Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2614/1

Statistics 2

Tuesday **12 JUNE 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

HN/2
1356

1 A medical statistician wishes to carry out a hypothesis test to see if there is any correlation between the head circumference and body length of newly-born babies.

(i) State appropriate null and alternative hypotheses for the test. [2]

A random sample of 20 newly-born babies have their head circumference, x cm, and body length, y cm, measured. This bivariate sample is illustrated in Fig. 1.

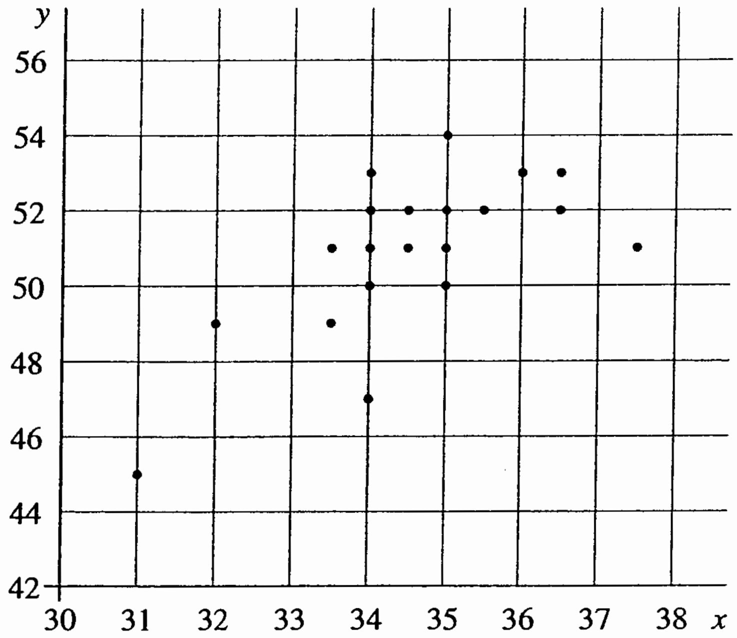

Fig. 1

Summary statistics for this data set are as follows:

$$n = 20,\quad \sum x = 691,\quad \sum y = 1018,\quad \sum x^2 = 23917,\quad \sum y^2 = 51904,\quad \sum xy = 35212.5.$$

(ii) Calculate the product-moment correlation coefficient for the data. Carry out the hypothesis test at the 1% significance level, stating the conclusion clearly. What assumption is necessary for the test to be valid? [8]

Originally, the point $x = 34$, $y = 51$ had been recorded incorrectly as $x = 51$, $y = 34$.

(iii) Calculate the values of the summary statistics if this error had gone undetected. [3]

Using the uncorrected summary statistics, the value of the product-moment correlation coefficient is -0.681 (to 3 significant figures).

(iv) How is it that this one error produces such a large change in the value of the correlation coefficient and also changes its sign? [2]

1356

2 *Extralite* are testing a new long-life bulb. The life-times, in hours, are assumed to be Normally distributed with mean μ and standard deviation σ. After extensive tests, they find that 19% of bulbs have a life-time exceeding 5000 hours, while 5% have a life-time under 4000 hours.

(i) Illustrate this information on a sketch. [2]

(ii) Show that $\sigma = 396$ and find the value of μ. [4]

In the remainder of this question take μ to be 4650 and σ to be 400.

(iii) Find the probability that a bulb chosen at random has a life-time between 4250 and 4750 hours. [3]

(iv) *Extralite* wish to quote a life-time which will be exceeded by 99% of bulbs. What time, correct to the nearest 100 hours, should they quote? [3]

A new school classroom has 6 light-fittings, each fitted with an *Extralite* long-life bulb.

(v) Find the probability that no more than one bulb needs to be replaced within the first 4250 hours of use. [3]

3 The numbers of goals per game scored by teams playing at home and away in the Premier League are modelled by independent Poisson distributions with means 1.63 and 1.17 respectively.

(i) Find the probability that, in a game chosen at random,

(*A*) the home team scores at least 2 goals, [3]

(*B*) the result is a 1-1 draw, [3]

(*C*) the teams score 5 goals between them. [3]

(ii) Give two reasons why the proposed model might not be suitable. [2]

The number of goals scored per game at home by *Rovers* is modelled by the Poisson distribution with mean 1.63. In a season they play 19 home games.

(iii) Use a suitable approximating distribution to find the probability that *Rovers* will score more than 35 goals in their home games. [4]

4 A team of five, the *Ambridge Archers*, takes part in an archery competition, where the objective is to hit the "bull" on the target. Each member of the team, independently, has a probability of 0.4 of hitting the "bull". A single round consists of each archer shooting once at the target. Let X represent the number of hits obtained by the team in one round.

(i) State the distribution of X. Copy and complete the following table. [3]

x	0	1	2	3	4	5
$P(X = x)$	0.0778					0.0102

The number of points, Y, scored by the team in the round is given by the formula $Y = \frac{1}{2}X(X + 1)$. [For example, if $X = 5$ then $Y = 15$.]

(ii) Obtain the probability distribution of Y and illustrate it in a sketch. [3]

(iii) Find $E(Y)$ and $Var(Y)$. [4]

(iv) Find the probability that after three rounds the team has scored exactly 1 point. [2]

The *Ambridge Archers* are competing against the five *Borset Bowmen*, each of whom also has probability 0.4 of hitting the "bull".

(v) Find the probability that after the first round the team scores are equal. [3]

Mark Scheme

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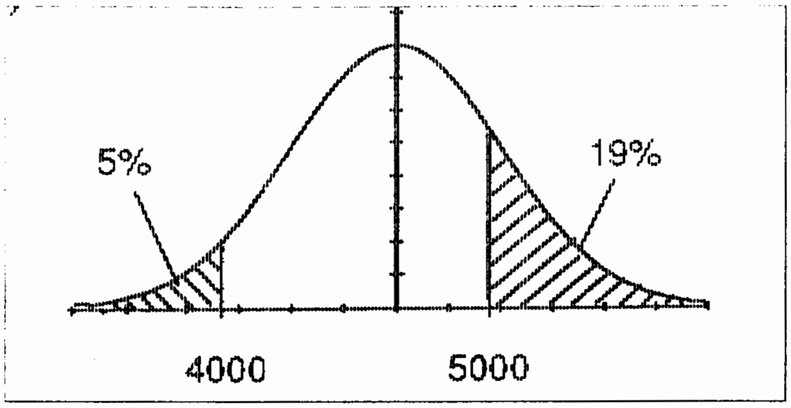
Question 1

(i)	H_0: $\rho = 0$, H_1: $\rho \neq 0$ [where ρ is the population correlation coefficient]	B1 for H_0 B1 for H_1	2
(ii)	$S_{xy} = \Sigma xy - n\bar{x}\bar{y} = 35212.5 - 20 \times 34.55 \times 50.9 = 40.6$ $S_{xx} = \Sigma x^2 - n\bar{x}^2 = 23917 - 20 \times 34.55^2 = 42.95$ $S_{yy} = \Sigma y^2 - n\bar{y}^2 = 51904 - 20 \times 50.9^2 = 87.8$ $r = \dfrac{40.6}{\sqrt{42.95 \times 87.8}}$ *or* $\dfrac{2.03}{\sqrt{2.1475}\sqrt{4.39}} = 0.66$ (2 s.f.)	M1 for attempt at S_{xy} *or* covariance M1 for attempt at finding *either* S_{xx} or S_{yy} *or* a variance M1 for structure of r A1 **cao**	4
	For $n = 20$, 1% critical value = 0.5614 Since $0.5614 < 0.661$ we reject H_0: There is sufficient evidence at the 1% significance level to suggest there is correlation between head circumferences and lengths of babies. Background population is *bivariate Normal.*	B1 for critical value M1 for comparison A1 for conclusion in words in context E1 for explanation	4
(iii)	$\Sigma x = 708$, $\Sigma y = 1001$, $\Sigma x^2 = 25362$, $\Sigma y^2 = 50459$, $n = 20$, $\Sigma xy = 35212.5$	B3 for all 6 correct [B2 for any 4 correct B1 for any 2 correct]	3
(iv)	The incorrect pair produce an *extreme* point to the *right and below* existing cluster, producing a negative correlation. ***Or*** There will be a large change in the summary statistics, which will make the covariance negative.	E1 for extreme point E1 for relative position ***Or*** E1 for large change E1 for negative cov.	2
			15

Question 2

(i)		G1 for left-hand tail G1 for right-hand tail, with area *larger* than the left-hand tail area	2
(ii)	$P(X > 5000) = 0.19 \Rightarrow 5000 = \mu + 0.8779\sigma$ $P(X < 4000) = 0.05 \Rightarrow 4000 = \mu - 1.645\sigma$ Solving: $1000 = 2.523\ \sigma$ $\Rightarrow \sigma = \dfrac{1000}{2.523} = 396$ (3 s.f.) Hence: $\mu = 4000 + 1.645 \times 396 = 4650$ (3 s.f.)	B1 for both z-values M1 for attempt at one equation with z-value M1 for attempt at finding σ A1 for both values	4
(iii)	$P(4250 < X < 4750) = P(-1 < Z < 0.25)$ $= 0.5987 - (1 - 0.8413)$ $= 0.44$ (2 s.f.)	M1 for standardisations M1 for probability calculations A1 **cao**	3
(iv)	$P(Z > -2.326) = 0.99$ $\Rightarrow x = 4650 - 2.326 \times 400 = 3719.6$ hence should quote 3700 hours	B1 for ± 2.326 M1 for calculation A1 **cao**	3
(v)	P(no more than one bulb needs replacing) $= 0.8413^6 + 6 \times 0.8413^5 \times 0.1587$ $= 0.76$ (2 s.f.)	M1 for 2nd product M1 for sum of 2 terms A1 **cao**	3
			15

Question 3

(i)	(*A*) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - e^{-1.63}(1 + 1.63)$ $= 1 - 0.515 = 0.485$ (3 s.f.)	M1 for sum of 2 probs. M1 for "1 – sum of 2 probs." A1	**3**
	(*B*) $P(X = 1) \times P(Y = 1)$ $= (e^{-1.63} \times 1.63) \times (e^{-1.17} \times 1.17)$ $= 0.116$ (3 s.f.)	M1 2 probabilities M1 for product A1	**3**
	(*C*) Using $\lambda = 1.63 + 1.17 = 2.8$: $P(X + Y = 5) = 0.9349 - 0.8477 = 0.087$ (2 s.f.) *or* $P(X + Y = 5) = e^{-2.8} \times \frac{2.8^5}{5!} = 0.087$ (2 s.f.)	B1 for $\lambda = 2.8$ M1 for calculation A1 **cao**	**3**
(ii)	Two reasons why proposed model might not be suitable: Poisson parameter unlikely to be same for each team; lack of independence between the variables	E1 for one reason E1 for second reason	**2**
(iii)	$\lambda = 19 \times 1.63 = 30.97$, hence suitable approximating distribution is N(30.97, 30.97) P(more than 35 goals in a season) $= P(X > 35.5) = P(Z > \frac{35.5 - 30.97}{\sqrt{30.97}})$ $= P(Z > 0.814)$ $= 1 - 0.792$ $= 0.208$ (3 s.f.)	B1 for use of Normal approx. B1 for continuity corr. M1 for calculation A1	**4**
			15

Question 4

(i)

$X \sim B(5, 0.4)$

x	0	1	2	3	4	5
$P(X=x)$	0.0778	0.2592	0.3456	0.2304	0.0768	0.0102

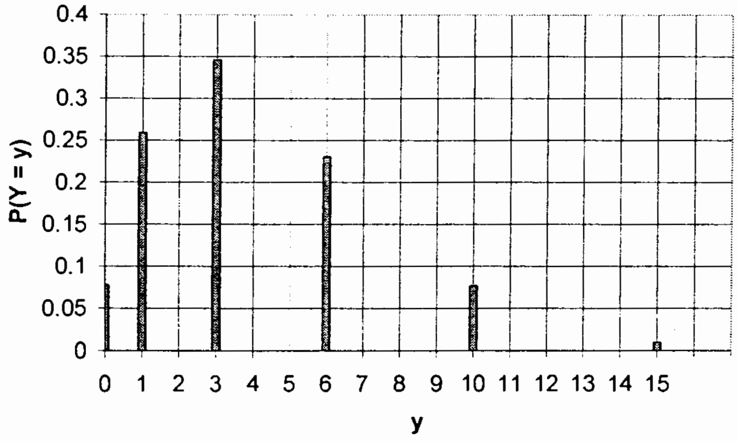
B1 for distribution

B2 for all probabilities (−1 for each error)

3

(ii)

y	0	1	3	6	10	15
$P(Y=y)$	0.0778	0.2592	0.3456	0.2304	0.0768	0.0102



B1 for table

G1 for lines in proportion

G1 for scaled axes, dependent on suitable diagram

3

(iii)

$E(Y) = \Sigma y\,P(Y=y)$
$= 0 \times 0.0778 + 1 \times 0.2592 + 3 \times 0.3456 + \ldots + 15 \times 0.0102$
$= 3.60$ (3 s.f.)

M1 for $E(Y)$

A1

$Var(Y) = E(Y^2) - [E(Y)]^2$
$= 0 \times 0.0778 + 1 \times 0.2592 + 9 \times 0.3456 + .. + 225 \times 0.0102 - 3.60^2$
$= 21.639 - 12.956 = 8.68$ (3 s.f.)

M1 for $E(Y^2)$

A1 **cao**

4

(iv)

P(score exactly 1 point)
$= 3 \times 0.0778^2 \times 0.2592 = 0.0047$ (2 s.f.)
or $= 15 \times 0.6^{14} \times 0.4 = 0.0047$ (2 s.f.)

M1 for either product

A1

2

(v)

$0.0778^2 + 0.2592^2 + 0.3456^2 + 0.2304^2 + 0.0768^2 + 0.0102^2 = 0.252$ (3 s.f.)

M1 for at least 3 squares

M1 for sum of 6 squares

A1 **cao**

3

15

Examiner’s Report

Statistics 2 (2614)

General Comments

There were 1280 candidates for this paper. The overall standard was slightly lower than the corresponding papers for 1999 and 2000 in the 5514 series. There was also a significantly greater spread of marks than we are accustomed to. A large proportion of candidates obtained high scores and an equally large proportion obtained fairly low scores. Relatively few candidates gained scores in the middle range from 25 to 40 marks. Candidates found the first three questions much more accessible than the fourth, in which even high-scoring candidates often gained very few marks. Throughout the paper, the discussion and interpretation questions caused candidates difficulty.

Comments on Individual Questions

Question 1 (Bivariate data: product moment correlation; body lengths and head circumferences of newly born babies)

Candidates were able score highly in this bivariate data question, compared with those of recent 5514 sessions. Particularly pleasing was the ability of many candidates to handle the 'rogue' pair of data in parts (iii) and (iv).

(i) Most hypotheses were given in terms of ρ, the population correlation coefficient. In future, it is expected that candidates should be able to define ρ, together with its relevance to an hypothesis test. Marks were lost by candidates who used a one tailed alternative hypothesis, who gave their hypotheses in words or who used another symbol in place of ρ.

(ii) Many fully correct answers were seen. However, some candidates were unable to deal correctly with n, either omitting n altogether or dividing by n at the wrong stage in the calculation. In some cases premature rounding caused inaccuracy in the final answer. A very small number of candidates wasted time by using the raw data on the scatter diagram to recalculate the summary statistics or to calculate r using their calculator statistical functions. The hypothesis test was often fully correct, with relatively few candidates failing to interpret their conclusion in context of the head circumferences and lengths of babies. Very few candidates were able to correctly identify the need for *bivariate Normality* in the population. Many suggested that the sample needed to be random, despite the question stating that a random sample had been taken.

(iii) The uncorrected summary statistics were usually found correctly, even by candidates who had, until this part, shown much misunderstanding of the basic ideas.

(iv) Few candidates gained full credit here, although a good number gained one mark by stating or implying that the incorrect point would be an outlier. Both marks could be obtained by also giving its position relative to the original scatter. Alternative solutions detailing the change in the value of the covariance were equally acceptable.

(i) H_0: $\rho = 0$, H_1: $\rho \neq 0$; (ii) $r = 0.661$ (3 s.f.), reject H_0 at 1% level, comment;
(iii) $n = 20$, $\Sigma x = 708$, $\Sigma y = 1001$, $\Sigma x^2 = 25362$, $\Sigma y^2 = 50459$, $\Sigma xy = 35212.5$;
(iv) explanation for negative correlation.

Question 2 (Normal distribution; lengths of life of electric light bulbs)

This question was generally tackled well by many candidates. However, significant problems occurred in part (ii), where they were unable to formulate and solve the simultaneous equations, in part (iv) where the wrong tail was often used, or in part (v) where the correct use of a binomial distribution was rare.

(i) Both marks were gained by the majority of candidates, although a number lost credit due to their 19% area being smaller than their 5% area, or their 5000 being further from the centre of the distribution than their 4000. Candidates need to be advised that a statistical sketch must 'look right', unlike the sketch of a function in pure mathematics, where scale is often ignored entirely.

(ii) Many candidates demonstrated a good understanding of the Normal distribution, producing clear and convincing working to justify the given value of σ and hence to find μ. In forming an equation involving the lower tail probability, the preferred method of equating $(x - \mu)/\sigma$ to a negative z value was more popular than the alternative of equating $(\mu - x)/\sigma$ to the corresponding positive z value, but both were awarded equal credit. A common error was to omit the negative sign from $z = -1.645$ and work with the equation $(4000 - \mu)/\sigma = 1.645$, leading either to a negative value of σ, or in some cases to the correct value of σ by incorrect algebra. Candidates who adopted the latter approach gained no credit for it. Some candidates used the given value of σ in their working. The value of σ was given in order to allow candidates to check that their equations were correct. They were not intended to use it within their working and lost credit if they did so.

(iii) Almost all candidates were able to gain credit for correct standardisation, but a surprising number were unable to evaluate the central probability successfully.

(iv) A reasonable number of correct solutions were seen, but a remarkable number of candidates found the z value of 2.326 from the table, but then omitted the negative sign and in effect found the life time which will be exceeded by 1% of bulbs.

(v) Many candidates successfully identified the probability that a given bulb needs replacing [0.1587], which had already been evaluated in the working for part (iii). To gain credit it was necessary to go beyond this stage and work with the upper tail of a binomial distribution based on this probability. This level of sophistication proved to be beyond the capabilities of most candidates.

(i) Normal distribution diagram; (ii) $\mu = 396$ (3 s.f.), $\sigma = 4650$ (3 s.f.);
(iii) 0.44 (2 s.f.); (iv) 3700 hours; (v) 0.76 (2 s.f.).

Question 3 (Poisson distribution; number of goals scored by football teams)

Most candidates seemed to find the context of this question appealing. However, fully correct answers to fairly basic Poisson calculations in part (i) were all too rare. The range of responses to the unsuitability of the model was great, as was that of the solutions to part (iii), where there was a very mixed response to the Normal approximation. A disappointingly high proportion of candidates failed to state and use the parameters correctly.

(i) (A) Most candidates realised that the use of tables was not appropriate since probabilities are not tabulated for $\lambda = 1.63$. Surprisingly few were able to correctly handle $P(X \geq 2) = 1 - P(X \leq 1)$, with a variety of errors here, the most common being $P(X \geq 2) = 1 - P(X = 1)$ and $P(X \geq 2) = 1 - P(X \leq 2)$. Those candidates who attempted to use tables with $\lambda = 1.6$ scored just the method mark.

(B) Candidates found this part of question 3 to be the most accessible, and many correct answers were seen. A frequent error was the addition rather than multiplication of the two correct probabilities.

(C) Those candidates who realised that they could simply add the two means to get the overall mean [$\lambda = 2.8$] for the total number of goals in a game were almost always able to score all three marks. Very often candidates failed to notice this and instead calculated and then summed the probabilities of the six possible outcomes of the match. This approach is correct and gained full credit if it was carried through with sufficient care to achieve the required degree of accuracy, although candidates often rounded prematurely and thus lost the final mark. Many omitted one or more of the probabilities and were therefore able to gain at most one mark. Candidates who adopted this method sometimes found themselves short of time to complete the paper after spending excessive time on this part.

(ii) A wide variety of responses was seen, with many candidates indicating some understanding of why the model might not be suitable. However, in order to score both marks, candidates were required to explain what features of the model might render it unsuitable and why they might do so. Candidates were rarely able to link these two ideas, either only discussing independence, randomness or uniformity, or only discussing the variety of standards, players, weather conditions and interaction between teams. Some had clearly learnt rote answers to 'when the Poisson distribution is a suitable model' and simply quoted these without reference to the context.

(iii) Candidates often recognised that a Normal approximation was needed, although some failed to use the correct parameters. As is usually the case the continuity correction was often omitted or applied in the wrong direction.

(i) (*A*) 0.485 (3 s.f.), (*B*) 0.116 (3 s.f.), (*C*) 0.087 (2 s.f.);
(ii) reasons for unsuitability; (iii) 0.208 (3 s.f.).

Question 4 (Discrete random variables; points scored by archery teams)

Candidates found this the most challenging question on the paper, and certainly harder than discrete random variable questions on recent 5514 papers. There was much confusion in the first three parts, but many candidates redeemed themselves by using common sense to calculate probabilities in parts (iv) and/or (v).

(i) Many candidates were able to calculate the binomial probabilities, although the $\binom{n}{r}$ term was often omitted. Some used a geometric distribution. Very few explicitly stated that the distribution was B(5, 0.4) as required for the third mark.

(ii) This part caused difficulties for the majority of candidates, with many producing a table of x and y values with no probabilities whatsoever, or with fractional probabilities with a denominator of 35, presumably found by adding the six y-values. A surprisingly large number applied the transformation to the probabilities of X. The graph was rarely drawn correctly, with many graphs simply being a plot of y against x or vice-versa. Candidates who did plot their probabilities often used x-values, or y-values with a non-uniform scale on the horizontal axis. Most candidates did attempt to draw a vertical line diagram, as expected for a discrete distribution, rather than a bar chart or a line graph.

(iii) Most candidates made a correct attempt to calculate expectation and variance, but often with x-values rather than y-values. Such candidates were able to score both method marks.

(iv) This proved to be challenging, with relatively few correct answers. Many candidates omitted the factor of 3 combinations in their answer.

(v) This was often well done, even when part (ii) had been completely misunderstood. Candidates who had used 35ths in part (ii) often reverted to the correct probabilities here. Even candidates who had made errors in the probabilities of X in part (i) could score both method marks.

(i) $X \sim B(5, 0.4)$, table of probabilities for X; (ii) table of probabilities for Y and vertical line chart; (iii) $E(Y) = 3.60$ (3 s.f.), $Var(Y) = 8.68$ (3 s.f.); (iv) 0.0047 (2 s.f.);
(v) 0.252 (3 s.f.).

(i) (A) Most candidates realised that the use of tables was not appropriate since probabilities are not tabulated for $\lambda = 1.63$. Surprisingly few were able to correctly handle $P(X \geq 2) = 1 - P(X \leq 1)$, with a variety of errors here, the most common being $P(X \geq 2) = 1 - P(X = 1)$ and $P(X \geq 2) = 1 - P(X \leq 2)$. Those candidates who attempted to use tables with $\lambda = 1.6$ scored just the method mark.

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(i) Many candidates were able to calculate the binomial probabilities, although the $\binom{n}{r}$ term was often omitted. Some used a geometric distribution. Very few explicitly stated that the distribution was B(5, 0.4) as required for the third mark.