OCR

RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Monday **21 JANUARY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

1 An engineer seeks to model the oscillatory motion of a suspension system as simple harmonic motion. She observes the motion of a point P which moves vertically about a mean position O. The displacement above O is x metres. At time t seconds the displacement of P above O is x metres. When $t = 0$, P is at its highest point.

The engineer records the distance between the highest and lowest points reached by P to be 0.1 m and the time for 20 oscillations to be 6.4 seconds.

(i) Write down the amplitude and the period of the motion. [2]

(ii) Calculate the velocity and acceleration of the point P when it is first 0.02 m below O, specifying the direction in each case. [6]

(iii) Sketch a graph of x against t for the first oscillation. [2]

(iv) Write down an expression for x in terms of t. Hence, or otherwise, calculate the time at which P first reaches the point 0.02 m below O while travelling upwards. [4]

[Total 14]

2 The end A of a light elastic string AB is fixed to a smooth horizontal table. A small bead of mass 0.05 kg is attached to the end B. Also attached to the end B is another light elastic string BC. The end C is fixed to a point on the table so that the strings lie in a horizontal straight line with the bead resting in equilibrium on the table and with AC equal to 2.3 metres, as shown in Fig. 2.

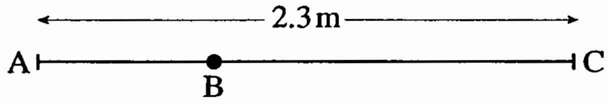


Fig. 2

The string AB has natural length 0.4 m and modulus of elasticity 4 N. The string BC has natural length 0.5 m and modulus of elasticity 2 N.

(i) The extension in AB is x_1 m and the extension in BC is x_2 m. Show that $5x_1 = 2x_2$ and calculate x_1 and x_2. [8]

The bead is held at A and released.

(ii) Determine whether the string BC goes slack. [4]

(iii) Calculate the maximum extension of the string AB. [3]

[Total 15]

3 A fairground ride consists of a number of small enclosed cars attached to a large wheel with centre O which rotates at constant angular speed ω, initially in a horizontal plane. Fig. 3.1 shows the wheel in a horizontal position and three of the cars. Each car is freely hinged at the point of attachment.

A person of mass m rides in one of the cars. Fig. 3.2 shows the forces acting on the rider. The reaction force exerted by the car on the rider, R, acts at ϕ to the vertical. The rider moves in a horizontal circle of radius r.

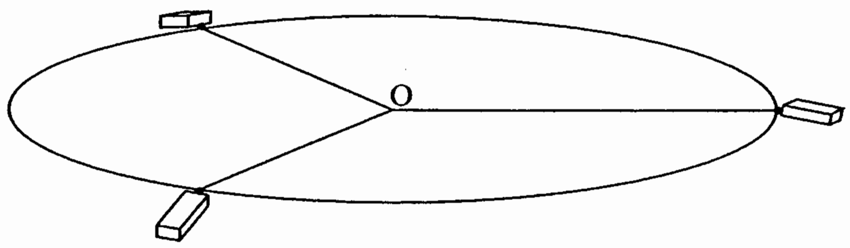


Fig. 3.1

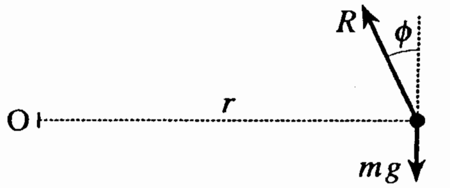


Fig. 3.2

(i) Write down the radial equation of motion and the vertical equilibrium equation for the rider. Show that each of these equations is dimensionally consistent. [6]

(ii) Given that the circle in which the rider moves has radius 10 m and and that the angular speed is 1.1 rad s^{-1}, calculate ϕ. [3]

During the ride, the wheel is moved into a vertical plane. The rider is now moving at a **constant** angular speed of 1.1 rad s^{-1} in a vertical circle of radius 10m. When the rider is at the point P, where OP is at an angle α to the horizontal, the reaction force of the car on the rider is at an angle θ to PO, as shown in Fig. 3.3.

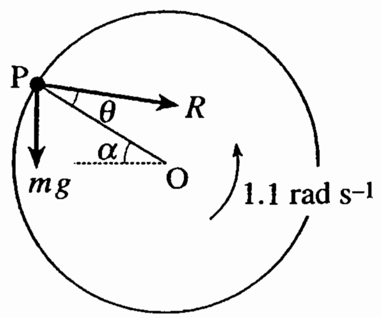


Fig. 3.3

(iii) Write down the radial and tangential equations of motion for the rider. Hence calculate the value of α for which the reaction force is horizontal. [6]

[Total 15]

[Turn over

4 A uniform lamina is in the shape of the quarter circle in the first quadrant bounded by the curve $x^2 + y^2 = 1$ as shown in Fig. 4.1.

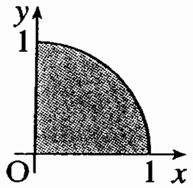


Fig. 4.1

(i) Show, by integration, that the y-coordinate of the centre of mass of the lamina is $\frac{4}{3\pi}$. (You may assume the standard result for the area of a circle). [5]

(ii) Write down the x-coordinate of the centre of mass. [1]

Another uniform lamina is in the shape of the region within the circle $x^2 + y^2 = 1$ in the second, third and fourth quadrants as shown in Fig. 4.2.

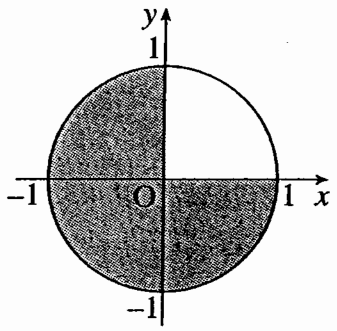


Fig. 4.2

(iii) Showing your method clearly, find the coordinates of the centre of mass of the lamina. [5]

This lamina is suspended by light vertical wires attached to the points A and B, where A is the point of the circumference on the line of symmetry and B is a point where a straight edge meets the circumference. The lamina hangs in a vertical plane with the axis of symmetry horizontal. This arrangement is shown in Fig. 4.3. The weight of the lamina is 50N.

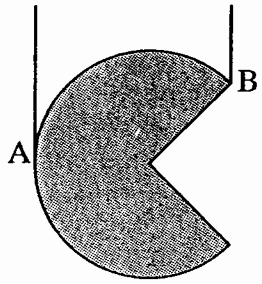
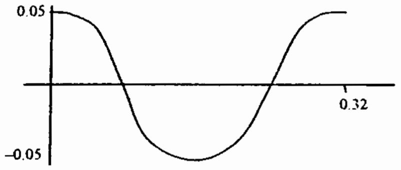


Fig. 4.3

(iv) Calculate the tensions in the wires, giving your answers correct to one decimal place. [5]

[Total 16]

Mark Scheme

1(i)	$a = 0.05$ m	B1		
	$T = 6.4/20 = 0.32$ s	B1		2
(ii)	$\omega = \frac{2\pi}{T} = 6.25\pi$	M1	2π/their T	
	$v^2 = (6.25\pi)^2(0.05^2 - (-0.02)^2)$	M1	use of formula (or calculate t and hence v)	
	$v = 0.900$ m s^{-1}	A1	cao	
	acc $= -\omega^2 x = -(6.25\pi)^2(-0.02)$	M1	use of formula (or calculate t and hence acc.)	
	$= 7.71$ m s^{-2}	A1	cao	
	velocity down, acceleration up	B1	both directions clearly stated	6
(iii)		B1 B1	shape scales	2
(iv)	$x = 0.05\cos 6.25\pi t$	B1	$a\cos\omega t$ or equivalent with their a, ω substituted	
	$-0.02 = 0.05\cos 6.25\pi t$	M1	equate to –0.02 and solve	
	$t = 0.32 - 0.101$	M1	solution corresponding to correct direction	
	$= 0.219$ s	A1	cao	4

Total 14

2(i)	$T_{AB} = \frac{4x_1}{0.4}$	M1 A1	use of Hooke's law	
	$T_{BC} = \frac{2x_2}{0.5}$	A1		
	$T_{AB} = T_{BC} \Rightarrow$	M1	equating tensions	
	$5x_1 = 2x_2$	E1		
	$0.4 + x_1 + 0.5 + x_2 = 2.3$	B1		
	$x_1 + x_2 = 1.4$			
	$x_1 + 2.5x_1 = 1.4$	M1	solving	
	$x_1 = 0.4, x_2 = 1$	A1	both	8
(ii)	initial energy $= \frac{2\times1.8^2}{2\times0.5} (= 6.48)$	B1	initial energy (need not be evaluated)	
	<u>either</u> if BC slack, max. extension of AB is given by $\frac{2\times1.8^2}{2\times0.5} = \frac{4x^2}{2\times0.4} \Rightarrow x = 1.138$	M1 A1	compare with energy at max. extension of AB or when BC goes slack deduce x or KE (or equivalent)	
	but 0.4 +1.138 + 0.5 < 2.3 so BC not slack	A1	complete argument , i.e. conclusion clearly and correctly deduced from calculations	
	<u>or</u> if BC slack, KE at point it goes slack is given by $\frac{2\times1.8^2}{2\times0.5} = \text{KE} + \frac{4\times1.4^2}{2\times0.4} \Rightarrow \text{KE} < 0$ which is impossible so BC not slack			4
(iii)	$\frac{2\times1.8^2}{2\times0.5} = \frac{4x^2}{2\times0.4} + \frac{2(1.4-x)^2}{2\times0.5}$	M1	energy equation involving EPE for both strings	
	$7x^2 - 5.6x - 2.56 = 0$	M1	solving (must be a 3-term quadratic)	
	$x = 1.125$	A1	cao	3

Total 15

3(i)	$R\sin\phi = mr\omega^2$	M1 A1	N2L with radial acceleration	
	$R\cos\phi = mg$	B1		
	$[\sin\phi]=[\cos\phi]=1 \Rightarrow$	B1	either component (and consider sin or cos)	
	$[R\sin\phi]=[R\cos\phi]=\text{MLT}^{-2}$			
	$[mg]=(\text{M})(\text{LT}^{-2})=\text{MLT}^{-2}$	E1		
	$[mr\omega^2]=(\text{M})(\text{L})(\text{T}^{-1})^2=\text{MLT}^{-2}$	E1	consider each quantity in radial equation	6
(ii)	$\tan\phi = \dfrac{r\omega^2}{g} = \dfrac{10\times 1.1^2}{9.8}$	M1 A1	eliminate R	
	$\phi = 51.0°\ (= 0.890\text{ rad})$	A1	cao	3
(iii)	$R\cos\theta + mg\sin\alpha = mr\omega^2$	M1 A1	attempt all terms	
	no tang. acc. $\Rightarrow R\sin\theta = mg\cos\alpha$	B1		
	horizontal reaction $\Rightarrow \theta = \alpha$	B1		
	$\dfrac{mg\cos\alpha}{\sin\alpha}\cos\alpha + mg\sin\alpha = m\times 10\times 1.1^2$	M1	eliminating R and substituting for θ from reasonable attempt at equations	
	$\Rightarrow \sin\alpha = \frac{g}{12.1} \Rightarrow \alpha = 54.1°\ (= 0.944\text{ rad})$	A1	cao	6
			Total	15

4(i)	$\frac{1}{4}\pi\bar{y} = \int_0^1 \frac{1}{2}y^2\,\text{d}x$	M1	attempt formula	
	$= \frac{1}{2}\int_0^1 (1-x^2)\,\text{d}x$	A1	both sides	
	$= \frac{1}{2}\left[x - \frac{1}{3}x^3\right]_0^1$	M1	integrate	
	$= \frac{1}{3}$	M1	limits (dependent on previous M marks)	
	$\bar{y} = \dfrac{4}{3\pi}$	E1		5
(ii)	(By symmetry,) $\bar{x} = \dfrac{4}{3\pi}$	B1		1
(iii)	By symmetry, $\bar{x} = \bar{y}$	B1		
	$\bar{x} = \bar{y} = \dfrac{\pi\cdot 0 - \frac{1}{4}\pi\cdot\frac{4}{3\pi}}{\pi - \frac{1}{4}\pi}$	M1 A1 M1	$\Sigma mx/\Sigma m$ or moments numerator denominator	
	or $\dfrac{\frac{1}{4}\pi(-\frac{4}{3\pi}) + \frac{1}{4}\pi(-\frac{4}{3\pi}) + \frac{1}{4}\pi(\frac{4}{3\pi})}{\frac{1}{4}\pi + \frac{1}{4}\pi + \frac{1}{4}\pi}$			
	$= -\dfrac{4}{9\pi}$	A1		5
(iv)	Taking moments about A	M1	moments equation	
	$(1 - \frac{4}{9\pi}\cdot\sqrt{2})\cdot 50 = (1 + \frac{1}{\sqrt{2}})T_B$	A1 A1	LHS RHS	
	$T_B = 23.4\text{ N}$	A1	cao (or T_A)	
	$T_A = 50 - T_B = 26.6\text{ N}$	F1		5
			Total	16

Examiner’s Report

Mechanics 3 (2609)

General Comments

Candidates generally showed a good understanding of most of the mechanics in this paper but it was disappointing to see poor levels of accuracy by a sizeable minority. In particular, premature rounding of intermediate answers (in some cases to only one significant figure) led to inaccuracies in final answers. There were some cases of candidates showing insufficient working, particularly (although not exclusively) when showing a given answer.

Comments on Individual Questions

Q1 (Simple Harmonic Motion)

This was often a well-answered question. Candidates generally knew what was required for all but the last part. However, careless errors and poor accuracy in the calculations was not uncommon, some candidates not even being able correctly to write down the amplitude and period. Inefficient methods were often used, e.g. calculating time and differentiating in part (ii) and using overcomplicated symmetries of the graph in part (iv). In the last part, many solved their equation for time without considering which solution was required.

[(i) 0.05 m, 0.32 s; (ii) 0.900 m s^{-1} down (3 s. f.), 7.71 m s^{-2} up (3 s. f.); (iv) $x = 0.05\cos 6.25\pi t$, 0.219 s (3 s. f.)]

Q2 (Hooke's Law and Energy)

Answers to the first part of this question were often correct. Some candidates attempted part (ii) erroneously by using ideas of amplitude and simple harmonic motion. Most candidates considered energy, but arguments to justify that BC was not slack were rarely complete. Few candidates seemed to realise that the form of the energy equation depended on the assumption of whether the strings were slack or stretched. Often the final conclusion was not clearly deduced. The calculation in part (iv) often used an energy equation which assumed BC slack, in spite of the fact that the candidate had just stated that BC did not go slack.

[(i) $x_1 = 0.4$, $x_2 = 1$; (ii) No; (iii) 1.125 m]

Q3 (Circular Motion)

In part (i) some candidates did not understand what was meant by a radial equation of motion, but most stated this equation and the vertical equation without much difficulty. However, the radial equation was frequently not in terms of the quantities given in the question and often a spurious minus sign appeared. There was also the common confusion caused by treating $mr\omega^2$ as a force, instead of using Newton's second law to relate the forces present to the mass and acceleration. The dimensional consistency was often poorly verified, with little more than $MLT^{-2} = MLT^{-2}$ being stated for both equations. Candidates should make it clear that they have considered the dimensions of each of the quantities. The best solutions stated the dimensions of all the relevant quantities and then showed that the dimensions of each side of the equations were the same. There were many good answers to the last part of the question, but also many poor ones. Resolving the forces was sometimes poorly done, with terms like $mg/\cos\alpha$ appearing.

[(ii) 51.0° (3 s. f.); (iii) 54.1° (3 s. f.)]

Q4 (Centre of mass of a lamina)

Most candidates were able to obtain the correct value for the y-coordinate of the centre of mass, but sometimes by spurious methods or by a method which actually was calculating the x-coordinate. Finding the position of the centre of mass of the compound lamina was often done well, although some candidates showed insufficient working to make it clear that they had used a correct method. Although the required coordinates were at the mean of the coordinates for each of the three quadrants, it was expected that candidates should indicate why this was the case here if they wanted to make use of this efficient method. In the last part, candidates generally realised that moments were necessary to calculate one of the tensions, but inaccuracies in the calculation of the necessary distances caused problems for many. Some candidates used the distance to the point of attachment at B rather than the perpendicular distance to the line of action of the tension.

[(ii) $\frac{4}{3\pi}$; (iii) $(-\frac{4}{9\pi}, -\frac{4}{9\pi})$; (iv) at A 26.6 N (3 s. f.), at B 23.4 N (3 s. f.)]