OCR

RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level

former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS 5515

Statistics 3

Monday **22 JANUARY 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
Students' Handbook

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/ answer booklet.

Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown on the answer paper to indicate that a correct method is being used.

This question paper consists of 3 printed pages and 1 blank page.

HN

1 Iron ore is loaded into railway wagons by a hopper. The nominal capacity of each wagon is 76 tonnes of ore. In fact, the amount of ore in tonnes delivered into each wagon is $75 + X$, where X is the continuous random variable having probability density function

$$f(x) = \begin{cases} kx^2(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{elsewhere.} \end{cases}$$

(i) Show that $k = 12$. [2]

(ii) Find the probability that the amount of ore delivered into a wagon exceeds $75\frac{2}{3}$ tonnes. [4]

(iii) Find the mean amount of ore delivered into a wagon. [4]

(iv) You are *given* that the standard deviation of the amount of ore delivered into a wagon is $\frac{1}{5}$ tonne. A train is made up of 30 wagons, considered as a random sample from the loaded wagons in a large yard. Find the probability that the average amount of ore per wagon in the train exceeds $75\frac{2}{3}$ tonnes. [5]

2 The label on a particular size of milk bottle states that it holds 1.136 litres of milk. In an investigation at the bottling plant, the contents x litres of 100 such bottles are carefully measured. The data are summarised by

$$\sum x = 112.4, \quad \sum x^2 = 126.80.$$

(i) Estimate the variance of the underlying population. [2]

(ii) Provide a 90% confidence interval for the mean of the underlying population, stating the assumptions you have made. [7]

(iii) A manager states that "the probability that the population mean lies in the calculated interval is 90%". Explain why this interpretation is wrong. Give the correct interpretation of the interval. [4]

(iv) Use the calculated interval to explain whether it appears that the target of 1.136 litres in a bottle is being met. [2]

3 A trial is being made of a new diet for feeding pigs. Ten pigs are selected and their increases in weight (in kg) are measured, over a certain period using the new diet. The data are as follows.

15.2 13.8 14.6 15.8 13.1 14.9 17.2 15.1 14.9 15.2

The underlying population can be assumed to be Normally distributed.

(i) Using an established diet, the mean increase in weight of pigs over the period is known to be 14.0 kg. Test at the 5% level of significance whether the new diet is an improvement, stating carefully your null and alternative hypotheses and your conclusion. [8]

(ii) Provide a 95% confidence interval for the mean increase in weight using the new diet. [4]

(iii) Little information about the conduct of the trial is given in the opening paragraph of the question. Comment on *two* aspects of how the trial should have been conducted. [3]

4 An organisation monitors "visits" to its website. Records show that, on average, 30% of visitors have never visited the website previously. As part of an inspection of the records for a certain period, a manager selects 100 random samples of 12 consecutive visits at intervals during the period and notes the values of X, the number in the sample who had never visited previously. The results are as follows.

x	0	1	2	3	4	5	$\geqslant 6$
frequency	6	10	16	15	16	21	16

(i) Under what conditions would X be modelled by a binomial distribution? [2]

(ii) Carry out a test, at the 5% significance level, to examine whether the binomial model with $p = 0.3$ fits the data well. [9]

(iii) Discuss your conclusions, referring both to the fit of the model and the conditions identified in your answer to part (i). [4]

Mark Scheme

1	(i)	$f(x) = Kx^2(1-x) \quad 0 \leqslant x \leqslant 1$ $1 = \int_0^1 f(x)\,dx$ $= K\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = K\left[\frac{1}{3} - \frac{1}{4}\right] = K.\frac{1}{12} \quad \therefore \underline{K = 12}$	M1 1	Beware printed answer	2
	(ii)	Want $P\left(X > \frac{2}{3}\right) = \int_{\frac{2}{3}}^1 f(x)\,dx$ $= \int_{\frac{2}{3}}^1 12x^2(1-x)\,dx$ $= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{\frac{2}{3}}^1$ $= 12\left\{\left[\frac{1}{3} - \frac{1}{4}\right] - \left[\frac{8}{81} - \frac{4}{81}\right]\right\}$ $= 12\left\{\frac{1}{12} - \frac{4}{81}\right\} = 1 - \frac{48}{81}$ $\underline{\underline{= \frac{33}{81}\ (= 0.4074) \text{ or } \frac{11}{27}}}$	M1 1 1		4
	(iii)	$E[x] = \int_0^1 x f(x)\,dx$ $= 12\int_0^1 (x^3 - x^4)\,dx$ $= 12\left(\frac{1}{4} - \frac{1}{5}\right)$ $= \frac{12}{20} = \frac{3}{5}$ $\therefore$ mean amount in wagon = $\underline{\underline{75\frac{3}{5}\ (= 75.6)}}$ (tonnes)	M1 1 1 1		4
	(iv)	$\bar{x} \sim \text{approx } N\left(\mu, \frac{\sigma^2}{n}\right)$ i.e. approx $N\left(0.6, \frac{1}{25 \times 30} = \frac{1}{750} = 0.001\dot{3}\right)$ Giving $P\left(\text{average} > 75\frac{2}{3}\right)$ $= P\left(x > \frac{2}{3}\right)$ $= P\left(Z > \frac{0.06666\ldots}{0.0365414} = 1.826\right) = 1 - 0.9660$ $= 0.034$	M1 A1 M1 1 A1	(slightly different answers are acceptable, depending on candidate's rounding	5

2	(i)	$S_{n-1}^2 = \frac{1}{99}\left(126.80 - \frac{112.4^2}{100}\right)$ $= \frac{0.4624}{99} = 0.00467(1)$	M1 A1	With divisor 100, $s^2 = 0.004624$. If this has been done, explicitly or implicitly, do NOT award this M1 A1, but FT into rest of question.	2
	(ii)	$\bar{x} = \frac{112.4}{100} = 1.124$ 1.124 ± 1.645 $\frac{\sqrt{0.00467}}{\sqrt{100}}$ = 1.124 1.645 0.00683(4) = 1.124 0.0112(42) = (1.112(758), 1.135(242)) Assumptions: random sample 0.00467 can be taken as σ^2	 M1 B1 M1 A1 A1 cao 1 1	Allow use of 1.660 (t_{99}; accepted as the t_{100}pt in tables) 1.660 leads to 1.124 0.0113(45) = (1.112(655), 1.135(345)) If 1.660, this mark then becomes 'normality'	7
	(iii)	Statement is wrong becomes 'μ', though unknown, is fixed and not a random variable – so it is not meaningful to attach a probability statement to it. Correct interpretation is that 90% of all such intervals that could arise in repeated sampling will contain the population mean.	E2 E2		4
	(iv)	[FT into this part from candidate's interval] Not reasonable to suppose that target of m = 1.136 is being set Allow 'reject', 'evidence against', or equivalent statement – because 1.136 is not in the interval.	1 1		2

3		$\bar{x}=14.98 \quad S_{n-1}^2=1.199(56) \quad S_{n-1}=1.095(24)$		allow $S_n^2 = 1.0796$, $S_n = 1.039(04)$ but ONLY if correctly used in sequence	
	(i)	$H_0 : \mu = 14.0$	1	Do NOT allow $\mu \leq 14.0$	
		$H_1 : \mu > 14.0$	1	Deduct 1 from any marks awarded here if 'μ' is not defined in words.	
		Where μ = mean increase in weight for new diet			
		Test statistic is $\dfrac{14.98-14.0}{\frac{1.095}{\sqrt{10}}}$	M1	Allow this M1 and FT inc do NOT allow the A1 for numerator of 14.0 – 14.98	
		= 2.83 (2.8295)	A1		
		Refer to t_9	1	May be awarded even if test statistic is wrong, but NO f.t. if this is wrong.	
		Upper 5% pt is 1.833	1	No f.t. if wrong. Must be –1.833 if test statistics was –2.83	
		Reject H_0	1		
		Seems new diet does give increased mean weight increase	1		8
	(ii)	CI given by 14.98	M1	Allow candidate's $\bar{x}$	
		± 2.262	B1		
		$\times \frac{1.095}{\sqrt{10}} = 14.98 \pm 0.78(34)$	M1	Allow candidate's $\frac{s}{\sqrt{n}}$	
		= (14.20, 15.76), [14.1966][15.7634]	A1 cao	Zero out of 4 if not same dist as used for test	4
	(iii)	Allow any two sensible comments, e.g.: - sample should be random [whatever that means here…] - pigs should in some sense be 'similar' to each other, e.g. in terms of initial weight - pigs are kept in controlled conditions, e.g. in respect of exercise	E3		3

4	(i)	X counts the number of 'successes' in n(12) trials – a binomial model would be appropriate if 'p' is constant	E1		
		And if trials are independent	E1		2
	(ii)	Model is X ~ B (12, 0.3)	M1		

X	0	1	2	3	4	5	≥6	
P(X = x)	0.0138	0.0712	0.1678	0.2397	0.2312	0.1585	0.1178	
Expected freq	1.38	7.12	16.78	23.97	23.12	15.85	11.78	A2
	8.50 ← grouping:							1
	16							
Observed freq	6	10	16	15	16	21	16	
Contrib to X^2	6.5176		0.0363	3.3567	2.1927	1.6733	1.5117	

		$X^2 = 15.38(84)$	1	f.t. from here if incorrect	
		Refer to χ_5^2	1	Allow f.t. from candidate's table. No f.t. if this does NOT agree with candidate's table	
		Upper 5% pt is 11.07	1	No f.t. if incorrect critical point used.	
		Significant	1		
		Seems model is not a good fit	1	[allow a mark for this if not already awarded]	9
	(iii)	These are considerably more low and high values of X than would have been expected, considerably fewer intermediate values	E2		
		Maybe p is not remaining constant and/or there is a lack of independence	E2		4

Examiner’s Report

Statistics 3 (5515)

General Comments

The overall performance of the candidates on this paper was quite pleasing with many scoring high marks. As in the past the candidates tackled comfortably the parts of questions to do with process and calculation. However once again it was clear that they were less comfortable with the parts requiring comment or interpretation. Their explanations were often woolly and imprecise and this made it difficult to know what candidates actually meant. For instance it was fairly common for candidates to recite a list of possible comments and/or assumptions with little obvious regard for their likely suitability, but in the hope that they might say something which could gain credit.

Q.1 was found to be particularly accessible and it should have given early encouragement to many candidates. Questions 2 and 3 proved to be quite high scoring for well prepared candidates whereas marks achieved on Question 4 were quite disappointing.

Invariably all four questions were attempted but there was evidence to suggest that many candidates were running out of time when they came to Question 4, which may account for the poorer marks.

Comments on Individual Questions

Question 1 (Continuous random variables; amount of iron ore in railway wagons.)

(i) This was well done with limits being used correctly and the integral equated to 1.

(ii) Most candidates answered this part correctly. A few found the complementary probability, a few had difficulty using their limits and a few attempted to use the Normal distribution (finding the mean and variance first).

(iii) This part was usually correct except when candidates forgot to add 75 to the mean of X.

(iv) Usually candidates coped well with this part. However many were surprisingly unsure about the mean and variance of the sample mean and this led to a considerable amount of muddled work. Most knew that they had to use the Normal distribution, but, as in previous years, far too many candidates were seen to switch from $\overline{X}$ to ΣX (which could give the same result) without it being clear that they knew what they were doing.

(ii) 0.4074; (iii) mean 75.6; (iv) 0.034.

Question 2 (Confidence interval for population mean from a large sample; contents of milk bottles.)

(i) As in the past there was a disappointingly large number of candidates who did not know how to use the information given to find the unbiased estimate of the population variance; many seemed content merely to use the version with divisor n.

(ii) In general most candidates were able to show that they knew how to construct a confidence interval. The most usual errors were neglecting to take the square root of the estimated variance or forgetting to divide by the square root of the sample size.

The statement of the required assumptions was poor, with candidates listing everything they could think of without considering what was really appropriate in the circumstances.

(iii) Answers to this part were quite poor, and full marks were rarely earned. Many candidates adopted the approach of repeating themselves: 'The manager is wrong because he should have said " ... " The correct interpretation is " ... " '. In many cases the "correct interpretation" given was imprecise and suggested that a form of words was being used without much understanding.

(iv) Attempts at this part were usually successful though candidates often appeared unwilling to trust the simple, obvious response and so went on to say more than was necessary.

(i) 0.00467; (ii) (1.112, 1.135).

Question 3 (Hypothesis test and confidence interval for population mean from a small sample; weight increases of pigs.)

(i) It was clear that candidates knew in broad terms what was expected here, but all too often their work was spoilt by a lack of attention to detail and this frequently resulted in loss of marks. The statement of the hypotheses usually lacked a satisfactory definition of the symbol used (i.e. a fully satisfactory verbal definition such as "μ = population mean increase in weight"). On many occasions the test statistic was incorrect because the wrong variance had been used. Care was needed also when it came to reading the tables of the *t*-distribution: a critical value of 2.262 instead of 1.833 was often seen. The use of the Normal distribution cropped up rather often.

(ii) As in Question 2, candidates knew about confidence intervals, but the same sort of errors abounded. It was not uncommon to see the use of a different distribution from the one used in part (i).

(iii) Although answers given to this part were usually good enough to secure marks it was often clear that candidates had not read the detail of the question thoroughly. In addition to legitimate suggestions there was also tendency to comment on aspects which were already covered in the opening paragraph. It was quite common for candidates to recommend that the pigs chosen should constitute a random sample which should also be representative or unbiased.

(i) test statistic 2.830, critical value 1.833; (ii) (14.20, 15.76).

Question 4 (Chi-squared test for goodness of fit of a Binomial model; visits to website.)

(i) Explanations about why the situation described might be modelled by a Binomial distribution were poor. Hardly anyone mentioned either the independence of trials or the need for a constant value of p. In fact it was rare for candidates to say anything of value. A very popular response was that a Binomial distribution required "large n and small p". Many referred to the independence of the events, but it was not clear that they understood what constituted an event on this occasion.

(ii) In this part of the question fully correct working was seen regularly. Only occasionally did candidates use the wrong parameters for the calculation of the expected frequencies, and they were not caught out by finding the frequency for 6 new visitors rather than *6 or more*. A high proportion of candidates neglected to combine the first two cells. The calculation of the test statistic, X^2, could almost always be relied on to follow correctly. A sizeable minority of candidates misjudged the number of degrees of freedom and hence looked up the wrong critical value.

(iii) In general candidates appeared not to pay proper attention to the detail of this question. It was the norm for candidates not to include any discussion of the quality of the fit of the model to the data. They were expected to point out the substantial discrepancies between the observed and calculated frequencies which would account for the outcome of the test. Several tried in vain to reject their "conditions" given in part (i), but failed to do so because of the inadequate and inappropriate nature of what they had said there.

(ii) Expected frequencies 8.50, 16.78, 23.97, 23.12, 15.85, 11.78; $X^2 = 15.38$; $\nu = 5$, critical value 11.07.