3$, $4\pi/3$, $5\pi/3$ –1 once in qn for: deg: 59.9, 120.1, 239.9, 301.1 o.e. rads: 1.05, 2.09, 4.19, 5.24 grads: 66.7, 133, 267, 333 –1 once from B4 for extra answers in range	4
6.	$a^2 + b^2 = c^2$ $\frac{a^2+b^2}{c^2} \to \frac{a^2}{c^2} + \frac{b^2}{c^2}$ or v.v. $\sin\theta = a/c$; $\cos\theta = b/c$ P →A → T or v.v.	P1 A1 T1 M1	Pythag statement Algebra Trig Complete method	4
7.	3.2	B3	M2 for $0.5/2 \times \{2 + 3.8 + 2(1.2 + 2.3)\}$o.e.; M1 for $k/2 \times \{2 + 3.8 + 2(1.2 + 2.3)\}$o.e. , $k \neq 0.5$ or for two separate correct traps, or correct formula with wrong / omitted brackets. B1 for 6.4 or 1.6	3
8.	(1/3, 0) or (–2,7)	B4	M1 for equating or subst for x M1 for $(3x - 1)(x + 2)=0$ or formula used correctly or for $y^2 - 7y = 0$ A1 for 1 pair of coords correct, or both x coords. or both y coords SC2 for both (–1/3, 2) and (2, –5) or both (–1/3, –4/3) and (2, 15)	4
9.	$(\pi)\int_0^3 [y + 4]dy$ $\left[\frac{y^2}{2} + 4y\right]_0^3$ = 16.5π or 51.8... volume [of bowl]	M1 A1 A1 E1	for subst of $y + 4$ for x^2 condone omission of limits and / or π for both Ms SC1 for $y^2/2 - 4y$ following $y - 4$ o.e. accept 'volume of revolution'	4
			Total Section A	30

	Section B				
10.	(i)	attempt at $(x-4)^2 + (y-1)^2$ $(x-4)^2 + (y-1)^2 = 17$ or $x^2 - 8x + 16 + y^2 - 2y + 1 = r^2$	M1 A2	B1 for correct expn. of $(x-4)^2$ or $(y-1)^2$	
		radius = $\sqrt{17}$ or 4.1(23..)	B1		
		or			
		clear use of $x^2 + 2gx$ ... eqn.	M1		
		centre is $(-g, -f) = (4, 1)$	A1	NB answer given	
		use of $r = \sqrt{(g^2 + f^2 - c)}$	M1		
		radius = $\sqrt{17}$ or 4.1(23..)	B1		4
	(ii)	P = (8,0)	B1	for 8	
		Q = (0,2)	B1	for 2; condone wrong labels	2
	(iii)	$y = -4x + 17$	B4	B1 for [grad OC] = ¼ s.o.i. M1 for grad AB = -4 or ft [implies previous mark if correct] M1 for $(y-1)$ = their -4 $(x-4)$ or $1 = 4$(their $-4) + c$	4
	(iv)	tan (0.5OCQ) = ¼ o.e.	M1	or correct use of cos rule NB M0 for $\theta = 2/\sqrt{17}$	
		$1/2 \times r^2 \times 0.49$	M1		
		$1/2 \times r^2 \times \sin 0.49$	M1	or ½ × 4 × 2	
		sector – triangle	M1	ft their clear attempts	
		0.16(4..) or 017	A1		5
11.	(i)	f(1) used	M1	or M1 long div as far as $x^2 + kx$	
		f(1) = 0 shown calculated	A1	A2 long div as far as $x^2 - 2x - 8$	
		one of $(x-4)$ and $(x+2)$	B1	B2 for $(x-4)$ and $(x+2)$	
		the other	B2	[mixed methods: mark one or other to adv of cand.]	
		sketch of correct shape – any cubic through (1, 0), correct way up and extending beyond axes	G1		
		at least two intns marked, correct or ft from their factors	G1		7
	(ii)	$f'(x) = 3x^2 - 6x - 6$	M1	allow where seen in any part	
		$x = 3$ grad = 3	A1	NB not B mark	
		$x = 3, y = -10$	B1		
		$y = 3x - 19$ o.e.	B1	eg B1 for $y + 10 = 3(x-3)$ isw	4
	(iii)	$f'(x) = 0$ s.o.i.	M1	allow marks for (iii) in (i)	
		use of formula in their $x^2 - 2x - 2 = 0$	M1	eg $\frac{2 \pm \sqrt{4+8}}{2}$	
		$1 \pm \sqrt{3}$ or $-0.7(3.)$. and 2.7(3..)	A2	1 each NB not B marks	4
				Total Section B	30

Examiner’s Report

2601 Pure Mathematics 1

General Comments

The full range of marks on this paper proved to be accessible to candidates, with full marks being scored occasionally and many excellent scripts seen. However, there were also, as last year, weak candidates being entered by some centres. Such candidates, for instance those scoring less than 10 out of 60, had little idea of how to attempt questions at this level.

The lack of basic manipulative algebra skills of some candidates was evident in questions such as Q.6, Q.8 and Q.10. A number of candidates also lost marks through not having learnt formulae thoroughly.

All candidates had sufficient time to complete the paper. Candidates used the accessibility of different parts of the long questions well, with failure to factorise the cubic in Q.11 often followed by full marks for later parts, for example.

Comments on Individual Questions

Q.1 Well done, but even some good candidates omitted the constant of integration. A few confused the rules for differentiation and integration.

$\frac{2}{5}x^5 + c$.

Q.2 Many who knew that they had to multiply by $\frac{180}{\pi}$ omitted the π from $\frac{7\pi}{5}$, but the correct answer was frequently seen.

252.

Q.3 Many ignored the modulus and drew only $y = 2x - 3$. Some stopped their graphs at the y – axis.

Graph of $y = |2x - 3|$.

Q.4 There were many accurate solutions, although some needed to multiply out the brackets, not recognising the use of the binomial expansion. The common error was to use $(2x)^3 = 2x^3$ etc.

$1 - 8x + 24x^2 - 32x^3 + 16x^4$.

Q.5 Most candidates gave two solutions, usually 60 and 120, although occasionally 60 and 300. Only better students recognised the possibility of the negative square root.

60, 120, 240, 300.

Q.6 There were some excellent presentations of this proof, but it was beyond many candidates, who were unable to progress beyond correct statements for Pythagoras' theorem and trigonometric ratios. There were many errors in algebraic manipulation of fractions.

Q.7 This was a simple application of the trapezium rule, which most candidates completed correctly, although problems in dealing with brackets were also frequent.

3.2.

Q.8 Some did not know how to start this problem. Of those who chose the simpler method of equating the y expressions, many made a mistake in the next line in rearranging the equation. Others who correctly obtained the x values frequently omitted the y values, although good solutions were often seen from the better candidates. Those who substituted for y usually made errors in coping with the fractions – correct solutions from this method were very rare.

$\left(\frac{1}{3}, 0\right)$ or $(-2, 7)$.

Q.9 Weaker candidates often obtained $\pi\frac{x^3}{3}$ as the integral. Better candidates usually coped well and there were many correct solutions. Interpretation of the integral as representing the volume was usually correct, but by no means always so.

16.5π or 51.8..., volume of bowl.

Q.10 Full marks were obtained on Q.10 by an encouraging number of candidates, who were able to apply a range of techniques accurately.

(i) Some completed the square and convincingly obtained $(x-4)^2+(y-1)^2=17$. Many more began with $(x-4)^2+(y-1)^2=r^2$ and expanded to $x^2-8x+16+y^2-2y+1=r^2$. They were then often unsure what to do with the 17 and r^2, sometimes obtaining $r^2=-17$.

(ii) The coordinates of P and Q were usually obtained correctly. Some used symmetry to state them directly, whilst some others took a whole page of calculation and the quadratic formula to obtain and solve $x^2-8x=0$.

(iii) Finding the equation of the line was usually done well.

(iv) Many candidates omitted the first part. The most popular method for finding angle OCQ was the cosine formula – surprisingly few used $\tan^{-1}\left(\frac{1}{4}\right)$ to find the half angle. Most candidates appreciated that to find the area of the segment they needed the difference between the sector and triangle areas, although some confused segment and sector. The sector area was often found correctly but the triangle calculation was often wrong or clumsy, with few using $\frac{1}{2}\times4\times2$.

(i) radius = $\sqrt{17}$, (ii) P = (8, 0) Q = (0, 2), (iii) $y=-4x+17$, (iv) 0.165.

Q.11 This was well-received by the majority of candidates with many scoring full marks.

(i) Many candidates calculated f(1) successfully and proceeded to attempt to find the other factors by various methods. Of these, the least successful was for those who wrote down the missing quadratic factor as ax^2+bx+c and became embroiled in simultaneous equations instead of using the more sensible x^2+Ax-8. Many candidates attempted poor plottings of the graph instead of the sketch that was requested. Weaker candidates sometimes interpreted the factor $(x+2)$ as implying the intersection (2, 0).

(ii) Some weaker candidates could not find the gradient of the tangent, but the majority successfully differentiated and found $f'(3)=3$. A small minority then went on to use $-\frac{1}{3}$ as the gradient.

(iii) The majority knew they had to solve $f'(x)=0$ and many were successful. There were many slips in using the quadratic formula and some weaker candidates mistakenly found rational factors. Some candidates went beyond the request to find also the value of the y-coordinates – clearly they had sufficient time for this. Some better candidates found these turning points earlier, to assist them in their graph sketch.

(i) $(x-1)(x-4)(x+2)$ found, sketch of this cubic, (ii) $y=3x-19$, (iii) $1\pm\sqrt{3}$ or −0.73 and 2.73.