OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Thursday **30 MAY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- There is an **Insert** for use in Questions **1** and **3**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages, 2 blank pages and an insert.

[Turn over

Section A

1 [There is an insert for use in this question.]

The weights on the network in Fig. 1 represent distances.

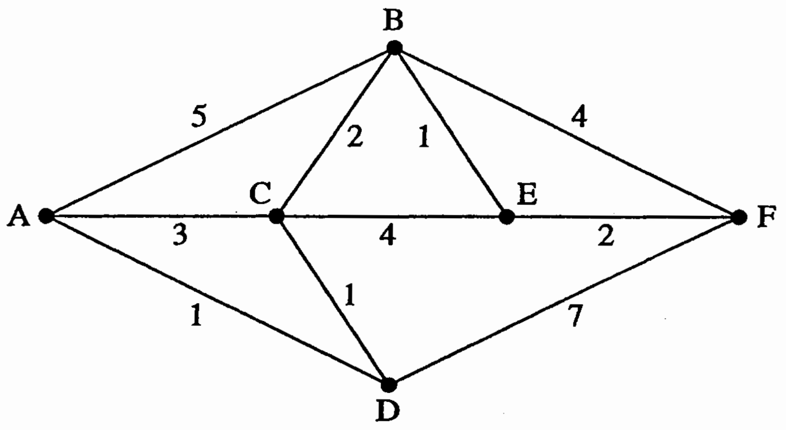

Fig. 1

(i) Using the insert, apply Dijkstra's algorithm to find the shortest route from A to F. [4]

(ii) Give the shortest route and its length. [1]

[Total 5]

2 The following steps define an algorithm which operates on two numbers.

Step 1: Write down the two numbers side by side on the same line.

Step 2: Beneath the left-hand number write down double the left-hand number.
Beneath the right-hand number write down half of the right-hand number, ignoring any remainder.

Step 3: Repeat Step 2 until the right-hand number is 1.

Step 4: Delete those rows where the number in the right-hand column is even.
Add up the remaining numbers in the left-hand column. This is the result.

(i) Apply the algorithm to the numbers 50 and 56. [3]

(ii) Use your result from part (i), and any other simpler examples you may choose, to write down what the algorithm achieves. [1]

[Total 4]

3 [There is an insert for use in this question.]

Six tasks, A, B, C, D, E and F, are to be carried out by six people, numbered 1, 2, 3, 4, 5 and 6. Each task requires a different combination of people to carry it out.

Table 3 shows for which tasks each person is required.

Person	1	2	3	4	5	6
Tasks	A	C	A	D	B	B
	C	D	E	E	E	C
	F	F	F	F	F	F

Table 3

Each person can only work on one task at a time, and this constrains which tasks can be under way at the same time.

(i) The situation is to be represented by a graph in which vertices represent tasks. Two vertices are to be joined if the tasks they represent require a person in common. For example, A and C should be joined because they both need person 1.

Complete this graph in part (i) of the insert. [2]

(ii) On part (ii) of the insert draw the complement graph, i.e. the graph with the same vertices, but with those arcs which are NOT in your graph in part (i). [2]

(iii) Use the complement graph to organise the tasks in an efficient way. [2]

[Total 6]

[Turn over

Section B

4 A small ferry boat has spaces for 12 passengers. It arrives at an isolated beach at 3 p.m. every afternoon, and sets off back to the neighbouring holiday resort at 4 p.m. For each of the four consecutive 15 minute periods that it waits, the number of people arriving at the boat is a realisation of a random variable. These random variables are X_1, X_2, X_3 and X_4 respectively. Their distributions are given in the tables.

3:00 – 3:15

x	0	1	2	3	4
$P(X_1 = x)$	0.2	0.2	0.3	0.2	0.1

3:15 – 3:30

x	0	1	2	3	4
$P(X_2 = x)$	0.1	0.2	0.2	0.3	0.2

3:30 – 3:45

x	0	1	2	3	4	5
$P(X_3 = x)$	0	0.1	0.1	0.2	0.3	0.3

3:45 – 4:00

x	0	1	2	3	4	5	6
$P(X_4 = x)$	0	0	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

(i) Give a rule to use 2-digit random numbers to simulate realisations of X_1. Use the following random numbers to simulate 5 realisations of X_1.

62 74 91 03 34 [2]

(ii) Give a rule to use 2-digit random numbers to simulate realisations of X_2. Use the following random numbers to simulate 5 realisations of X_2.

60 85 22 50 76 [2]

(iii) Give a rule to use 2-digit random numbers to simulate realisations of X_3. Use the following random numbers to simulate 5 realisations of X_3.

69 01 36 03 93 [2]

(iv) Give a rule to use 2-digit random numbers to simulate realisations of X_4. Use the following random numbers to simulate 5 realisations of X_4.

33 01 99 75 73 89 [3]

(v) Use your values from parts (i) to (iv) to simulate the number of passengers the boat carries back on each of 5 afternoons.

Hence estimate the mean number carried back.

Estimate also the mean number of people per afternoon who fail to secure a place in the boat. [5]

(vi) How might you improve your estimates in part (v)? [1]

[Total 15]

5 A construction project involves nine activities. Their immediate predecessors and durations are listed in Table 5.1.

Activity	Immediate predecessors	Duration (days)
A	–	5
B	–	3
C	–	6
D	A	2
E	A, B	3
F	C, D	5
G	C	1
H	E	2
I	E, G	4

Table 5.1

(i) Draw an activity-on-arc network for the project. [5]

(ii) Show on your network the early time and the late time for each event. [4]

(iii) Give the critical activities and the minimum duration of the project. [2]

(iv) Some activity durations can be reduced at extra cost, as shown in Table 5.2.

Activity	Amount by which duration can be reduced	Extra cost
A	1 or 2 days	£1000 per day
B	1 or 2 days	£500 per day
C	1 or 2 days	£600 per day
D	1 day	£250
E	1 day	£250

Table 5.2

Show that the cheapest way of reducing the minimum project duration by one day involves reducing the durations of **two** activities. Give these activities and the extra cost.

Show how to reduce the minimum project duration by a further day, and give the extra cost of this reduction. [4]

[Total 15]

6 A company bottles and sells mountain spring water. It produces one-litre bottles which sell for 50p, and half-litre bottles which sell for 30p.

The spring produces 5000 litres per day.

There is a demand for up to 4000 one-litre bottles per day, and up to 3000 half-litre bottles per day.

The company wishes to maximise its income from the sale of bottles of water.

(i) Choose variables for the number of one-litre bottles and the number of half-litre bottles the company should produce per day.

Give an expression for the daily income in terms of your variables.

Write down an inequality which models the constraint imposed by the availabillity of spring water.

Complete the formulation of a linear programming problem to find how many of each type of bottle the company should produce per day. You will need two more inequalities for this. [5]

(ii) Use a graphical method to solve the problem. [6]

(iii) The company's marketing manager advises that half-litre bottles be reduced in price to 28p to increase demand for half-litre bottles. Verify that the company must sell more than 5000 half-litre bottles per day for income to be increased. [2]

(iv) After a wet winter the spring produces more water. At the original prices, how much extra water per day could the company use, and how much extra income would this generate? [2]

[Total 15]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

INSERT

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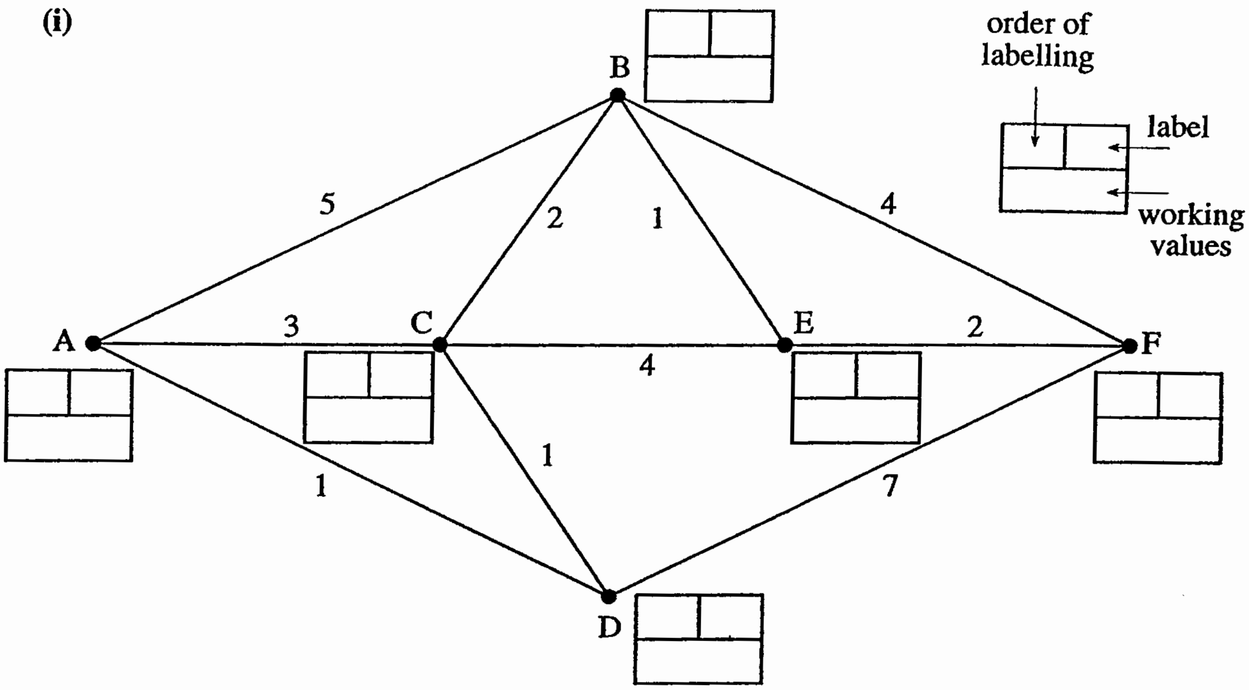
INSTRUCTIONS TO CANDIDATES

- This insert should be used in Questions **1** and **3**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

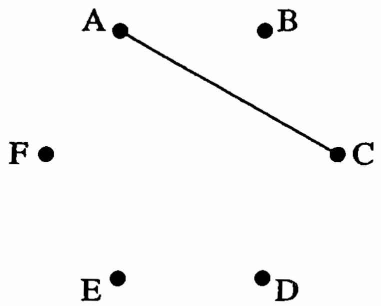
1 (i)



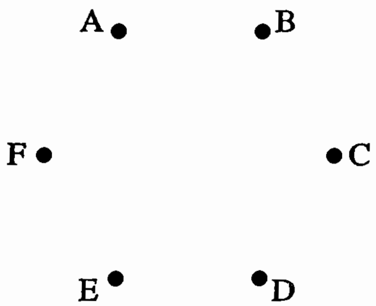
(ii) Shortest route: ....

Length: ....

3 (i)



(ii)



(iii) ....

....

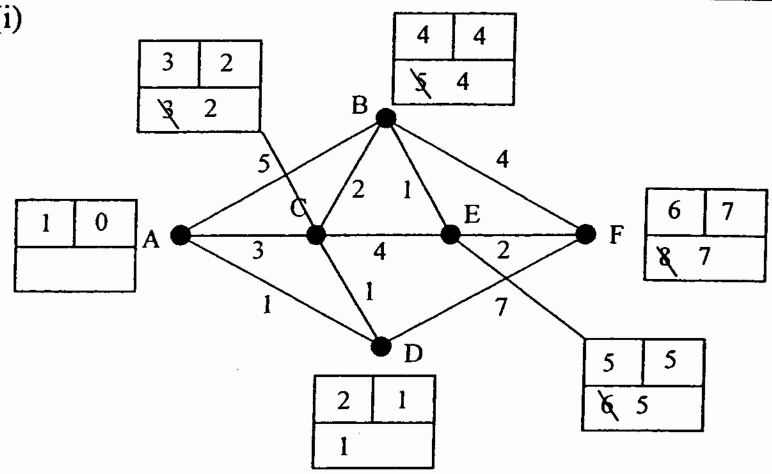
....

....

Mark Scheme

1.

(i)



(ii) ADCBEF 7

M1 Dijkstra
A1 working values at D, B, C
A1 working values at E, F
A1 labels

B1 ADCBEF

2.

(i)

~~50~~	~~56~~
~~100~~	~~28~~
~~200~~	~~14~~
400	7
800	3
1600	1

2800

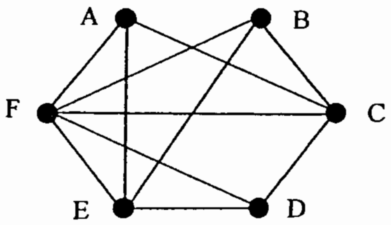
(ii) multiplication

M1 sca
A1 halving and doubling
A1 crossing and adding

B1

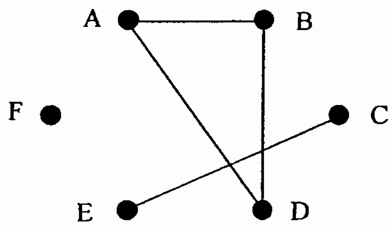
3.

(i)



M1
A1

(ii)



M1
A1

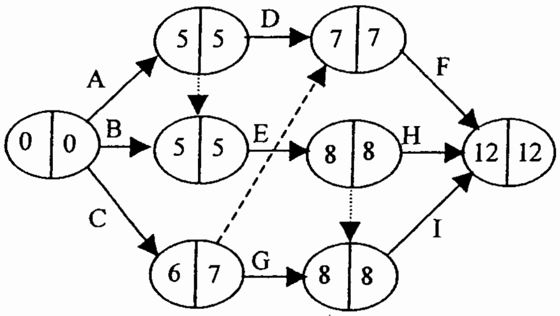
(iii) Schedule ABD together and schedule C and E together.

B1 B1

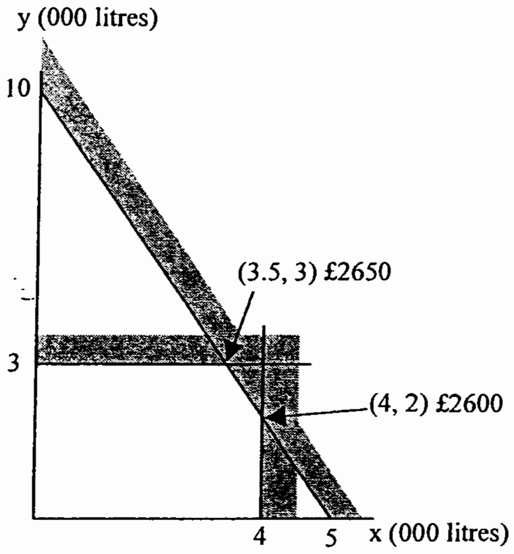
4.

(i) 00–19 → 0 20–39 → 1 40–69 → 2 70–89 → 3 90–99 → 4	B1
2, 3, 4, 0, 1	B1
(ii) 00–09 → 0 10–29 → 1 30–49 → 2 50–79 → 3 80–99 → 4	B1
3, 4, 1, 3, 3	B1
(iii) 00–09 → 1 10–19 → 2 20–39 → 3 40–69 → 4 70–99 → 5	B1
4, 1, 3, 1, 5	B1
(iv) 00–15 → 4 16–63 → 5 64–95 → 6 96, 97, 98, 99 ignore	M1 some ignored A1
5, 4, 6, 6, 6	A1
(v) 14, 12, 14, 10, 15 ↓ ↓ ↓ ↓ ↓ 12, 12, 12, 10, 12 giving a mean of 11.6	B1 B1 B1√
2, 0, 2, 0, 3 giving a mean of 1.4	B1 B1
(vi) more repetitions	B1

5.

(i) & (ii) 	M1 sca at activity-on-arc A1 single start and end + A, B, C A1 D, G, H M1 dummies A1 E, F, I M1 A1 forward pass M1 A1 backward pass
(iii) A, D, E, F, I 12 days	M1 A1 critical activities
(iv) To save 1 day crash D and E each by 1 day at cost of £500.	M1 A1
To save another day crash A and C at a cost of £1600.	M1 A1
(Total crashing cost = £2100.)	

6.

(i)	Let x be number (of thousands) of one litre bottles and y be ...	B1 explicit
	Max $500x + 300y$ s.t. $x + 0.5y \le 5$ $x \le 4$ $y \le 3$	M1 A1 objective A1 spring capacity A1 both
(ii)		M1 A1 line A1 line A1 line A1 shading
	Produce 3500 litre bottles and 3000 half litre bottles, giving a daily income of £2650.	A1
(iii)	Best soln. still at intersection of $x + 0.5y \le 5$ and $y = \ldots$ Thus y = 5(000) gives x = 2.5(000), producing an income of $(500\times 2.5) + (280\times 5) = 2650$	B1 reference to spring capacity B1 showing income regained
(iv)	Can use 5500 litres per day, giving a daily income of £2900 – £250 extra.	B1 B1

Examiner’s Report

2620 Decision and Discrete Mathematics I

General Comments

Despite the entry of a number candidates who were not in a position to perform well, the overall standard on this paper was good.

Comments on Individual Questions

Q.1 Over the years examiners for decision and discrete have come to expect to see a large proportion of candidates who are not able to demonstrate knowledge of Dijkstra's algorithm. This sitting was no exception. Correct applications of the algorithm produce the published solution, complete with the given working values and no others.

(ii) ADCBEF 7.

Q.2 This straightforward application of Russian peasant multiplication gave few candidates any problems in part (i). In part (ii) not all were able correctly to identify that it was an algorithm for multiplication.

(i) 2800
(ii) multiplication.

Q.3 This was a difficult question for section A, yet most candidates were able to pick up marks in parts (i) and (ii). Relatively few were able to see through part (iii).

(i) All arcs save AB, BD, DA and CE.
(ii) AB, BD, DA and CE
(iii) Schedule three sessions. In one session do F. In another session do C and E.
In the third session do A, B and D.

Q.4 Candidates who were prepared for this examination were, for the most part, able to score the first 9 marks for parts (i) to (iv). Thereafter the question proved to be surprisingly testing. It had been expected that in part (v) some candidates would be seen to be accumulating their answers within parts, rather than across parts, but it was surprising that so many did this. Such candidates tended to soldier on, even though they had only 4 results.

(i) 2, 3, 4, 0, 1, (ii) 3, 4, 1, 3, 3, (iii) 4, 1, 3, 1, 5, (iv) 5, 4, 6, 6, 6
(v) Boarding 12, 12, 12, 10, 12, with mean of 11.6 Left behind 2, 0, 2, 0, 3, with mean of 1.4.

Q.5 The activity network for this question was quite difficult. Candidates did well in managing its construction and analysis in parts (i), (ii) and (iii). In part (iv) fewer candidates demonstrated that the existing critical paths needed to be shortened to save one day. Fewer still realised that subsequently there needed to be a check for new critical paths.

(iii) A, D, E, F, I; 12 days
(iv) To save 1 day crash D and E each by 1 day at cost of £500.
To save another day crash A and C at a cost of £1600.

Q.6 In part (i) very few candidates earned the mark for identifying their variables, and many paid a greater price for their omission by consequential failures in formulation. The constraint on the availability of spring water caused many difficulties, with the half-litre variable often attracting a coefficient of 2 instead of 0.5.

Many candidates were able to get started on using the graphical approach, but not all were able to take it through to finding the vertex representing the optimal solution. Only better candidates were able to make significant progress with parts (iii) and (iv).

(ii) Produce 3500 litre bottles and 3000 half litre bottles, giving a daily income of £2650.
(iv) Can use 5500 litres per day, giving a daily income of £2900 – £250 extra.