OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2604**

Pure Mathematics 4

Tuesday **5 JUNE 2001** Mornng 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 A curve has equation $y = \dfrac{(x-1)(x+4)}{x^2+4}$.

(i) Write down the equation of the asymptote parallel to the x-axis. [1]

(ii) Find $\dfrac{dy}{dx}$. Hence find the coordinates of the stationary points. [6]

(iii) Sketch the curve. [4]

(iv) On a separate diagram, sketch the curve with equation $y = \left|\dfrac{(x-1)(x+4)}{x^2+4}\right|$.

Give the coordinates of the stationary points on this curve. [4]

(v) Solve the inequality $\left|\dfrac{(x-1)(x+4)}{x^2+4}\right| < 1$. [5]

2 **(a)** Find $\sum_{r=1}^{n}(6r-1)(4r-1)$, giving your answer in its simplest form. [5]

(b) Express $\dfrac{3r-2}{r(r+1)(r+2)}$ in partial fractions.

Hence find the sum of the first n terms of the series

$$\frac{1}{1\times2\times3} + \frac{4}{2\times3\times4} + \frac{7}{3\times4\times5} + \dots .$$ [8]

(c) A sequence of real numbers $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = -1 \quad \text{and} \quad u_{n+1} = \frac{3u_n}{5u_n+6} \quad \text{for } n \geqslant 1.$$

Prove by induction that $u_n = \dfrac{3}{2^n-5}$. [7]

3 You are given that the complex number $\alpha = 1 + 4\text{j}$ satisfies the cubic equation

$$z^3 + 5z^2 + kz + m = 0,$$

where k and m are real constants.

(i) Find α^2 and α^3 in the form $a + b\text{j}$. [3]

(ii) Find the value of k, and show that $m = 119$. [4]

(iii) Find the other two roots of the cubic equation. Give the arguments of all three roots. [7]

(iv) Verify that there is a constant c such that all three roots of the cubic equation satisfy

$$|z + 2| = c.$$

Draw an Argand diagram showing the locus of points representing all complex numbers z for which $|z + 2| = c$. Mark the points corresponding to the three roots of the cubic equation. [6]

4 **(a)** Two straight lines L and M have equations

$$L: \frac{x-7}{8} = \frac{y-13}{6} = \frac{z-14}{1}, \qquad M: \frac{x-5}{-2} = \frac{y+8}{5} = \frac{z+9}{3}.$$

(i) Show that the lines L and M do not intersect. [5]

(ii) Find a vector which is perpendicular to both lines L and M. [3]

(iii) Find, in the form $ax + by + cz + d = 0$, the equation of the plane which contains the line L and is parallel to the line M. [3]

(b) The matrix $\begin{pmatrix} 4 & 2 \\ 6 & k \end{pmatrix}$, where k is a constant, defines a transformation in the (x, y)-plane.

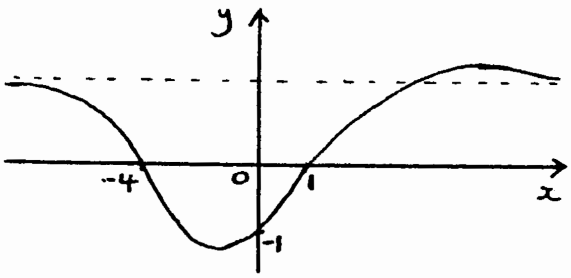
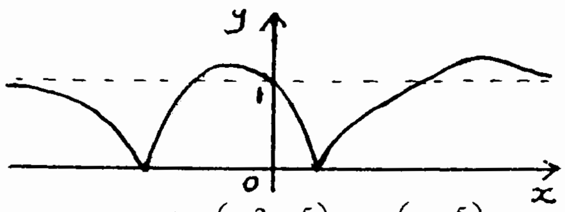
(i) Find the set of invariant points of the transformation

(A) when $k = 4$,

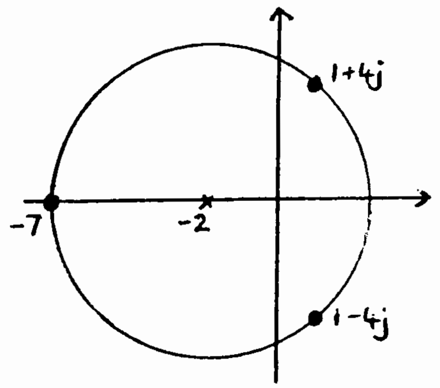
(B) when $k = 5$. [5]

(ii) When $k = 5$, verify that $y = 2x - 3$ is an invariant line of the transformation. [4]

Mark Scheme

1 (i)	$y = 1$	B1 1	*Accept* $y \to 1$
(ii)	$\dfrac{dy}{dx} = \dfrac{(x^2+4)(2x+3)-(x^2+3x-4)(2x)}{(x^2+4)^2}$ $= \dfrac{-3x^2+16x+12}{(x^2+4)^2}$ $\dfrac{dy}{dx} = 0$ when $-3x^2+16x+12=0$ $x = -\dfrac{2}{3},\ 6$ Stationary points are $\left(-\dfrac{2}{3},\ -\dfrac{5}{4}\right)$ and $\left(6,\ \dfrac{5}{4}\right)$	M1 A1 M1 M1 A2 cao 6	Attempt to use quotient rule (or equivalent) Obtaining a quadratic equation Solving quadratic to obtain at least one value of x Give A1 for one point correct, or for both values of x correct
(iii)		B1 B1 B1 ft B1 4	Curve through $(-4, 0)$ with negative gradient and through $(1, 0)$ with positive gradient. (B0 if any other intersections with x-axis.) Curve passing through $(0, -1)$ Maximum and minimum on curve (and no more turning points) Curve of correct shape approaching the asymptote correctly
(iv)	 Stationary points are $\left(-\dfrac{2}{3},\ \dfrac{5}{4}\right)$ and $\left(6,\ \dfrac{5}{4}\right)$	B2 ft B1 B1 ft 4	Curve with negative section(s) reflected. Give B1 if one minor error. Two sharp points on the x-axis *Ignore* $(-4, 0)$ *and* $(1, 0)$ *Accept coordinates clearly indicated on graph*
(v)	$\dfrac{(x-1)(x+4)}{x^2+4} = 1$ when $x = \dfrac{8}{3}$ $\dfrac{(x-1)(x+4)}{x^2+4} = -1$ when $x = 0,\ -\dfrac{3}{2}$ $\left\lvert\dfrac{(x-1)(x+4)}{x^2+4}\right\rvert < 1$ when $x < -\dfrac{3}{2},\quad 0 < x < \dfrac{8}{3}$	B1 B1 M1 A2 cao 5	Considering intervals defined by three critical values (obtained from $\lvert y\rvert = 1$) Give A1 for $\le$ instead of $<$, or for a minor transcription error. When critical values a, 0, b have been obtained, $a < 0$ and $b > 1$, give A1 ft for $x < a$, $0 < x < b$

2 (a)	$\sum(24r^2 - 10r + 1)$ $= 4n(n+1)(2n+1) - 5n(n+1) + n$ $= 8n^3 + 7n^2$	B1 B1B1B1 ft B1 cao 5	
(b)	$\dfrac{3r-2}{r(r+1)(r+2)} = -\dfrac{1}{r} + \dfrac{5}{r+1} - \dfrac{4}{r+2}$	M1 A1	Method for finding partial fractions (with linear denominators)
	Sum is $\left(-\frac{1}{1}+\frac{5}{2}-\frac{4}{3}\right)+\left(-\frac{1}{2}+\frac{5}{3}-\frac{4}{4}\right)+\left(-\frac{1}{3}+\frac{5}{4}-\frac{4}{5}\right)$ $+\ldots+\left(-\frac{1}{n-1}+\frac{5}{n}-\frac{4}{n+1}\right)+\left(-\frac{1}{n}+\frac{5}{n+1}-\frac{4}{n+2}\right)$	M1 A1 ft M1	Using partial fractions (two terms) Three terms correct Pattern of cancelling
	$= -1 + \dfrac{5}{2} - \dfrac{1}{2} - \dfrac{4}{n+1} + \dfrac{5}{n+1} - \dfrac{4}{n+2}$	M1	Three fractions left at beginning, three at the end
	$= 1 + \dfrac{1}{n+1} - \dfrac{4}{n+2} \quad \left(= \dfrac{n^2}{(n+1)(n+2)}\right)$	A2 cao 8	Give A1 for an unsimplified correct expression (allowing r instead of n)
(c)	$\dfrac{3}{2-5} = -1$ So it is true when $n = 1$ Assuming it is true for $n = k$,	B1	
	$u_{k+1} = \dfrac{3\left(\dfrac{3}{2^k - 5}\right)}{5\left(\dfrac{3}{2^k-5}\right)+6}$	M1A1	
	$= \dfrac{9}{15 + 6(2^k - 5)}$	M1	Simplification to form $\dfrac{a}{b + c2^k}$
	$= \dfrac{3}{5 + 2(2^k - 5)}$	M1	Use of $2(2^k) = 2^{k+1}$
	$= \dfrac{3}{2^{k+1} - 5}$	A1	Correctly obtained
	True for $n = k \Rightarrow$ True for $n = k + 1$ Hence it is true for all positive integers n	A1 7	Stated or clearly implied *Dependent on previous 5 marks*

3 (i)	$\alpha^2 = -15 + 8j$ $\alpha^3 = (1+4j)(-15+8j) = -15 - 52j - 32$ $= -47 - 52j$	B1 M1 A1 cao 3	 Multiplying out and using $j^2 = -1$
(ii)	$-47 - 52j + 5(-15+8j) + k(1+4j) + m = 0$ Equating imaginary parts, $-52 + 40 + 4k = 0$ $k = 3$ Equating real parts, $-47 - 75 + k + m = 0$ $m = 119$	 M1 A1 cao M1 A1 (ag) 4	
(iii)	Another root is $1 - 4j$	B1	
	$(z-1-4j)(z-1+4j) = z^2 - 2z + 17$ Cubic is $(z^2 - 2z + 17)(z+7)$	M1 M1	Quadratic with real coefficients
	OR Sum of roots $(1+4j) + (1-4j) + \gamma = -5$ M2		
	OR Product of roots $(1+4j)(1-4j)\gamma = -m$ M2		
	Third root is -7 $\arg(1+4j) = 1.33$ (or $76°$, 0.42π, $\arctan 4$) $\arg(1-4j) = -1.33$ (or 4.96, $284°$, 1.58π) $\arg(-7) = \pi$ (or $180°$, 3.1, $-\pi$)	A1 cao B1 B1 ft B1 cao 7	-7 always earns B3 Allow 1.3 , 1.32 – 1.33 Can be earned from any negative root
(iv)	$\lvert 1+4j+2 \rvert = \lvert 3+4j \rvert = 5$ $\lvert 1-4j+2 \rvert = \lvert 3-4j \rvert = 5$ $\lvert -7+2 \rvert = 5$  	 B2 B2 B2 cao 6	 Give B1 for two correct A single circle with centre -2 and enclosing O (Give B1 for any circle with centre on the negative real axis) Roots correctly shown on locus Give B1 for two correct *Or* give B1 ft for three roots in approx correct positions (disregard locus)

4(a)(i)	If they intersect, $7+8\lambda = 5-2\mu$ (1) $13+6\lambda = -8+5\mu$ (2) $14+\lambda = -9+3\mu$ (3) Solving (1) and (2), $\lambda=-1$, $\mu=3$ Check in (3), LHS $=13$, RHS $=0$ Hence the lines do not intersect	M1 A1 A1 cao M1 A1 cao 5	Equating (at least two) components, using different parameters Two correct equations Or (e.g.) $\lambda=4$, $\mu=9$ from (2) & (3) $\lambda=-2$, $\mu=7$ from (1) & (3) Checking consistency *Dependent on previous 4 marks*
(ii)	$\begin{pmatrix}8\\6\\1\end{pmatrix}\times\begin{pmatrix}-2\\5\\3\end{pmatrix}=\begin{pmatrix}13\\-26\\52\end{pmatrix}$	M1 A2 3	Attempt to evaluate vector product of direction vectors (or other valid method) Give A1 if just one error
(iii)	Plane is $x-2y+4z+d=0$ Contains $(7,13,14)$, so $7-26+56+d=0$ Equation is $x-2y+4z-37=0$	M1 M1 A1 cao 3	For $x-2y+4z$ Using a point on *L* to find the constant *Dependent on previous M1* Accept $13x-26y+52z=481$ etc
	OR $x=7+8\lambda-2\mu$ $y=13+6\lambda+5\mu$ $z=14+\lambda+3\mu$ Eliminating λ and μ, M2 $x-2y+4z-37=0$ A1		Obtaining a linear equation
(b)(i)	(*A*) $\begin{pmatrix}4&2\\6&4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4x+2y\\6x+4y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$ The only invariant point is $(0,0)$ (*B*) $\begin{pmatrix}4&2\\6&5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4x+2y\\6x+5y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$ Set of invariant points is the line $y=-\frac{3}{2}x$	M1 A2 M1 A1 5	
(ii)	$\begin{pmatrix}4&2\\6&5\end{pmatrix}\begin{pmatrix}t\\2t-3\end{pmatrix}$ $=\begin{pmatrix}8t-6\\16t-15\end{pmatrix}$ $2(8t-6)-3=16t-15$, so the line $y=2x-3$ is invariant	M1 A1 M1 A1 4	Or finding images of *two* points on $y=2x-3$ Or images of two points correct Checking image(s) in $y'=2x'-3$ *Or* finding equation of image line
	OR The line $y=mx+c$ is invariant if $6x+5(mx+c)=m[4x+2(mx+c)]+c$ M2 which is satisfied by $m=2$, $c=-3$ A2		Properly shown

Examiner’s Report

Pure Mathematics 4 (2604)

General Comments

The general standard of work on this paper was high, with about 30% of candidates scoring 50 marks or more (out of 60) and a fair number obtaining full marks. Only about 20% scored less than 30 marks. Question 1 was the most popular, and question 4 was the least popular.

Comments on Individual Questions

Question 1 (Curve sketching and inequalities)

This question was attempted by almost every candidate, and it was (along with question 3) one of the best answered. Half of the candidates scored 16 marks or more (out of 20), and about one fifth scored full marks. Most found the equation of the asymptote in part (i), and the stationary points in part (ii), correctly. The graphs in parts (iii) and (iv) were sketched well; in particular the sharp points on the modulus graph were usually clearly shown. The inequality in part (v) caused problems for a significant proportion of candidates, although those who referred to the graph were often successful. Many attempted a purely algebraic method, usually beginning by squaring both sides, and it was rare for all three critical values to be found by such a method.

(i) $y=1$; (ii) $\frac{dy}{dx}=\frac{-3x^2+16x+12}{(x^2+4)^2}$, $\left(-\frac{2}{3}, -\frac{5}{4}\right)$ and $\left(6, \frac{5}{4}\right)$;

(iv) $\left(-\frac{2}{3}, \frac{5}{4}\right)$ and $\left(6, \frac{5}{4}\right)$; (v) $x<-\frac{3}{2}$, $0<x<\frac{8}{3}$.

Question 2 (Series and Induction)

The series in part (a) was usually summed correctly, the most common error being failure to deal with the constant term. Many lost the final mark for simplifying the expression, often because of poor manipulation of brackets and signs. The technique for using partial fractions and telescoping a sum in part (b) was generally well understood and very often completed correctly. The proof by induction in part (c) was done confidently by very many candidates; the main errors were starting with $n=2$ instead of $n=1$, and attempting to add on the next term as if it were a series.

(a) $n^2(8n+7)$; (b) $1+\frac{1}{n+1}-\frac{4}{n+2}$.

Question 3 (Complex numbers)

About a quarter of the attempts at this question scored full marks. Parts (i) and (ii) caused few problems. In part (iii), almost all candidates stated that the complex conjugate of α was a root, but the real root was frequently given as 7 or $(z+7)$ instead of -7; even when -7 had been correctly obtained, its argument was often given as 0 instead of π. In part (iv), most candidates knew that the locus was a circle, although it was often centred at 0 or at 2 instead of -2.

(i) $\alpha^2=-15+8j$, $\alpha^3=-47-52j$; (ii) $k=3$;
(iii) $1-4j$, -7, Arguments 1.33, -1.33, π; (iv) $c=5$.

Question 4 (Vectors and Matrices)

This question was attempted by about half of the candidates, and with a mean mark of about 12 it was the worst answered question. In part (a)(i), most candidates knew how to show that the two lines were skew, although surprisingly many made arithmetic errors when solving simultaneous equations. Most found the perpendicular vector in part (a)(ii) correctly, and used this to find the equation of the plane in part (a)(iii). However, a fair number used the direction vector of M as the normal vector of the plane. In part (b)(i)(*A*), most obtained the equations $4x+2y=x$, $6x+4y=y$, but few drew the correct conclusion from these; some gave two lines of invariant points, and some stated that there were no invariant points. Part (b)(i)(*B*), in which both equations give the same line, was more often answered correctly. Establishing the invariant line in the final part was generally well understood by those who attempted it.

(a)(ii) $\begin{pmatrix}1\\-2\\4\end{pmatrix}$; (iii) $x-2y+4z-37=0$; (b)(i)(*A*) (0, 0) only, (*B*) $y=-\frac{3}{2}x$.