Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Thursday 14 JUNE 2001 Morning 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

An **INSERT** is provided for Question **5** parts **(a) (i)** and **(a) (ii)** and Question **6** part **(a) (iii)**.

This question paper consists of 6 printed pages, 2 blank pages and an insert.

Section A

1 The table below shows the numbers of vertices of orders 1, 2 and 3 in three graphs. Draw an example of each of these graphs.

Order of vertex	1	2	3
Graph 1	3	0	1
Graph 2	2	2	0
Graph 3	1	1	1

[Total: 5]

2 (i) Two-digit random numbers are to be used to simulate realisations of the random variable which is defined by the following table.

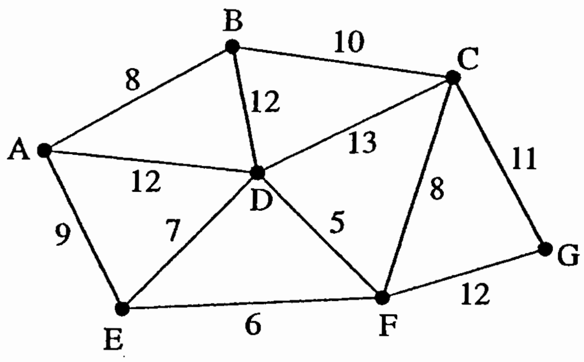
Value	1	2	3	4	5
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$

Produce a table showing which random numbers are to be associated with each value. You should ensure that your table leads to an efficient use of random numbers. [4]

(ii) Explain whether or not three-digit random numbers would be more efficient. [2]

[Total: 6]

3 Use an algorithm to find a minimum connector for the following network. Give the name of your algorithm. Explain the first three steps in this case. Give the total weight of your minimum connector.



[Total: 5]

Section B

4 The Chief Executive of Leschester City Football Club plc has up to £4 million to spend following a good cup run. He has to decide on spending priorities. Money needs to be spent on strengthening the playing squad and on extra support facilities (i.e. non-playing staff and stadium facilities).

The Coach, who is popular with the fans, has said that he will resign unless he gets at least £2 million to spend on new players.

The authorities require that at least £0.6 million be spent to remedy stadium deficiencies affecting crowd safety.

Club policy is that the amount to be spent on support facilities must be at least one quarter of the amount to be spent on the playing squad.

(i) Let £x million be the amount to be spent on the playing squad and let £y million be the amount to be spent on support facilities. Write down four inequalities in terms of x and y representing constraints on spending. [4]

(ii) Draw a graph to illustrate your inequalities. [4]

(iii) Find the maximum amount which may be spent on the playing squad. [2]

A report commissioned from a market research company indicates that fans regard both team performance and facilities as being important. The report states that the function $0.8x + 0.2y$ gives a measure of satisfaction with extra expenditure.

The Chief Executive proposes to spend £2.5 million on the playing squad and £1.5 million on support facilities.

(iv) Calculate the measure of satisfaction corresponding to the Chief Executive's proposals. [1]

(v) Add to your graph the line $0.8x + 0.2y = 2.3$, and explain what points on this line represent. [2]

(vi) The Coach argues that the Chief Executive can achieve the same satisfaction score by spending less in total, but more on the playing squad. How much less and how much more? [2]

[Total: 15]

[Turn over

5 **An insert is provided for this question.**

(a) The coach of a netball team has to arrange three pre-season training sessions, each of length 90 minutes. She wants to schedule the activities which are listed below. Some are to be scheduled more than once.

Activity		Duration (mins)	Number of times activity is to be scheduled
A	shooting practice	10	3
B	passing practice	15	3
C	blocking practice	12	3
D	sprinting	5	3
E	intermediate distance running	14	2
F	long distance running	20	1
G	team games	12	3
H	4-a-side practice game	20	2
I	full scale practice game	20	1

(i) Use the first fit decreasing algorithm to allocate activites to each of the three training sessions, giving your answers in Table 5.1 on the insert. [3]

(ii) The solution given by first fit decreasing is not satisfactory since it leads to repeated activities in the same session. The first fit decreasing algorithm is modified so that the next activity is placed in the first available session only if it will fit and if the same activity has not already been placed in that session.

Apply this modified algorithm until it fails, using Table 5.2 on the insert. [3]

(iii) Prove that it is not possible to fit the activities into the three sessions so that no session contains a repeated activity. [2]

Part (b) of this question is on the facing page.

(b) At the end of pre-season training the coach has allocated points to ten players. She will choose the seven highest-scoring players for her team for the first match.

Player	A	B	C	D	E	F	G	H	I	J
Points	81	92	76	43	82	45	51	93	71	62

(i) Count the number of comparisons she would have to make if she were to:

1 check along the points table from left to right, comparing the first number with the second, then the larger with the third, and so on to find the player with the largest score;

2 choose that player for the team and delete the player's entry from the points table;

3 repeat the process on the reduced points table until the team is chosen. [2]

(ii) Instead she executes a bubble sort on the numbers in the list, starting from the left and with smaller numbers moving to the right. She does this only until she can be sure that the three lowest scores are in the three rightmost positions.

Show the steps of this sort, and state the number of comparisons that are made. [5]

[Total:15]

6 **An insert is provided for this question.**

(a) The activities involved in clearing and covering an outside court at a major tennis tournament are shown in the table below. Also shown are their immediate predecessors and the times required to complete them.

Activity		Immediate predecessor(s)	Duration (mins)
A	escort players and officials off court	–	1
B	disconnect electronic sensors	–	0.5
C	remove umpire's chair	A, B	1
D	remove 8 line judges' chairs	A, B	0.5
E	remove net	B	0.5
F	remove the two net posts	E	1
G	pull out waterproof cover	C, D, F	0.5

(i) Draw an activity on arc network for these activities. [3]

(ii) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. [6]

(iii) Use the grid provided on the insert to produce a cascade chart, given that all activities are scheduled to start as early as possible. [2]

(b) The numbers of staff required for activities A, B, C and E are 1, 1, 2 and 2 respectively.

It takes one person 0.5 min to remove one line judge's chair. It takes one person 1 min to remove one net post. To pull out the waterproof cover takes eight people 0.5 min, or takes four people 1 min.

(i) Give the minimum number of staff needed to complete the clearing and covering of the court in the minimum total time. Which activity can be resourced at less than the maximum level to achieve this? [2]

(ii) Give the minimum time in which the clearing and covering can be completed if only 4 staff are available. (You may use the spare grid on the insert if you wish.) [2]

[Total: 15]

Candidate Name	Centre Number	Candidate Number

OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

INSERT

Thursday 14 June 2001 Morning 1 hour 20 minutes

Instructions to candidates

This insert should be used in Question **5** parts **(a)(i)** and **(a)(ii)** and Question **6** part **(a)(iii)**.

Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.

Insert for Question 5

Session	Activity									
first										
second										
third										

Table 5.1

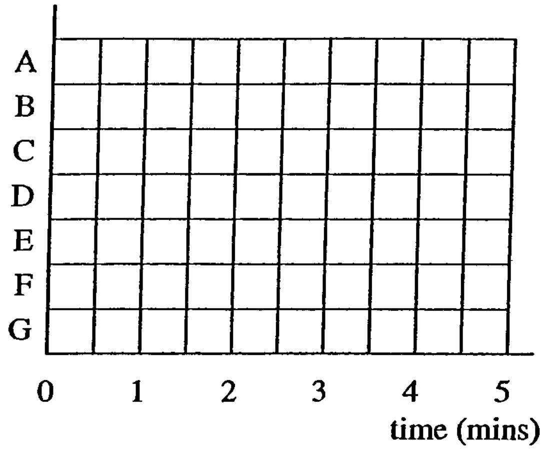
Session	Activity									
first										
second										
third										

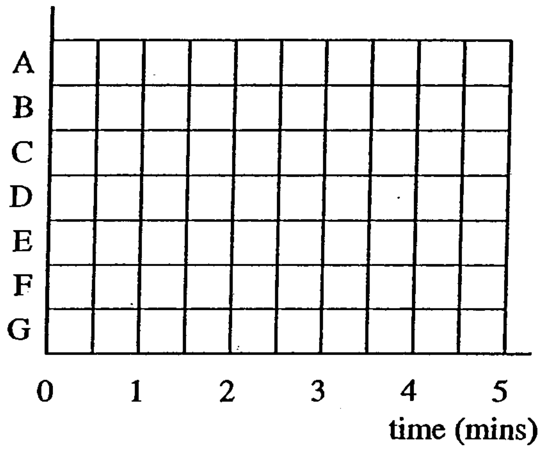
Table 5.2

This insert consists of 2 printed pages.

[Turn over

Insert for Question 6





Mark Scheme

OCR

General Certificate Examination

Advanced Level

MEI STRUCTURED MATHEMATICS

2620: Decision and Discrete Mathematics I

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

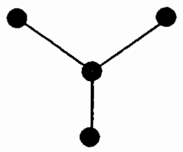
M marks can be implied by correct answers.

A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

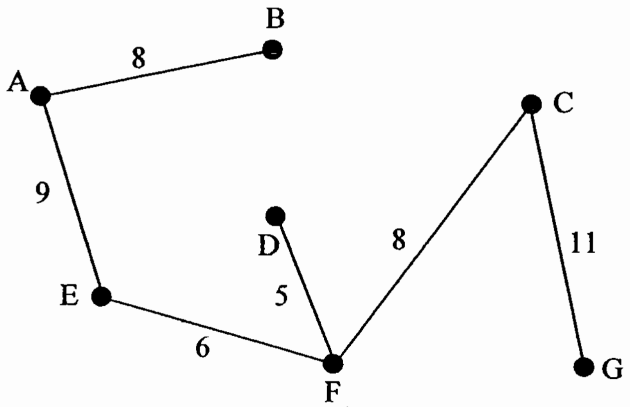
B marks are for specific results or statements, and are independent of method.

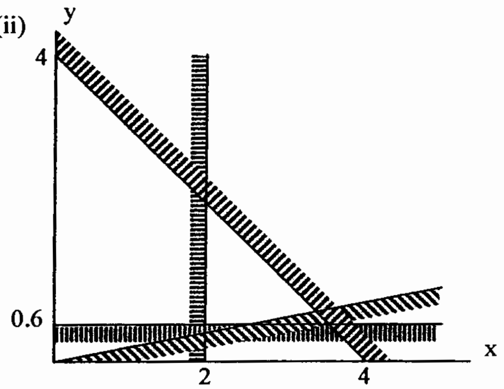
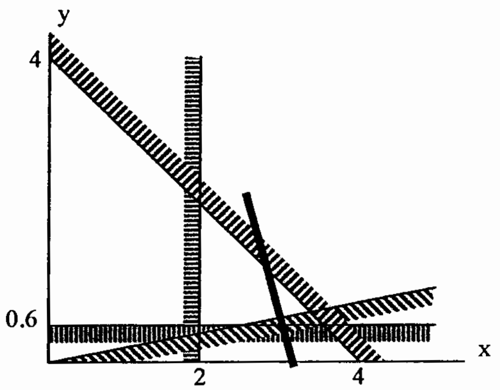
√ marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of û1, û2 or û3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

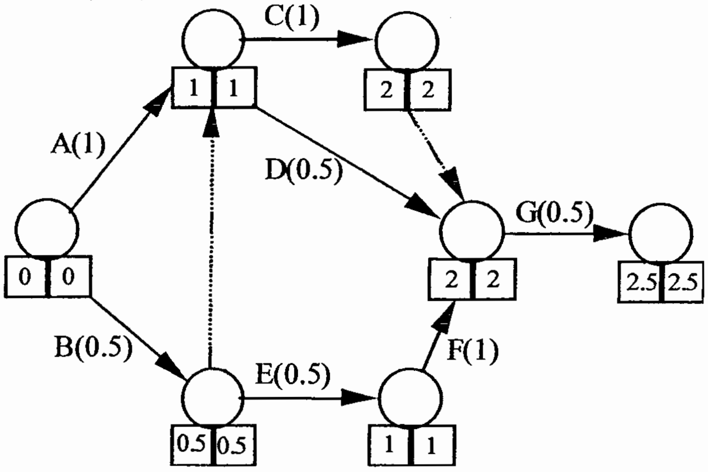
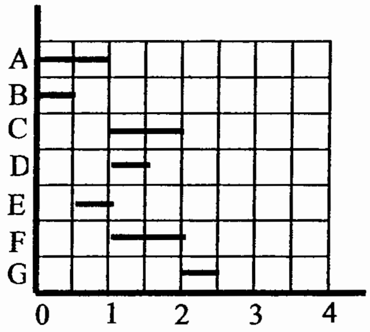
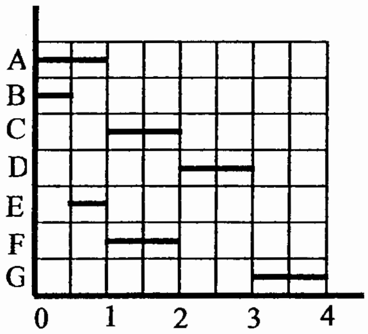
1. e.g.	
Graph 1 	M1 sca
	A1
Graph 2 	A1
Graph 3 	A1

2. e.g.		
00 – 07	1	M1 some numbers ignored
08 – 23	2	A2 (–1 each error)
24 – 47	3	A1 efficient use of numbers (use of 96 numbers)
48 – 79	4	
80 – 95	5	
96 – 99	ignore and re-draw	
Lower proportion wasted (4 out of 100)		M1 a reasonable (but possibly incomplete) explanation + 3 figures better
(B1 allowed for "More digits used")		A1 correct explanation
		Special case
		B1 an argument that 3 fig worse than 2 as it uses more numbers

3. 	B1 tree B1 correct min connector B1 total weight = 47 M1 demonstration or explanation of algorithm A1 name

4. (i) $x+y \le 4$ $x \ge 2$ $y \ge 0.6$ $y \ge 0.25x$	B1 B1 B1 B1
(ii) 	B1 axes labelled and scaled M1 lines A1 B1 shading
(iii) £3.2million	M1 A1 cao
(iv) 2.3 = 0.8*2.5 + 0.2*1.5 	B1 B1 line B1 points of equal (fan) satisfaction
(vi) £3,382,000 in total (£618,000 less) £2,706,000 on the playing squad (£206,000 more)	M1 A1

5. (a) (i) session 1 F H H I A session 2 B B B E E C D session 3 C C G G G A A D D	M1 sca - in particular, **first** fit A1 order correct within sessions A1 activities in correct sessions
(ii) session 1 F H I B E session 2 H B E C session 3 B C	M1 sca - **first** fit A1 correct up to second C A1 terminating at third C
(iii) e.g. Each session must have A, B, C (D) and G, leaving 41 (36) mins. But there are four 20 minute activities, so one must have two of these. That leaves one minute, so there will not be a fit.	M1 A1
(b) (i) 9+8+7+6+5+4+3 = 42	M1 A1
(ii) B A C E F G H I J D 92 81 76 82 45 51 93 71 62 43 B A E C G H I J F D 92 81 82 76 51 93 71 62 45 43 B E A C H I J G F D 92 82 81 76 93 71 62 51 45 43	M1 bubble sort A1 first pass correct A1 two more passes correct A1 stop after three passes
9+8+7 = 24 comparisons	B1 cao

6. (a) (i)&(ii)	
	M1 sca A1 correct dummies A1 rest correct M1 forward pass A1 M1 backward pass A1
time - 2.5 mins critical - A; B; C; E; F; G	B1 cao B1 cao
(iii) 	M1 A1
(b) (i) 8 staff activity D (by using 4 staff to shift chairs over 1 min)	B1 cao B1 cao
(ii) 4 mins, e.g. 	M1 A1

Examiner’s Report

Decision and Discrete Mathematics 1 (2620)

General Comments

Compared to January, fewer candidates gained high marks and there was a definite skew towards the bottom end. There was less differentiation between average and weaker candidates.

Of the weaker candidates, quite a number scored very poorly indeed. Of these, some did very little of the paper. Others seemed to have very little knowledge on which to base their answers. In some centres identical errors cropped up in every script.

The paper was generally considered to be too long, with too much writing, there being evidence of some reasonable candidates not finishing. This may have been at least partly due to the change of format, although similar problems were not apparent in January. There was no such evidence of time pressures on candidates for 5519, which shared 50% of the material (algorithms and CPA), but on which candidates were provided with spaces for answers on the paper.

Comments on Individual Questions

Question 1 (graphs)

It is clear that students were ill-prepared in general for a graphs question. There were very few fully correct answers. The most common response gained a method mark but lost all accuracy marks for the question.

Too many candidates do not know the definition of a graph. Some either gave no response at all or thought that a graph needed axes.

Question 2 (simulation)

A good proportion of candidates realised that 96 values were needed. Most used 00-95, some 01-96. A few correctly proportioned the random numbers from 00 as required but forgot that the last value would be 95 and quoted 96. Quite a number incorrectly used 100 random numbers. Yet others correctly used multiples of eight numbers for some fractions, but for the range with probability of 0.25 used 25 numbers, thereby unbalancing the ratio between the ranges.

Part (ii) was not done very well at all. Some spotted that the same number of values would be wasted but very few realised that this would be a smaller proportion. Many tried to discuss the situation in terms of the accuracy of the values being used – three figures being thought to be more accurate than two!

Question 3 (networks)

There were many good answers to this, the best done of the short questions. A few reached for Dijkstra. Of those who had a better idea of what they were doing, some made slips, and some described Prim but used Kruskal or vice versa.

Minimum connector has total weight 47.

Question 4 (LP)

Some candidates did well on this question, but many found it difficult to sort out the appropriate inequalities. However, they were often able to earn marks on follow through. Not many were able to describe what the points of the line represented in part (v). Only the most able candidates were successful with part (vi).

(iii) £3.2million (vi) £618 000 less in total, and £206 000 more on the playing squad

Question 5 (algorithms)

Part (a) was very disappointing. Too many candidates failed to use first fit decreasing. Very few could produce a proof in part (a)(iii).

In part (b) most candidates got 42 or 45 for part (i), but very heavy weather was made of the bubble sort. Many candidates failed to follow the specific instructions. Others pursued the algorithm through to its conclusion to give a complete re-ordering. Some candidates used Shuttle Sort or Interchange Sort. Yet others gave an explanation of the process of the bubble rather than the sequence after each pass. Not many obtained the correct value of 24 for the number of comparisons.

(b)(i) 42 (ii) 24 comparisons

Question 6 (CPA)

In part (a) many good diagrams were seen with correct forward passes and backward passes, although the C/D dummy was often missed. A surprising number omitted to give the duration or the critical activities, and the critical activities when given were very often not complete. Nearly everyone managed the correct answer to part (iii), even if it did not match their network!

In part (b) candidates fared less well. Few correct answers were seen to (b)(i), and few correct answers of 4 minutes were seen in the final part.

This was the best answered question. Even weak candidates could often draw a plausible network and do the passes, even if they couldn't answer part (b).

There were, however, some cases in which activity on node was used. It is made clear in the specification that activity on arc is the only acceptable method. Other subjects may use activity on node, and centres may find it useful to check if the confusion with students is arising from this source.

(a)(ii) 2.5 minutes; activities A, B, C, E, F and G (b)(i) 8 staff; activity D (ii) 4 minutes

Decision and Discrete Mathematics 2 (2621)

General Comments

This was the first sitting of this examination, and there was only a small number of candidates. Performances were generally good.

Comments on Individual Questions

Question 1 (logic)

This question hinged on understanding that "if ... then ..." is expressed as $\Rightarrow$. Follow through was applied as far as possible to the large numbers of candidates who did not make that step, but subsequent work was not always of much value.

It would help the markers, and some candidates, if truth tables were organised with rows in binary order. They were often not, and lines were sometimes omitted as a consequence.

(i) $(\sim s \Rightarrow \sim n) \wedge n$ (iii) no snow and no north wind

Question 2 (decision analysis)

Most candidates were successful with part (i), weaker candidates using a verification approach.

Part (ii) was also answered well, although the expected number of errors were seen in the handling of chance and choice nodes. Few managed the final calculation.

(ii) value of advice = 6; 50 would have to increase to 80

Decision and Discrete Mathematics 1 (2620)

Some of the points in this report have been made before, on 5519 and on 2620 in January. They are repeated for obvious reasons.

General

The vast majority of the projects were on the CPA or Simulation syllabus areas. There were only a few cases of off-syllabus work, in each case where the problem identified was a TSP or Route Inspection problem. This is something that centres need to watch very carefully if candidates are attempting networks projects. The problem identified must not be a 2621 problem.

Candidates who had been given very prescriptive advice or even fully specified problems tended to produce much less successful projects. Such candidates cannot score much for their problem identification.

Some candidates still do not explain how they obtained or justified their data (for example activity durations and/or precedences). However, this aspect is improving. Candidates also often fail to consider the quality of their data and the implications of its inaccuracy.

There were few cases this time of unnecessary out-of-context theory being quoted, but it did happen. The candidate can assume that the reader is familiar with the workings of the standard algorithms. What the reader needs to see is evidence of the candidate's correct application of the algorithm in the context of the problem identified. Too often a candidate claims to have used, say, Prim's algorithm but provides absolutely no evidence of the order in which the vertices were selected. Only the final MST appears - and as far as the moderator can see it has been obtained by inspection.

Candidates aiming for high marks need to consider their extension ideas sensibly. Too often a claimed extension does not result in any extension of the mathematics, and sometimes in fact the analysis is repeated more simply. For example, a candidate with a Dijkstra problem contrives to remove a couple of nodes (often for a reason which seems spurious) and then repeats Dijkstra on an easier network.

Networks

There were relatively few projects in this area, and most of these were rather straightforward. There were some notable exceptions including good extensions involving modifications of Dijkstra's algorithm, and thoughtful modelling of times.

In this area particularly, candidates often omit evidence of their use of algorithms (see the general points above). The reader cannot see if Prim's algorithm has been correctly applied if all that is presented is the final MST. Some claimed uses of Dijkstra looked like inspection to the moderator.

Moderators commented that large networks are necessary to provide the complexity needed for earning high marks in this area.

Linear Programming

Several of these were seen, sometimes from all of the candidates in a centre. Some of them were pleasing, and clearly rooted in real-life experience. We also saw quite a lot of quite trivial text-book type problems about buns and cakes, with a surprising interest in production of chocolate brownies! Although the mathematical models are all quite similar, the best candidates had clear problem identification and were able to do some detailed and interesting interpretation.

It is expected that this will be a growth area as centres become more confident with the new syllabus.

Simulation

Much of the work in this area was very good indeed, with some excellent use of spreadsheets. In some cases weaker candidates used arbitrary simulation rules. There are also those who are determined to do a simulation where there is no evidence of a problem.

Apart from these issues, weaknesses fell into three general types. There were those who modelled and simulated, but did not compare with the real situation before varying the conditions. There were those who modelled but did not even simulate the existing situation before varying the conditions. And there were those whose only objective was to simulate the real situation, so they did compare, but they did not vary the conditions.

Some good modelling and simulation was spoiled by poor or non-existent interpretation.

There were some original games simulations, restaurant seating with a number of factors (such as group size), and also stock control and traffic light sequences in addition to the usual shop and petrol queues.

CPA

Most networks contained at least 20 activities, although there were a few centres where almost all candidates chose projects with far fewer activities and simple precedences. These produced very linear or parallel networks, often with nearly every activity critical, and very little scope for worthwhile resource levelling. Such candidates need to be steered into more sophisticated work.

More candidates than previously took the time to discuss their activities and the relationships between them.

There were still problems with dummy activities. Dummies are needed either to model the required precedences or to ensure that no two activities have the same i and j event.

Most candidates carried out CPA right through to resource levelling, and some very good discussions of the related practicalities were seen. More candidates were prepared to tackle multi-person activities. There were several instances where unoccupied people were moved to job-sharing roles so that critical activity durations could be reduced.

Graphs

No moderator reported seeing work in this area. One centre did label their work Graph Theory - but the projects were on Networks.

Decision and Discrete Mathematics 2 (2621)

With such a small entry for this first sitting, it is difficult to make any general points. Most of the candidates were from experienced D&D centres, and there were no surprises. No moderating adjustments were made.

Overwhelmingly the projects were on the Networks section, with just a handful of exceptions. The exceptions were too few in number to generate any advice.

The few Route Inspection projects were generally very sound, with appropriate attention paid to data sources.

There were far more TSP projects, and some of them indicated confusion between the classical and the practical problem. Candidates' problems are nearly always practical problems, with initially incomplete networks and with no constraint on revisiting vertices. In the classical problem, the network is complete, and no vertex may be revisited.

The approach adopted in the syllabus is to convert the practical problem into the classical problem, with a complete network of shortest distances (typically some of these shortest distances will be via other nodes). The method for producing a lower bound is by deleting a vertex and its edges, finding the length of the minumum spanning tree of the remaining network, and then adding in the lengths of the two shortest deleted