OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2608/1

Mechanics 2

Thursday **7 JUNE 2001** Afternoon 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.

The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

1 Two young skaters, Percy of mass 55 kg and Queenie of mass 45 kg, are moving on a smooth horizontal plane of ice. You may assume that there are no external forces acting on the skaters in this plane. Percy and Queenie are moving with speeds of $2\ \mathrm{m\,s^{-1}}$ and $\frac{4}{3}\ \mathrm{m\,s^{-1}}$ respectively **towards** one another in the same line of motion. When they meet they embrace.

(i) Calculate the common velocity of the two skaters after they meet and the magnitude of the impulse between them in the collision. [5]

Percy and Queenie, still together, collide directly with a moving skater, Roger, of mass 60 kg. The coefficient of restitution in the collision is 0.2. After the collision, Percy and Queenie have a speed of $0.1\ \mathrm{m\,s^{-1}}$ in the same direction as before the collision.

(ii) Calculate Roger's velocity before the collision and his velocity after it. [6]

While moving at $0.1\ \mathrm{m\,s^{-1}}$ horizontally, Percy drops a small ball. The ball has zero vertical speed initially and drops 0.4 m onto the ice. The coefficient of restitution in the collision between the ball and the ice is 0.5.

(iii) At what angle to the horizontal does the ball leave the ice as it bounces? [4]

2 A parcel of mass 20 kg slides down a slope at 35° to the horizontal. Its speed increases from $4\ \mathrm{m\,s^{-1}}$ to $6\ \mathrm{m\,s^{-1}}$ while sliding a distance of 5 m down the slope.

(i) Calculate the work done against the resistance to motion. [4]

(ii) Assuming that a constant frictional force between the parcel and the slope is the only resistance to motion, show that the coefficient of friction between the parcel and the slope is 0.45, correct to two significant figures. [4]

(iii) For what value of the coefficient of friction would the parcel slide down the slope at a constant speed? [2]

The parcel is sliding down the slope and the coefficient of friction is 0.45. A force, applied parallel to the slope, does 520 J of work and brings the parcel to rest from $6\ \mathrm{m\,s^{-1}}$ in x m.

(iv) Calculate the value of x. [5]

3

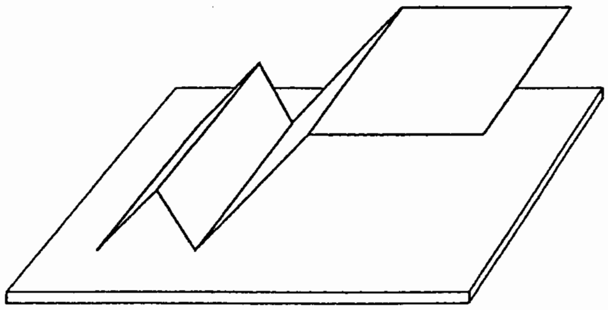


Fig. 3.1

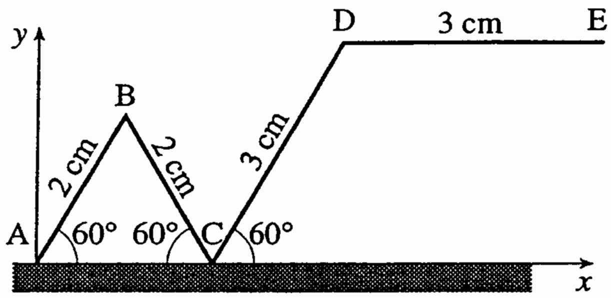


Fig. 3.2

A uniform, rectangular lamina of mass 25 kg is folded and placed on a horizontal floor, as shown in Fig. 3.1. Fig. 3.2 shows the cross-section ABCDE of the folded lamina. The dimensions and angles of the cross-section are given in Fig. 3.2 and DE is horizontal.

(i) Show that the x-coordinate of the centre of mass of the lamina is 2.725, referred to the axes shown in Fig. 3.2. Calculate also the y-coordinate, referred to the same axes, giving your answer correct to three decimal places. [6]

(ii) Explain briefly why the lamina cannot be in equilibrium in the position shown without the application of an additional force. [2]

(iii) What is the least vertical force that must be applied to the lamina at A so that it will stay in equilibrium in the position shown? [4]

Instead of applying the vertical force at A, a horizontal force is applied to the lamina at E.

(iv) Assuming that the lamina does not slide on the floor, calculate the greatest value the horizontal force at E can take without the lamina turning *anti*-clockwise. [3]

[Turn over

4

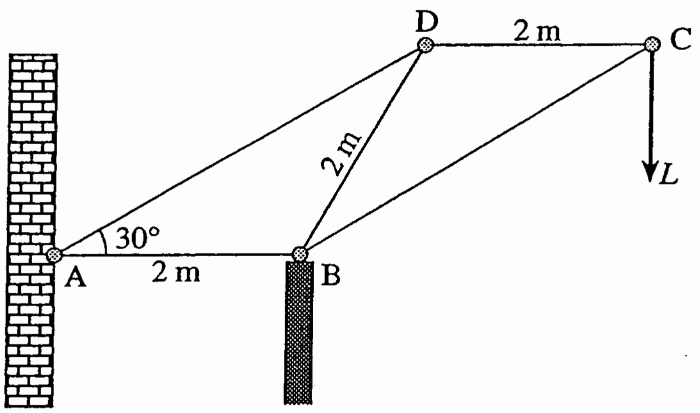


Fig. 4

Fig. 4 shows a vertical framework made from the light, rigid rods AB, BC, CD, DA and BD which are freely pin-jointed at A, B, C and D. ABCD is a parallelogram with AB = CD = BD = 2 m; angle DAB = 30°. The rod AB is horizontal. The framework is freely hinged to a wall at A and rests on a smooth support at B. A vertical load of L N is attached at C.

(i) Show that the support at B exerts a force of $\frac{5}{2}L$ N upwards on the framework and that the wall exerts a force of $\frac{3}{2}L$ N downwards on the framework at A. [5]

(ii) Draw a diagram showing all the forces acting on each of the pin-joints of the framework, including those due to the internal forces in the rods. Calculate the magnitudes of the internal forces in the rods and state for each rod whether it is in tension or compression. [You may express your answers in terms of surds where appropriate.] [10]

Mark Scheme

1. (i) Before P → 2 m s^{-1}, ← Q $\frac{4}{3}$ m s^{-1}		
After PQ → v m s^{-1}		
PCLM		
$55 \times 2 - 45 \times \frac{4}{3} = 100v$	M1 PCLM applied B1 signs consistent	
$v = 0.5$ so 0.5 m s^{-1} in original direction of Percy	A1 F1 Either explicit or implied by diagram	
Impulse is $\lvert 55(2 - 0.5) \rvert = 82.5$ Ns	F1	[5]
(ii) Before PQ → 0.5 m s^{-1}, R → v m s^{-1}		
After PQ → 0.1 m s^{-1}, R → v' m s^{-1}		
PCLM $50 + 60v = 10 + 60v'$ $3v' - 3v = 2$	M1 PCLM A1 Any form	
NEL $\frac{v' - 0.1}{v - 0.5} = -0.2$ $v' + 0.2v = 0.2$	M1 Including consistent use of signs A1 Any form	
Solving $v = -\frac{7}{18}$, $v' = \frac{5}{18}$		
So before, $-\frac{7}{18}$ m s^{-1} (opp dir to PQ) after, $\frac{5}{18}$ m s^{-1} (same dir as PQ)	A1 A1 (Award max A1 for final answers unless directions both specified or implied by Diagram)	[6]
(iii) Ball hits ice at vert speed $\sqrt{2 \times 0.4 \times 9.8}$ $= 2.8$ m s^{-1}	B1	
Linear momentum conserved horiz NEL on vert cpt gives 1.4 m s^{-1} up so after bounce 0.1 m s^{-1} horiz and 1.4 m s^{-1} up	M1 May be implied e.g. in diagram B1	
Angle is $\arctan(\frac{1.4}{0.1}) \approx 86°$	A1	[4]

[Total 15]

Solutions and mark scheme

2. (i)	$20 \times 9.8 \times 5 \times \sin 35 - \frac{1}{2} \times 20 \times (6^2 - 4^2)$	M1 Difference in GPE and KE	
		B1 GPE term	
		B1 Either KE term	
	$= 362.104..$ so 362 J (3sf)	A1 Accept 2 sf	[4]
(ii)	$5F = 362.104...$ so $F = 72.4209...$	B1	
	$R = 20 \times 9.8 \times \cos 35$	B1	
	$\mu = 0.4510...$ so 0.45 (2sf)	M1 Use of $F = \mu R$	
		E1	[4]
(iii)	$\mu mg \cos 35 = mg \sin 35$	M1	
	$\mu = 0.70$ (2sf)	A1 (Accept WW for both marks)	[2]
(iv)	$72.2492.. \times x + 520 - 20gx \sin 35$	M1 Use of work-energy, allow 1 term missing	
	$= \frac{1}{2} \times 20 \times 6^2$	B1 Equation contains GPE term	
		M1 All relevant terms present	
		A1 Signs correct	
	$x = 3.982...$ so 3.98 m (3 sf)	A1 cao	[5]
		[FT original μ from (ii) for full marks]	
		[Total 15]	

Alternate schemes using N2L

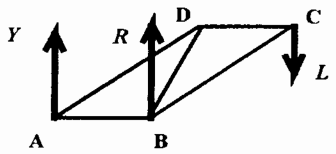
(i)	N2L +'*uvast*'	M1
	$a = 2$	B1
	N2L equation with all terms correct	B1
	362 J (3sf)	A1
(ii)	N2L	M1
	R	B1
	Use of $F = \mu R$	M1
	0.45 (2.s.f.)	E1
(iv)	N2L +'*uvast*'	M1
	Force is $\frac{520}{x}$	B1
	N2L equation with all relevant terms present	M1
	Correct acceleration expression	A1
	$x = 3.982...$ so 3.98 m (3 sf)	A1 cao

3. (i) $10\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 2\begin{pmatrix}\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix} + 2\begin{pmatrix}\frac{3}{2}\\ \frac{\sqrt{3}}{2}\end{pmatrix} + 3\begin{pmatrix}2.75\\ \frac{3\sqrt{3}}{4}\end{pmatrix} + 3\begin{pmatrix}5\\ \frac{3\sqrt{3}}{2}\end{pmatrix}$

	M1 Appropriate method	
	B1 Correct masses	
	B1 At least two x cpts correct	
	B1 At least two y cpts correct	
(2.725, 1.516)	E1 A1	[6]
(ii) cm gives a clockwise moment about C Reaction at A cannot give an a.c. moment	E1 Considering moments	
	E1 Complete argument	[2]
(iii) Moments about C	M1	
$2w = 25g \times 0.725$	A1	
	B1 Use of weight	
$w =$ 88.8125 so about 88.81 N	A1 cao	[4]
(iv) Moments about A	M1	
$3\frac{\sqrt{3}}{2}F = 25g \times 2.725$	A1 Must be a correct statement	
$F = 256.968..$ so about 257 N	A1 Any reasonable accuracy	[3]

[Total 15]

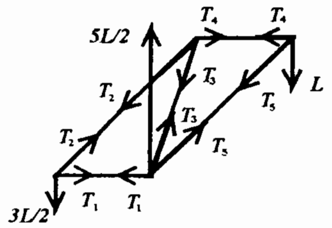
4. (i)



For the whole framework

$\curvearrowright$A $2R = (2 + 2\sin 30 + 2)L$	M1 Taking moments B1 RHS (Award M1 B0 if no supporting evidence for $2R = 5L$)	
$\uparrow R + Y = L$	B1 Resolve or take moments again	
so $R = \frac{5L}{2}$ and $Y = -\frac{3L}{2}$	E1	
so upwards at B and downwards at A	E1 Directions clearly established (accept a clear diagram)	[5]

(ii)



B1 All forces present with arrows and labels

Equilibrium at the pin-joints — M1 Attempt at equilibrium at a pin-joint

A$\uparrow$ $T_2 \cos 60 + Y = 0$
$T_2 = 3L$ — M1 Systematic attempts at three or more equilibria

A$\rightarrow$ $T_2 \cos 30 + T_1 = 0$ — B1 At least one equilibrium correct
$T_1 = -\frac{3\sqrt{3}\,L}{2}$

C$\downarrow$ $T_5 \cos 60 + L = 0$
$T_5 = -2L$

C$\leftarrow$ $T_4 + T_5 \cos 30 = 0$
$T_4 = \sqrt{3}\,L$

B$\rightarrow$ $T_5 \cos 30 + T_3 \cos 60 - T_1 = 0$
$T_3 = -\sqrt{3}\,L$ — B1 Correct interpretation of signs to give T and C

AB	$\frac{3\sqrt{3}\,L}{2}$ N	(C)	A1	
BC	$2L$ N	(C)	F1	
CD	$\sqrt{3}\,L$	(T)	F1	
DA	$3L$	(T)	F1	
BD	$\sqrt{3}\,L$	(C)	F1	[10]

[Total 15]

Examiner’s Report

Mechanics 2 (2608)

General Comments

The paper seemed to be of about the correct length and many candidates made good attempts at all of the questions. Although most of the candidates from some centres did very well there were a few other centres where most obtained rather low marks and did not seem to be ready for the examination.

The standard of the diagrams seen was quite good but many candidates did not give essential diagrams and this often caused sign errors. Those parts of the questions that required explanation or interpretation were often not done well.

Comments on Individual Questions

Question 1 (Impulse and momentum)

Part (i) was meant to be straightforward but some candidates did not give a diagram or indicate a positive direction and this often produced a sign error. Few candidates knew how to calculate the impulse and the mark for this was rarely scored.

In part (ii), the principle of conservation of linear momentum was usually applied correctly, although there were some sign errors. Newton's experimental law often contained sign errors, usually in the absence of a diagram. There were many errors seen in the solution of the simultaneous equations.

Part (iii) was on the item new to the syllabus and most of the candidates knew what to do and did it well. A few candidates applied Newton's experimental law to both velocity components.

(i) 0.5 m s^{-1} in the original direction of Percy, 82.5 Ns; (ii) before $-\frac{7}{18}$ m s^{-1}, after $\frac{5}{18}$ m s^{-1} both referred to the original direction of Percy + Queenie; (iii) 85.9° (3 s. f.)

Question 2 (Work and energy)

Candidates who used an energy approach to part (i) usually did well, although some did not seem to understand exactly what was required of them. Those using Newton's second law usually found the correct acceleration but many then showed a lack of understanding and simply argued that the frictional force must be *ma*, giving 200 J.

Part (ii) was usually done well, even by those who could not find the correct frictional force in part (i). Obviously the given answer helped. Only a few candidates then amended their answer to part (i), having now obtained the correct frictional force in this part (confirmed by the given result).

Part (iii) was usually done well although a few candidates thought that $\mu = 0$.

It was rare to see a complete argument in part (iv) and this was a place where strong candidates could distinguish themselves. Most partial attempts used Newton's second law and the constant acceleration formulae but candidates either forgot a term (usually the friction) or made calculation errors or could not deal with the force that did 520 J of work. These attempts often took up much more time and space than the complete solution using a work-energy approach.

(i) 362 J; (iii) 0.700 (3 s. f.); (iv) 3.98 m (3 s. f.)

Question 3 (Centre of mass)

Part (i) was generally well understood and done fairly well; the mistakes were usually arithmetical or failure to read the question, which asked for 3 decimal places.

Part (ii) required an explanation and few of these were sufficiently complete to merit both marks; it was necessary both to consider the moments about C and also to recognise that the normal reaction at A can only act upwards.

In part (ii), most candidates knew that they should take moments about C, but very common errors were

to use mass instead of weight in the moment calculation,

wrongly to use 10 kg for the mass,

incorrectly to find the horizontal distance of C from the centre of mass.

Many candidates in this part (and the next) considered the elements of the structure from first principles and so recalculated the x component of the centre of mass.

Part (iv) was often done badly. Many candidates tried a different set of moments about C and those who took moments about A often used the horizontal instead of the vertical distance of E from A.

(i) 1.516 (3 d. p.); (iii) 88.8 N (3 s. f.); (iv) 257 N (3 s. f.)

Question 4 (Freely pin-jointed light framework)

In part (i), most candidates obtained answers, but since they were given, complete arguments were needed for full marks; many candidates failed to justify the lengths used or did not make the directions clear.

Virtually everyone knew what to do in part (ii) but there were a lot of errors; these were mainly arithmetical but sometimes a component was forgotten or wrongly resolved. A very common error was made at C where the resolution $T_{BC} \sin 30 + L = 0$ or $T_{BC} \cos 60 + L = 0$ was solved to give $T_{BC} = -\frac{1}{2}L$ instead of $-2L$. This mistake was seen from candidates of all abilities.

(ii) rod AB, $\frac{3\sqrt{3}}{2}L$ N (C); rod BC, $2L$ N (C); rod CD, $\sqrt{3}L$ N (T);

rod DA, $3L$ N (T); rod BD, $\sqrt{3}L$ N (C)