Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Monday **21 JANUARY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- There is an **insert** for use in Question **4**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages, 3 blank pages and an insert.

[Turn over

Section A

1 Vertices of the graph shown in Fig.1 represent objects. Some arcs have been drawn to connect vertices representing objects which are the same colour.

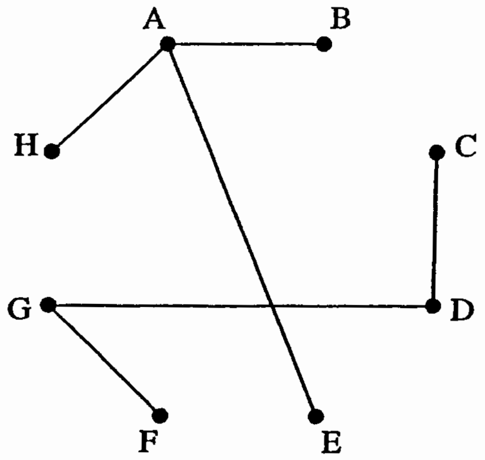


Fig. 1

(i) Copy Fig.1 and draw in whichever arcs you can be sure should be added. [2]

(ii) How many arcs would be needed in total if you were also told that the objects represented by B and F were the same colour? [2]

[Total 4]

2 (i) The possible outcomes of an experiment are the values 10, 20 and 30. These occur with the probabilities shown in Table 2.1.

Outcome	10	20	30
Probability	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Table 2.1

Give a rule for using a fair six-sided die to simulate outcomes of the experiment. [2]

(ii) In a different experiment the probabilities are as shown in Table 2.2.

Outcome	10	20	30
Probability	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{5}$

Table 2.2

Describe how to simulate outcomes of this experiment using a fair six-sided die together with a fair coin. [4]

[Total 6]

3 The following six steps define an algorithm.

Step 1: Think of a positive whole number and call it X.

Step 2: Write X out in words (i.e. using letters, not numbers).

Step 3: Let Y be the number of letters used.

Step 4: If $Y = X$ then stop.

Step 5: Replace X by Y.

Step 6: Go to step 2.

(i) Apply the algorithm with $X = 62$. [2]

(ii) Show that for all values of X between 1 and 99 the algorithm produces the same answer. (You may use the fact that, when written out, numbers between 1 and 99 all have twelve or fewer letters.) [3]

[Total 5]

Section B

4 [There is an insert for use in this question.]

A network has ten vertices, A to J. Table 4 shows the distance between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I	J
A		3				4				
B	3		2			7		1	6	
C		2				3		6		
D					2		2			5
E				2						4
F	4	7	3					1		
G				2						3
H		1	6			1				
I		6								
J				5	4		3			

Table 4

(i) Use Table 4.1 on the insert to apply the tabular form of Prim's algorithm to the network, starting at vertex A. Show that the algorithm terminates when vertices A, B, C, F, H and I are connected. Draw your minimum connector for these vertices on Fig. 4.3 on the insert, and give its total length. [7]

(ii) Using Table 4.2 on the insert, restart Prim's algorithm at vertex D to find a minimum connector for the remaining vertices. Draw your minimum connector on Fig. 4.3 on the insert, and give its total length. [6]

(iii) Arcs AG, of length 1, and EI, of length 2, are added to the network. Show that the minimum connector for the new connected network is not given by taking your two minimum connectors together with AG. [2]

[Total 15]

[Turn over

5 Claire wants to prepare and eat her breakfast in the minimum time. The activities involved, their immediate predecessors and their durations are shown in Table 5.

	Activity	Immediate Predecessors	Duration (mins)
F	Fill kettle	–	0.5
I	Put instant coffee in cup	–	0.5
W	Boil water	F	10
G	Grill toast	–	7
D	Dish out cereal	–	0.5
O	Fetch and open milk	–	0.5
M	Make coffee	I, W	0.5
B	Butter toast	G	0.5
E	Eat cereal and milk	D, O	3
T	Eat toast	E, B	5
C	Drink coffee	M, T	3

Table 5

(i) Draw an activity-on-arc network for these activities. Do not take account of the fact that Claire can do only one thing at a time. [5]

(ii) Show on your network the early time and the late time for each event. [4]

(iii) Give the critical activities and the minimum time needed for Claire to complete her breakfast, again taking no account of the fact that she can do only one thing at a time. [2]

(iv) Activities W and G do not require Claire's attention. For all the other activities, Claire can do only one thing at a time.

Produce a schedule for Claire starting at 7 a.m.

At what time does she actually finish her breakfast? [4]

[Total 15]

6 An airline needs to decide how many rows of seats in its new plane will be club class, and how many will be economy class. The plane can have up to 30 rows of seats in total. Every club class row has 4 seats and every economy class row has 6 seats.

The airline does not believe that it can sell more than 10 rows of club class seats.

Club class tickets sell at 25% above the price of economy class tickets.

On long flights the plane can carry no more than 150 passengers.

The airline wishes to maximise its income from ticket sales.

(i) Explain why this can be formulated as the following linear programming problem.

$$\begin{array}{ll} \text{Maximise} & 5x + 6y \\ \text{subject to} & x + y \leqslant 30, \\ & 4x + 6y \leqslant 150, \\ & x \leqslant 10, \\ & x \geqslant 0,\ y \geqslant 0. \end{array}$$

[5]

(ii) Solve the problem graphically. [8]

(iii) On short flights less fuel is needed and the plane can carry more passengers. What must be the capacity of the plane if the constraint $x + y \leqslant 30$ is not redundant? [2]

[Total 15]

Candidate Name	Centre Number	Candidate Number

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

INSERT

Monday 21 JANUARY 2002 Morning 1 hour 20 minutes

Instructions to candidates

- This insert should be used in Question **4**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

HN/2

[Turn over

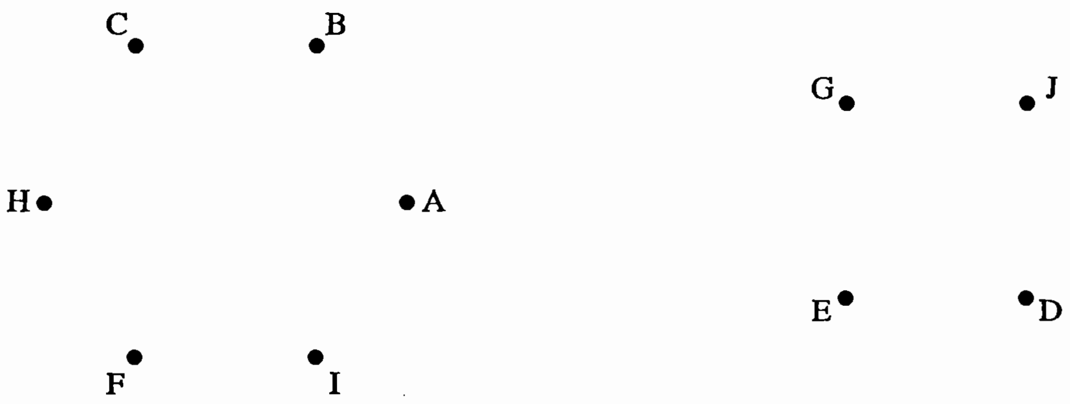
4

	A	B	C	D	E	F	G	H	I	J
A		3				4				
B	3		2			7		1	6	
C		2				3		6		
D					2		2			5
E				2						4
F	4	7	3					1		
G				2						3
H		1	6			1				
I		6								
J				5	4		3			

Table 4.1

	A	B	C	D	E	F	G	H	I	J
A		3				4				
B	3		2			7		1	6	
C		2				3		6		
D					2		2			5
E				2						4
F	4	7	3					1		
G				2						3
H		1	6			1				
I		6								
J				5	4		3			

Table 4.2



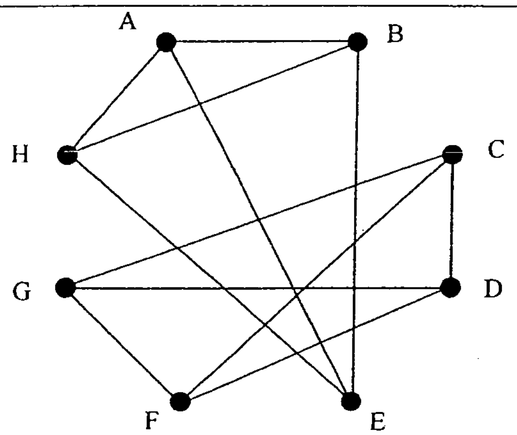
Length of connector = ...

Length of connector = ...

Fig. 4.3

Mark Scheme

1.

(i) 	M1 A1
(ii) $^8C_2 = 28$	M1 A1

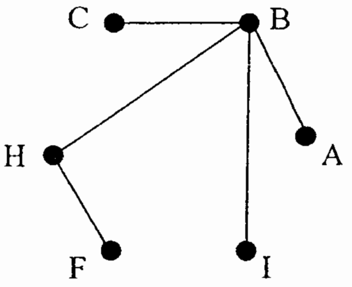
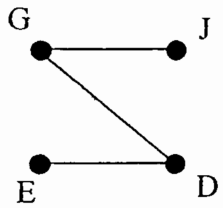
2.

(i) e.g. $1, 2 \to 10$ $3, 4, 5 \to 20$ $6 \to 30$	M1 A1
(ii) coin and die used together e.g. T1, T2, T3 $\to$ 10 T4, T5, T6, H1, H2 $\to$ 20 H3, H4 $\to$ 30 H5, H6 ignore and repeat.	B1 M1 equally likely set M1 some ignored A1 (dep. on both Ms)

3.

(i) sixty two 8 eight 5 five 4 four	M1 A1
(ii) $\{1, 2, 6, 10\} \to 3, 5, 4$ $\{4, 5, 9\} \to 4$ $\{3, 7, 8) \to 5, 4$ $\{11, 12\} \to 6, 3, 5, 4$	M1 enumeration A1 4 classes A1

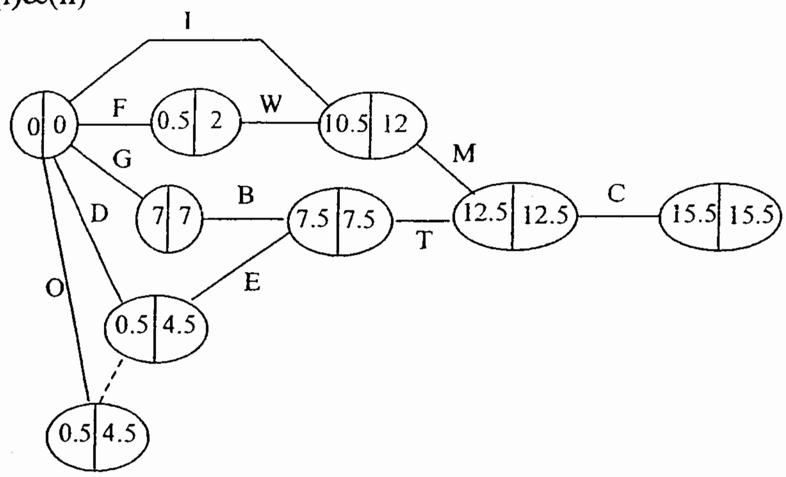
4.

(i) See below Stalls when no entries left in selected columns.  length = 13	M1 Prim A1 selecting entries A1 handling rows A1 handling columns B1 stalling B1 connector B1 length
(ii) See below.  length = 7	M1 Prim for 2nd time A1 selecting entries A1 handling rows A1 handling columns B1 connector B1 length
(iii) Use AG and EI, and delete BI.	B1 B1

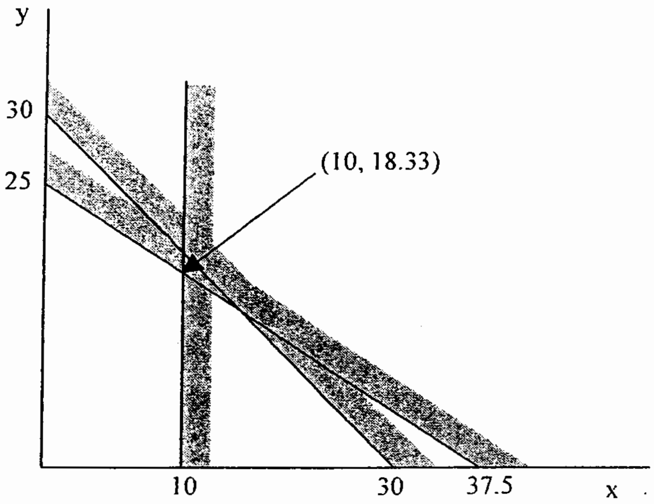
	A^1	B^2	C^5	D	E	F^4	G	H^3	I^6	J
~~A~~		~~3~~				~~4~~				
B	3		~~2~~			~~7~~		~~1~~	~~6~~	
C		2				~~3~~		~~6~~		
D					2		2			5
E				2						4
F	~~4~~	~~7~~	~~3~~					1		
G				2						3
H		1	~~6~~			~~1~~				
I		6								
J				5	4		3			

	A	B	C	D^1	E^2	F	G^3	H	I	J^4
A		3				4				
B	3		2			7		1	6	
C		2				3		6		
D					~~2~~		~~2~~			~~5~~
E				2						~~4~~
F	4	7	3					1		
G				2						~~3~~
H		1	6			1				
I		6								
J				~~5~~	~~4~~		3			

5.

(i)&(ii) 	M1 sca, act.-on-arc A1 I, F, W, M A1 G, B, T, C M1 dummy A1 D, O, E M1 A1 forward pass M1 A1 backward pass
(iii) G, B, T, C 15.5 mins	B1 B1
(iv) 0700 Put toast on to grill Fill kettle 0700.5 Put kettle on to boil Dish out cereal 0701 Put coffee in cup 0701.5 Fetch and open milk 0702 Eat cereal 0707 Butter toast 0707.5 Eat toast 0712.5 Make coffee 0713 Drink coffee Finishes at 0716.	 M1 A2(−1) B1 (cao)

6.

(i) x is the number of club class rows …	B1
Objective is to maximise income ($5 = 4*1.25$).	B1
$x + y \leq 30$ is the constraint on the number of rows.	
$4x + 6y \leq 150$ is the constraint on the no. of passengers.	
$x \leq 10$ is the constraint on the no. of club class rows.	B2 (−1 each error)
(ii) 	M1 A1 line A1 line A1 line A1 shading
(10, 18) gives income = 158	M1 top right of feasible region
(9, 19) gives income = 159	
(8, 19) gives income = 154	M1 integer solution
(7, 20) gives income = 155	A1 two or more lattice points
etc.	
Best is 9 rows of club and 19 rows of economy.	A1
(iii) $x + y = 30$ and $x = 10$ intersect at (10, 20)	M1 A1
$4*10 + 6*20 = 160$	

Examiner’s Report

Decision and Discrete Mathematics I (2620)

General Comments

Performances on this paper were generally much better than in June 2001. There were fewer very weak candidates. Conversely, although there were many candidates with high marks, few managed to score a high proportion of the "grade A" marks.

In June 2001 there was evidence that the paper was too long. This was not the case with this paper. However, there remains the feeling that many of the candidates are making heavy weather of Section A, where the questions are intended to be quick to answer.

Comments on Individual Questions

Question 1 (Graphs)

A surprising number of candidates were unable to answer part (i) of this question. In part (ii) candidates could score the method mark by indicating that the graph would now be complete. It was expected that weaker candidates would struggle with the arc count, and that proved to be the case.

> [(ii) 28 arcs in the complete graph]

Question 2 (Simulation)

Most, but not all, candidates gave a correct rule for part (i). A pleasingly high proportion were also successful with part (ii). On the other hand, quite a large number of candidates gave answers involving two-digit random numbers!

> [(i) e.g. 1,2→10; 3,4,5→20; 6→30
> (ii) e.g. T1,T2,T3→10; T4,T5,T6,H1,H2→20; H3,H4→30; H5,H6→reject and repeat]

Question 3 (Algorithms)

Most candidates were able to apply the algorithm in part (i). Very few were able to mount a proof in part (ii). Many waffled. Some scored a method mark by listing all twelve possibilities and noting that only the number four has the property that it is the number of its letters. They did not realise that they needed also to demonstrate convergence to this from the other numbers. Some very nice diagrammatic representations of the convergent paths were seen.

Question 4 (Networks)

Those that knew the tabular form of Prim's algorithm did well on this question. Those that did not could not score well. In part (i) few candidates bothered to make a comment about the algorithm terminating before all of the nodes were connected – a mark was reserved for such a comment. Very few were able to score 1 or 2 of the final two "grade A" marks in part (iii). Some that attempted it misread it as "Show that the two minimum connectors together with AG and EI do not form a minimum connector for the new connected network".

[(i) Min connector is of total length 13; (ii) Min connector is of total length 7;
(iii) Use AG and EI, and delete BI]

Question 5 (CPA)

Parts (i), (ii) and (iii) were routine and were generally answered well. The most common error, predictably, was the failure to use a dummy activity to model the dependency of E on D and O. There were also a number of candidates who failed to use a unique start event. Errors were often made as a result of candidates not labelling their activities – activity C was sometimes forgotten.

Part (iii) caused a great deal of difficulty, not least to the markers! Apart from general confusion and lack of order, some candidates failed to grill their toast or boil their water, thus demonstrating a lack of understanding of the relationship between logical precedences and resourcing.

[(iii) G, B, T, C; 15.5 mins; (iv) Claire can finish her breakfast at 07.16]

Question 6 (LP)

In part (i) good candidates started by identifying the variables as representing numbers of rows, identified the "5" in the objective function as 4×1.25, and identified the constraints as referring to rows and passengers/seats as appropriate. Poor candidates waffled, and failed to distinguish between rows and seats. Some thought that the objective function should have been $5x + 4y$, and that a mistake had been made.

Part (ii) was relatively easy, since the LP was given, and it was answered well – up to finding the solution. At this point many failed to realise that integer solutions were needed, or gave an integer solution without realising that there was an issue. Some weaker candidates reversed axes and/or interchanged the intercepts with the axes of the $4x + 6y = 150$ line.

Only a few very good candidates managed to score the final difficult marks in part (iii)

[(ii) 9 rows of club and 19 rows of economy, with an objective function value of 159
(iii) At least 160 passengers]

Numerical Methods (2623)

General Comments

Overall, this paper attracted some very good attempts. There were a few candidates who were clearly out o their depth, but there were some who had an excellent grasp of the concepts involved. Routine numerica processes were carried out accurately for the most part. Analysis was more challenging.