OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2608/1

Mechanics 2

Thursday **23 MAY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\,\text{m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages and 2 blank pages.

1

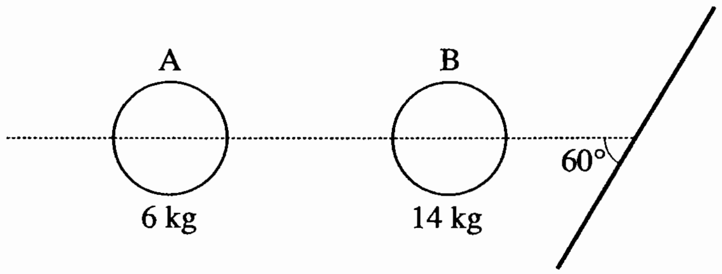


Fig. 1

Two circular discs slide on a smooth horizontal surface. Disc A has mass 6 kg and disc B has mass 14 kg and both are initially at rest. A force of 12 N acts on disc A for 4 seconds and this disc then collides directly with disc B. The coefficient of restitution between the two discs is 0.25.

(i) Calculate the velocity of disc A before the collision. [2]

(ii) Show that, after the collision, disc B has speed $3\,\text{m}\,\text{s}^{-1}$ and find the new speed of disc A. [6]

(iii) Calculate the impulse on disc A in this collision. [2]

Disc B now collides with a smooth plane at 60° to the line of its motion, as shown in Fig. 1. The speed of the disc after the collision is $1.6\,\text{m}\,\text{s}^{-1}$.

(iv) Calculate the coefficient of restitution between the disc and the plane. [5]

[Total 15]

2

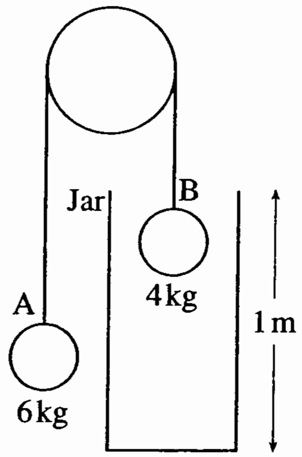


Fig. 2

Two objects A and B are connected by a light, inextensible string which passes over a smooth fixed pulley. The mass of A is 6kg and of B is 4kg.

The object B is inside a fixed empty jar 1 m tall. At no time does B reach the pulley or touch the sides of the jar.

Object B is held at rest and then the system is released. Object A falls a distance of 0.8m.

(i) Calculate the change in gravitational potential energy of the system. [2]

(ii) Calculate the speed of the objects when A has fallen the 0.8m. [3]

The jar is now filled with liquid which may be assumed to provide a constant resistance of 12N to the motion of B. Object B is initially held at rest at the bottom of the jar and then the system is released. When A has dropped 0.8m, the string breaks at B.

(iii) Calculate

(*A*) the kinetic energy of B when the string breaks, [3]

(*B*) the distance B rises after the string breaks, [3]

(*C*) the speed of B when it reaches the bottom of the jar. [3]

[Total 14]

[Turn over

3

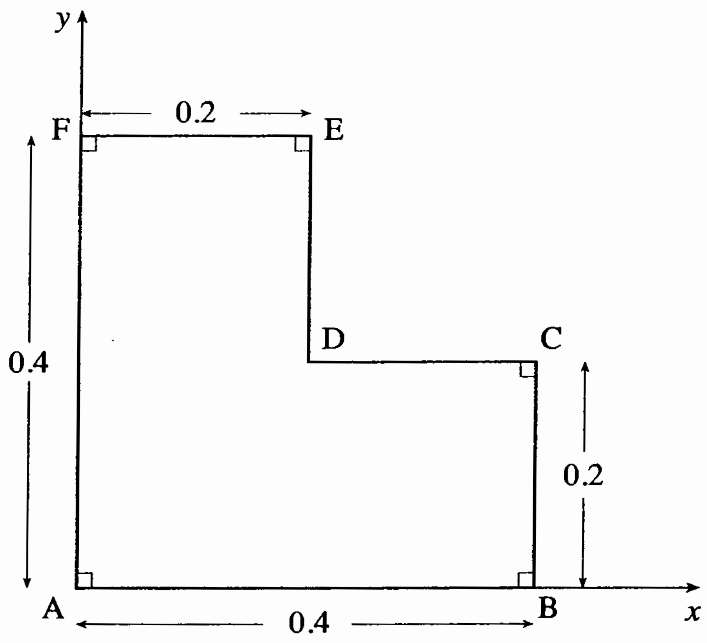


Fig. 3.1

In this question, the units on the axes are metres. Answers should be given in exact numbers or correct to three significant figures. Coordinates are referred to the axes shown in Fig. 3.1.

Fig. 3.1 shows a shape ABCDEF with the dimensions given.

(i) Regarding ABCDEF as a uniform lamina, show that the coordinates of its centre of mass are $(\frac{1}{6}, \frac{1}{6})$. [5]

(ii) Now regarding ABCDEF as a framework of uniform rods, show that the coordinates of its centre of mass are (0.175, 0.175). [5]

Question 3 continues on the next page.

A toolbox is shown in Fig. 3.2. This toolbox is a hollow prism with ends ABCDEF and GHIJKL perpendicular to its long axis. The toolbox is 0.6 m long and is made from thin, uniform material.

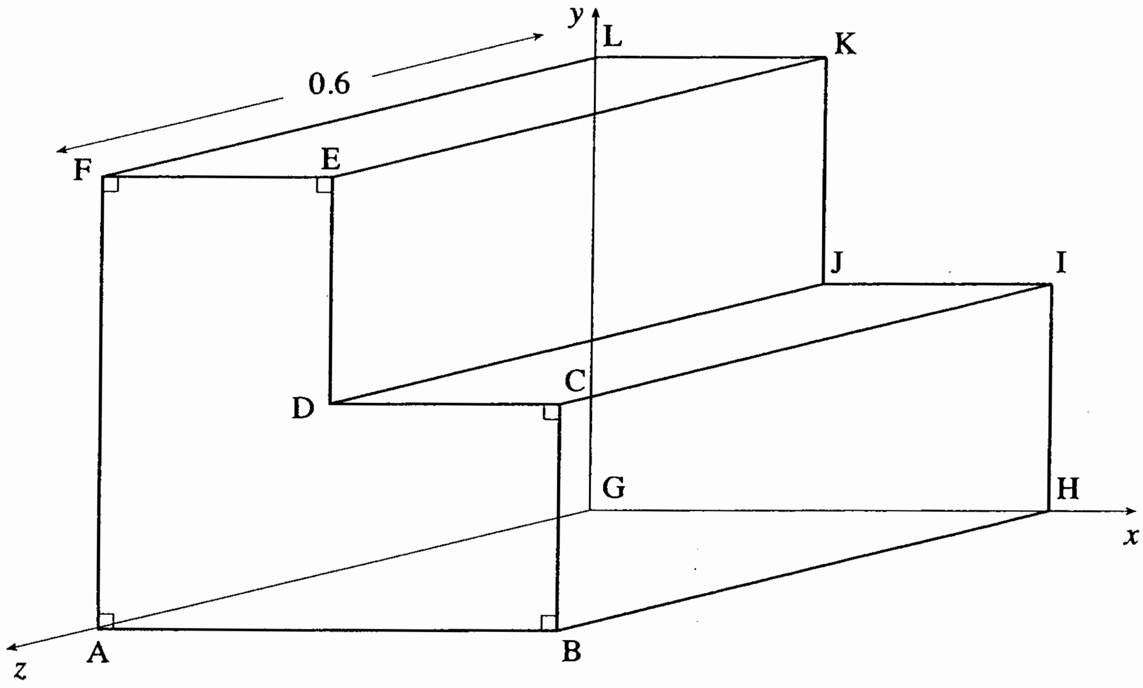


Fig. 3.2

(iii) By using your answers from parts **(i)** and **(ii)**, or otherwise, calculate the position of the centre of mass of the toolbox, referred to the axes shown in Fig. 3.2. [6]

[Total 16]

4 **(a)** A uniform rod AB of length 8 m and weight 180 N is held in horizontal equilibrium by two vertical wires. One wire is 1 m from A and the other 2 m from B, as shown in Fig. 4.1.

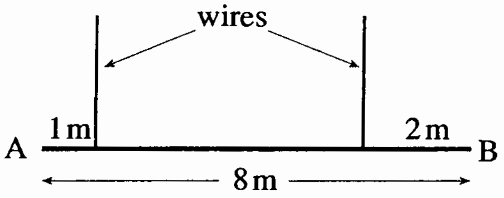


Fig. 4.1

(i) Draw a diagram showing all the forces acting on the rod. [1]

(ii) Calculate the tensions in the wires. [4]

(b)

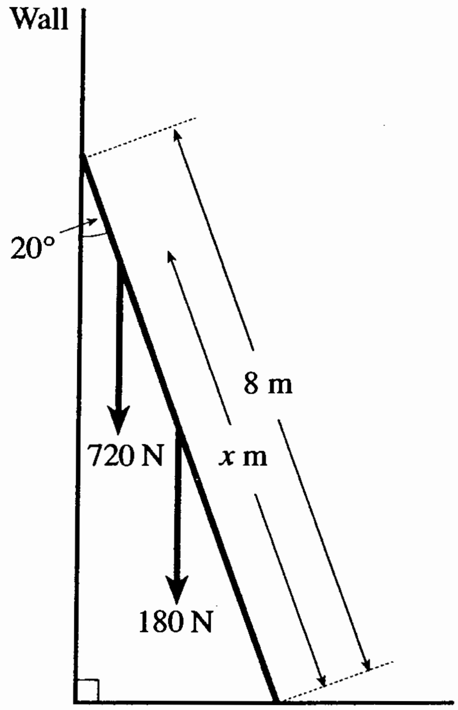


Fig. 4.2

A uniform ladder of length 8 m and weight 180 N rests against a smooth, vertical wall and stands on a rough, horizontal surface. A woman of weight 720 N stands on the ladder so that her weight acts at a distance x m from its lower end, as shown in Fig. 4.2.

The system is in equilibrium with the ladder at 20° to the vertical.

(i) Show that the frictional force between the ladder and the horizontal surface is F N, where

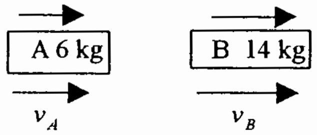
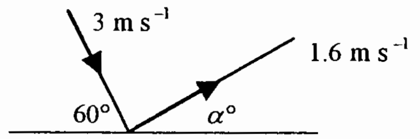
$$F = 90(1 + x)\tan 20°.$$ [5]

(ii) Deduce that F increases as x increases and hence find the values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. [5]

[Total 15]

Mark Scheme

Solutions and mark scheme

Q 1		Mark		
(i)	$6v = 12 \times 4$	M1	Impulse = Ft or $F = ma$ and $v = u + at$	
	$v = 8$ so 8 m s^{-1} in the direction of AB	A1	Direction not required	2
(ii)	before 8 0 			
	PCLM $48 = 6\ v_A + 14\ v_B$	M1	Use of PCLM	
		A1	Any form	
	NEL $\frac{v_B - v_A}{0-8} = -\frac{1}{4}$	M1	NEL. Accept sign errors.	
	$v_B - v_A = 2$	A1	Any form	
	Solving for v_B, $v_B = 3$	E1		
	$v_A = v_B - 2 = 1$	A1		6
			[If $v_B = 3$ assumed, M1 for NEL/PCLM and A1 for $v_A = 1$. Full marks if values justified in the other equation]	
(iii)	→ $6(1-8) = -42$ Ns opp direction to v_A	M1	Impulse.	
		A1	Direction must be clear. Accept –ve without explanation	2
(iv)				
	parallel to plane	M1	CLM parallel to the plane	
	$3\cos 60 = 1.6\cos\alpha$			
	$\alpha = 20.3641...$	A1		
	$e = \frac{1.6\sin\alpha}{3\sin 60}$	M1	Use of normal components	
		A1		
	$= 0.2143...$ so 0.214 (3 s.f.)	A1		5
			[Award M1 for attempt to use components]	
	If Pythagoras used then			tot
	$1.6^2 = (eu\sin 60)^2 + (u\cos 60)^2$	M1		15
		A1	Each term on RHS	
		A1		
	Correct substitution and manipulation	M1		
	$e = 0.214$	A1		

Solutions and mark scheme

Q 2		Mark		
(i)	GPE loss is $(6g-4g)\times 0.8 = 1.6g$ J (= 15.68 J)	M1 A1	GPE is *mgh* in any expression Accept either form	2
(ii)	$\frac{1}{2}(6+4)v^2 = 15.68$ so v = 1.7708... so 1.77 m s^{-1}	M1 B1 A1	Equate Δ KE to Δ GPE Use of 10 kg and 15.68 FT **their** GPE loss	3
	or Use of N2L Use of *uvast* to give a = 0.2g v = 1.7708... so 1.77 m s^{-1}	M1 M1 A1	Attempt to find acceleration. [M0 for $a = g$] Must be appropriate. Accept a wrong but not $a = g$.	
(iii) (A)	$15.68 - 0.8\times 12 = \frac{1}{2}\times 10\times v^2$	M1	Work-energy including WD against resistance, GPE and KE. Accept sign error.	
	$v^2 = 1.216$	A1	FT from (i)	
	KE of B is $\frac{1}{2}\times 4\times v^2 = 2.432$ J	A1	FT from (i)	3
	or N2L and *uvast* to give accn 0.76 m s^{-2} $v^2 = 1.216$ KE 2.432 J	M1 A1 A1	N2L must be used. Resistance must be included.	
(B)	Suppose B rises an extra distance x $2.432 = 12x + 4\times 9.8\times x$ so x = 0.0475	M1 B1 F1	Trying to find this distance. Award without weight term All terms present. Accept sign error. Accept **their** KE from (A) FT	3
	or N2L $-12-4g = 4a$ a = -12.8 x = 0.0475	M1 A1 A1		
(C)	For downward motion $\frac{1}{2}\times 4\times v^2 = 0.8475g\times 4 - 12\times 0.8475$ v = 3.39499... so 3.39 m s^{-1} (3 s.f.)	M1 B1 A1	Work-energy equation, all terms present. Signs correct. Use of 0.8 + **their** x cao	3
	or s = 0.8475 N2L and *uvast* giving accn 6.8 m s^{-2} v = 3.39499... so 3.39 m s^{-1} (3 s.f.)	B1 M1 A1	Award for 0.8 + **their** x N2L and appropriate *uvast* cao	Tot 14

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Solutions and mark scheme

Q 3		Mark		
(i)	$\bar{x} = \bar{y}$	B1	Symmetry; must be explicit.	
	For $\bar{x}$	M1	Method for c.m.	
	$(0.4^2 - 0.2^2)\bar{x} = 0.4^2 \times 0.2 - 0.2^2 \times 0.3$	B1	All masses correct	
		A1	At most one c.m. wrong. If vector used award if not more than one error in either x or y component	
	c.m. is at $\left(\frac{1}{6}, \frac{1}{6}\right)$	E1	[Award 1st B1 if other component calculated separately with same answer]	5
(ii)	$1.6\bar{x} = 0.4 \times 0.2 + 0.2 \times 0.4 + 0.2 \times 0.3 + 0.2^2 + 0.2 \times 0.1$	M1	Method for c.m.	
		B1	Masses correct	
		B1	At least two c.m. correct. If vector used then at least two c.m. have either x or y cpts correct.	
	$\bar{x} = \frac{0.28}{1.6} = 0.175$	E1		
	so (0.175, 0.175)	B1	Symmetry; must be explicit. [Award if other component calculated separately with same answer]	5
(iii)	For $\bar{x}$ of composite figure	M1	Method could lead to solution. Allow up to 2 parts of the figure missing.	
	$(1.6 \times 0.6 + 2 \times 0.12)\bar{x}$	B1	Total mass	
	$= 1.6 \times 0.6 \times 0.175 + 2 \times 0.12 \times \frac{1}{6}$	B1	First term [If divided into plane parts, award for 4 or equivalent parts correct]	
		B1	Second term [If divided into plane parts, award for 6 or equivalent parts correct]	
	so $\bar{x} = \frac{0.208}{1.2} = 0.17\dot{3}$ $\left(= \frac{13}{75}\right)$	A1	cao. Award for x or y correct	
	so (0.173, 0.173, 0.3)	F1	$\bar{y}$ or $\bar{z}$ by symmetry (may be implied)	6
				Tot 16

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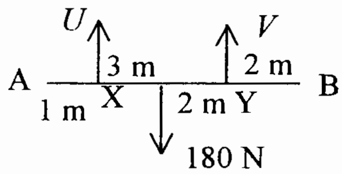
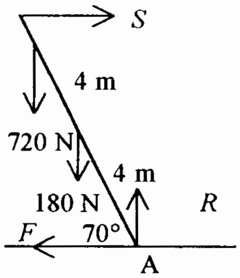
Solutions and mark scheme

Q 4		Mark		
(a) (i)		B1	Accept no or wrong dimensions Must have correct labels and arrows	1
(ii)	Moments about X			
	$3 \times 180 - 5v = 0$	M1	moments	
	$v = 108$ so 108 N	A1		
	Resolve vertically			
	$U + V = 180$	M1	Or take moments again	
	so $U = 72$ so 72 N	A1	[Deduct one mark if $180g$ instead of 180]	4
(b) (i)				
	$\rightarrow \quad F = S$	B1	Award for $R = 900$ if moments taken about top of ladder	
	$\hat{A} \quad 180 \times 4\cos 70 + 720 \times x\cos 70$	M1	Moments equation using angles with at least 2 terms	
		A1	One correct term	
	$= 8S\sin 70$	A1	Second correct term	
	$\Rightarrow F = 90\tan 20(1 + x)$	E1	Clearly shown [Award 4/5 if $F = S$ not established by resolution]	5
(ii)	All other terms constant so $F \uparrow$ as $x \uparrow$	E1	Clearly explained.	
	Hence worst case is $x = 8$	E1	[Award if $x = 8$ used without explanation]	
	giving $F = 810\tan 20$ with $R = 900$	B1	Values of both R and F.	
	Since $F \le F_{max} = \mu R$,	M1	Use of $F = \mu R$	
	$\mu \ge \frac{810\tan 20}{900} = \frac{9\tan 20}{10}$ $(= 0.3275...)$	A1	cao. Inequality required. Accept >. Accept inequality stated and not justified.	5
				Tot 15

Examiner’s Report

2608 Mechanics 2

General Comments

This paper appeared to be accessible to the majority of candidates and a high proportion of them were able to attempt every question. There was some evidence that weaker candidates found the paper long but this was often due to their use of inefficient methods in both Q2 and the last part of Q3. The presentation of scripts was generally good with many candidates setting out work clearly, a notable improvement on some previous sessions. A small but significant minority of candidates did not use adequate diagrams; this was particularly noticeable in Q4 where forces were omitted, not labelled or not attached to anything.

Comments on Individual Questions

Q.1 On the whole this question was answered well with a significant proportion of students obtaining good marks.

Part (i) was often done well but a few candidates failed to use $\mathbf{I} = \mathbf{F}t$ and instead used Newton's second law and $v = u + at$.

Part (ii) was answered well by most of the candidates but, as often happens in the absence of good diagrams, sign errors in the application of Newton's experimental law prevented many from establishing the given result.

The calculation of impulse in part (iii) was done well except that a large proportion of the candidates did not make the vector nature of the impulse explicit by giving a direction.

The last part of the question was often poorly done. Many candidates obviously did not understand the ideas involved with oblique impact and simply took the ratio of the speeds after and before the impact. Others realised that some resolution was required but did not know exactly what to do and just assumed that the approach and separation directions were equally inclined to the plane.

(i) 8 ms^{-1}; (ii) 1 ms^{-1}; (iii) 42 N s in the opposite direction to the initial velocity of disc A;
(iv) 0.214 (3 s. f.).

Q.2 This was the least well answered of the questions. Most candidates either did very well or quite badly with the majority in the latter category. It was common to see candidates use a mixture of methods involving energy and Newton's second law with little progress being made; generally candidates are better off using one approach or the other. Many candidates failed properly to analyse what was going on. A very common mistake was to assume that the acceleration of the system was g in every part of the motion; there were also many errors made because of confusion about when the mass of a part or of the whole system was required. Some other misconceptions about the relationship of work and energy were revealed.

(i) 15.68 J; (ii) 1.77 ms^{-1} (3 s.f.); (iii) (A) 2.432 J; (B) 0.0475 m; (C) 3.39 ms^{-1} (3 s.f.).

Q.3 The first two parts of this question caused very few problems to the candidates and the majority did well. Many did not argue from symmetry but instead worked out the centres of mass separately for the x and y directions; a minority did not explicitly say why the x and y components are the same but simply wrote them down as being so. The last part of the question caused some difficulties for all but the most able who had usually successfully used the hint in the question. Many students did not use the hint and split up the box into all of its component parts, a daunting task that led to a number of unforced errors. Others considered the box to be two plane ends joined by a framework or considered the box to be a solid or considered it to be a framework covered by a lamina.

(iii) (0.173, 0.173, 0.3) (3 s. f.).

Q.4 Many candidates scored well on this question, particularly those who drew clear diagrams and explained what they were doing at each stage of their solution. A few candidates took the weight to be180g but most of them read the question correctly.

As the result was given in part (ii), some candidates succumbed to the temptation to 'fiddle' their expressions instead of looking for their error(s); the most common mistakes were to take moments about the top of the ladder but omit the normal reaction at the ground, or to attempt to use tan70 or tan 20 throughout instead of resolving in the usual way. However, it was pleasing that there were fewer attempts to deceive than have been seen in recent sessions. Whether candidates took moments about the top or the bottom of the ladder, it was necessary to obtain at least one other equation and candidates were penalised if this equation was not given explicitly with a reason but just used in a substitution.

Few candidates scored full marks on the last part of the question as many of them could not adequately explain in a proper mathematical way why the friction increases as x increases. It was common to see the statement that F is proportional to x. Few candidates could deal convincingly with the inequality.

(a)(ii) 108 N and 72 N; (b)(ii) $\mu \geqslant 0.328$ (3 s. f.).