OCR
RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level
former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS **5503(A)**

Pure Mathematics 3

Section A

Wednesday **10 JANUARY 2001** Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
Students' Handbook

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown on the answer paper to indicate that a correct method is being used.

NOTE

This paper will be followed by **Section B: Comprehension**

This question paper consists of 4 printed pages.

1 The population of a city is P millions at time t years. When $t = 0$, $P = 1$.

(a) A simple model is given by the differential equation

$$\frac{dP}{dt} = kP,$$

where k is a constant.

(i) Verify that $P = Ae^{kt}$ satisfies this differential equation, and show that $A = 1$.

Given that $P = 1.24$ when $t = 1$, find k. [4]

(ii) Why is this model unsatisfactory in the long term? [1]

(b) An alternative model is given by the differential equation

$$4\frac{dP}{dt} = P(2 - P).$$

(i) Express $\dfrac{4}{P(2-P)}$ in partial fractions. [3]

(ii) Hence, by integration, show that

$$\frac{P}{2-P} = e^{\frac{1}{2}t}.$$ [4]

(iii) Express P in terms of t. Verify that, when $t = 1$, P is approximately 1.24. [3]

(iv) According to this model, what happens to the population of the city in the long term? [1]

[Total: 16]

2 (a) (i) Find $\int \sin 2x \, dx$. [2]

(ii) Hence or otherwise show that

$$\int_0^{\frac{1}{3}\pi} x \sin x \cos x \, dx = \frac{2\pi + 3\sqrt{3}}{48}.$$ [5]

(b) (i) Find the binomial expansion of $\frac{1}{\sqrt{1-4x^2}}$, up and including the term in x^4.

State the range of values of x for which the expansion is valid. [4]

(ii) Show that $\sqrt{\frac{1+2x}{1-2x}} = \frac{1+2x}{\sqrt{1-4x^2}}$.

Deduce the binomial expansion of $\sqrt{\frac{1+2x}{1-2x}}$, up and including the term in x^5. [4]

[Total: 15]

3 (i) Given that $\tan \alpha = 2$ and $0 < \alpha < \frac{1}{2}\pi$, find the exact values of $\sin \alpha$, $\cos \alpha$ and $\sin 2\alpha$. [3]

Fig. 3 shows a sketch of the curve whose parametric equations are

$$x = \cos t + 2\sin t, \qquad y = \sin 2t \qquad (0 \leqslant t \leqslant \pi).$$

The curve cuts the x-axis at the points A, B and D. The point on the curve where the x-coordinate attains its maximum value is C.

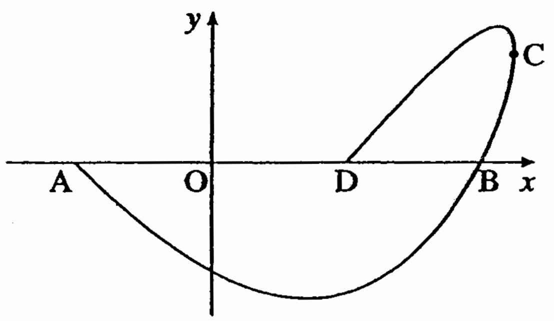


Fig. 3

(ii) Find the x-coordinates of the points A, B and D. [4]

(iii) Show that $\cos t + 2\sin t$ can be expressed in the form $R\cos(t - \alpha)$, where α is the angle given in part (i) and R is to be determined. Hence or otherwise find the *exact* coordinates of C. [4]

(iv) Find $\frac{dy}{dx}$ in terms of t. Deduce the value of the gradient of the curve at the point A. [4]

[Total: 15]

[Turn over

4 Fig. 4 illustrates the flight path of a helicopter H taking off from an airport. Coordinate axes $Oxyz$ are set up with the origin O at the base of the airport control tower. The x-axis is due east, the y-axis due north, and the z-axis vertical. The units of distance are kilometres throughout.

The helicopter takes off from the point G. The position vector $\mathbf{r}$ of the helicopter t minutes after take-off is given by

$$\mathbf{r} = (1 + t)\mathbf{i} + (0.5 + 2t)\mathbf{j} + 2t\,\mathbf{k}.$$

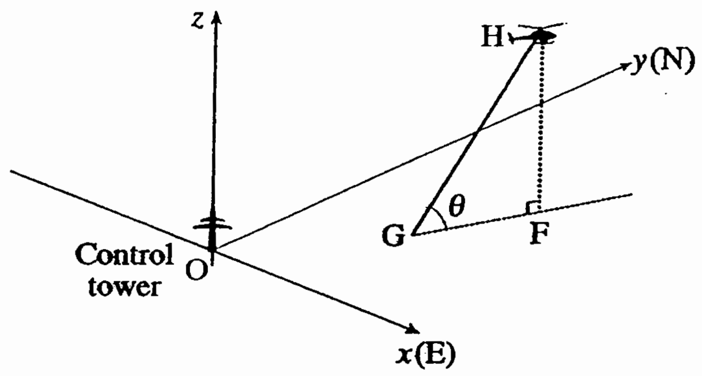


Fig. 4

(i) Write down the coordinates of G. [1]

(ii) Find the angle the flight path makes with the horizontal. (This angle is shown as θ in Fig. 4.) [3]

(iii) Find the bearing of the flight path. (This is the bearing of the line GF shown in Fig. 4.) [2]

(iv) The helicopter enters a cloud at a height of 2 km. Find the coordinates of the point where the helicopter enters the cloud. [3]

(v) A mountain top is situated at M (5, 4.5, 3). Find the value of t when HM is perpendicular to the flight path GH. Find the distance from the helicopter to the mountain top at this time. [5]

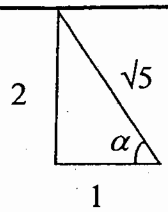
[Total: 14]

Mark Scheme

1(a)(i) $P = Ae^{kt}$ $\Rightarrow \dfrac{dP}{dt} = kA\,e^{kt} = kP$ when $t = 0, P = 1, \Rightarrow 1 = A\,e^0 = A$ when $t = 1, P = 1.24 = 1\,.\,e^k$ $\Rightarrow k = \ln 1.24 = 0.215$	B1 B1 M1 A1 [4]	verifying $\dfrac{dP}{dt} = kP$ or $\int\frac{dP}{P} = \int k dt$ $\Rightarrow \ln P = kt + c$ $\Rightarrow P = Ae^{kt}$ B1 $A = 1$ or $c = 0$ B1 verifying $A = 1$ (may come first) substitutes $t = 1$ 0.215.. Accept ln 1.24 or 0.22 or better
(ii) As $t \to \infty$, $P \to \infty$, so population grows without limit	B1 [1]	unlimited growth
(b)(i) $\dfrac{4}{P(2-P)} \equiv \dfrac{A}{P} + \dfrac{B}{2-P}$ $\Rightarrow 4 \equiv A(2-P) + BP$ $P = 0 \Rightarrow 4 = 2A$ $\Rightarrow A = 2$ $P = 2 \Rightarrow 4 = 2B$ $\Rightarrow B = 2$ so $\dfrac{4}{P(2-P)} \equiv \dfrac{2}{P} + \dfrac{2}{2-P}$	M1 A1 A1 [3]	$\dfrac{A}{P} + \dfrac{B}{2-P}$ $A = 2$ $B = 2$
(ii) $\int \dfrac{4}{P(2-P)} dP = \int dt$ $\Rightarrow 2\int (\dfrac{1}{P} + \dfrac{1}{2-P}) dP = \int dt$ $\Rightarrow 2\,[\ln P - \ln(2-P)] = t + c$ $\Rightarrow \ln\dfrac{P}{2-P} = \dfrac{1}{2}t + c$ when $t = 0, P = 1, \Rightarrow \ln 1 = c = 0$ $\Rightarrow \dfrac{P}{2-P} = e^{\frac{1}{2}t}$ *	M1 B1 DM1 E1 [4]	$\int \dfrac{4}{P(2-P)} dP = \int dt$ LHS $= 2\,[\ln P - \ln(2-P)]$ evaluating c at any stage deriving *
(iii) $\dfrac{P}{2-P} = e^{\frac{1}{2}t}$ $\Rightarrow P = (2-P)e^{\frac{1}{2}t} = 2e^{\frac{1}{2}t} - Pe^{\frac{1}{2}t}$ $\Rightarrow P(1 + e^{\frac{1}{2}t}) = 2e^{\frac{1}{2}t}$ $\Rightarrow P = \dfrac{2e^{\frac{1}{2}t}}{1 + e^{\frac{1}{2}t}}$ when $t = 1.24$, $P = 1.2449 \approx 1.24$	M1 A1 cao E1 [3]	multiplying through by $2 - P$ and expanding collecting P s $P = \dfrac{2e^{\frac{1}{2}t}}{1 + e^{\frac{1}{2}t}}$ or $P = \dfrac{2}{e^{-\frac{1}{2}t} + 1}$ $P = 1.2449$ or 1.245 Accept 1.24 or better SC Putting $t = 1$ and verifying $P = 1.24$ B1
(iv) As $t \to \infty$, $P \to 2$,	B1 [1]	$P \to 2$
	16	

2 (a) (i) $\int \sin 2x \,dx = -\frac{1}{2}\cos 2x + c$	M1 A1 [2]	$\pm\frac{1}{2}\cos 2x$ $-\frac{1}{2}\cos 2x + c$
(ii) $\int_0^{\frac{\pi}{3}} x\sin x\cos x \,dx = \int_0^{\frac{\pi}{3}} x.\frac{1}{2}\sin 2x \,dx$	B1	$\sin x \cos x = \frac{1}{2}\sin 2x$
$= \int_0^{\frac{\pi}{3}} x.\frac{d}{dx}[-\frac{1}{4}\cos 2x]\,dx$	M1	use of integration by parts
$= \left[-\frac{1}{4}x\cos 2x\right]_0^{\frac{\pi}{3}} + \int_0^{\frac{\pi}{3}} \frac{1}{4}\cos 2x \,dx$	A1	$\left[-\frac{1}{4}x\cos 2x\right]_{...}^{...} + \int_{...}^{...}\frac{1}{4}\cos 2x \,dx$
$= \frac{1}{8}.\frac{\pi}{3} + \frac{1}{8}[\sin 2x]_0^{\frac{\pi}{3}}$	A1 f.t.	$\frac{1}{8}\sin 2x$ f.t. their $k\cos 2x$ from the line above
$= \frac{\pi}{24} + \frac{1}{8}.\frac{\sqrt{3}}{2}$		
$= \frac{2\pi + 3\sqrt{3}}{48}$ *	E1 [5]	deriving *
or		
$\int_0^{\frac{\pi}{3}} x\sin x\cos x \,dx = \int_0^{\frac{\pi}{3}} x.\frac{d}{dx}(\frac{1}{2}\sin^2 x)\,dx$	M1	Attempt at integration by parts with $u = x$ and $v = \int \sin x\cos x dx$
$= \left[\frac{1}{2}x\sin^2 x\right]_0^{\frac{\pi}{3}} - \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2 x \,dx$	A1	$\left[\frac{1}{2}x\sin^2 x\right] - \frac{1}{2}\int \sin^2 x dx$
$= \frac{3}{8}.\frac{\pi}{3} - \frac{1}{4}\int_0^{\frac{\pi}{3}} (1-\cos 2x)\,dx$	B1	$\sin^2 x = \frac{1}{2}(1-\cos 2x)$
$= \frac{\pi}{8} - \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{3}}$	A1 f.t.	$\frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_{...}^{...}$ f.t. their $\frac{1}{4}(1 - \cos 2x)$ from the line above
$= \frac{\pi}{8} - \frac{1}{4}(\frac{\pi}{3} - \frac{\sqrt{3}}{4})$		
$= \frac{2\pi + 3\sqrt{3}}{48}$ *	E1 [5]	deriving *
OR $\int_0^{\frac{\pi}{3}} x\sin x\cos x dx = \int_0^{\frac{\pi}{3}} x\frac{d}{dx}\left(-\frac{1}{2}\cos^2 x\right)dx$		As above with appropriate changes

(b) (i) $\frac{1}{\sqrt{1-4x^2}} = (1-4x^2)^{-\frac{1}{2}}$	M1	$(1-4x^2)^{-\frac{1}{2}}$
$= 1 + (-\frac{1}{2})(-4x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-4x^2)^2 + \ldots$	A1	unsimplified expansion
$= 1 + 2x^2 + 6x^4 + \ldots$	A1 cao	
valid for $4x^2 < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	B1 [4]	$-\frac{1}{2} < x < \frac{1}{2}$
(ii) $\sqrt{\frac{1+2x}{1-2x}} = \frac{\sqrt{1+2x}\sqrt{1+2x}}{\sqrt{1-2x}\sqrt{1+2x}}$ $= \frac{1+2x}{\sqrt{1-4x^2}}$ *	E1	deriving *
so $\sqrt{\frac{1+2x}{1-2x}} = (1+2x)(1+2x^2+6x^4+\ldots)$	M1	$(1+2x)(1+2x^2+6x^4+\ldots)$
$= 1+2x^2+6x^4+2x+4x^3+12x^5+\ldots$	A1ft	expanding brackets f.t. their series from (i)
$= 1+2x+2x^2+4x^3+6x^4+12x^5+\ldots$	A1 cao [4]	$1+2x+2x^2+4x^3+6x^4+12x^5+\ldots$
	15	

3 (i) $\tan\alpha = 2$ $\Rightarrow \sin\alpha = 2/\sqrt{5}$ $\cos\alpha = 1/\sqrt{5}$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $= 4/5$		B1 B1 B1 [3]	$\sin\alpha = 2/\sqrt{5}$ or $\sqrt{0.8}$ $\cos\alpha = 1/\sqrt{5}$ or $\sqrt{0.8}/2$ $\sin 2\alpha = 4/5$ or 0.8 SC1 calculating α, then $\sin\alpha$, $\cos\alpha$ and $\sin 2\alpha$ approximately
(ii) $y = 0$ when $\sin 2t = 0$ $\Rightarrow 2t = 0, \pi, 2\pi$ $\Rightarrow t = 0, \pi/2, \pi$ $\Rightarrow x = 1, 2, -1$ so $x-$ coordinate of A is -1 $x-$ coordinate of B is 2 $x-$ coordinate of D is 1		M1 A1 A1 A1 [4]	$\sin 2t = 0$ s.o.i. or $\sin t\cos t = 0$ etc $x = 1$ $x = 2$, $x = -1$ condone wrong labels. Condone A = -1 etc except SC A = -1 with *no* relevant working B = 2 D = 1 B2, 1, 0. Deduct 1 for each wrong number
(iii) $R\cos(t-\alpha) = R\cos t\cos\alpha + R\sin t\sin\alpha$ $= R/\sqrt{5}\cos t + 2R/\sqrt{5}\sin t$ $= \cos t + 2\sin t$ $\Rightarrow R = \sqrt{5}$ At C, $x = \sqrt{5}$, $y = \sin 2\alpha = 4/5$		M1 B1 B1f.t. B1f.t. [4]	use of compound angle formula (stated) or $\sin\alpha = \frac{2}{R}$, or $\cos\alpha = \frac{1}{R}$ or $\tan\alpha = 2$ but $\sin\alpha = 2\cos\alpha = 1$ is M0 $R = \sqrt{5}$ $x = \sqrt{5}$ f.t. their R $y = 4/5$ f.t. their R
(iv) $\dfrac{dy}{dx} = \dfrac{2\cos 2t}{2\cos t - \sin t}$ at A, $t = \pi$, $\dfrac{dy}{dx} = -1$		M1 A1 A1 A1 [4]	$\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$ $2\cos 2t$ $2\cos t - \sin t$ -1 cao
		15	

4 (i) G is (1, 0.5, 0)	B1 [1]	(1, 0.5, 0) Accept:- $\begin{pmatrix}1\\0.5\\0\end{pmatrix}$, $i+\frac{1}{2}j$, (1, 0.5)
(ii) Direction of GH is $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ $\tan\theta = 2/\sqrt{5} \Rightarrow \theta = 42°$	M1 A1 A1 [3]	direction of GH $\tan\theta = 2/\sqrt{5}$ or equivalent 42°
(iii) Direction of GF is $\mathbf{i}+2\mathbf{j}$ angle with north is $\tan^{-1} 1/2 = 27°$ bearing is 027°	M1 A1 [2]	$i+2j$ or $\tan^{-1}$ ½ seen anywhere 27° or 027°
(iv) $z = 2$ when $t = 1$, $\mathbf{r} = 2\mathbf{i}+2.5\mathbf{j}+2\mathbf{k}$ coordinates are (2, 2.5, 2)	M1 A1 A1 [3]	$z = 2$ $\Rightarrow t = 1$ (2, 2.5, 2)
(v) $\vec{HM} = 5\mathbf{i}+4.5\mathbf{j}+3\mathbf{k} - [(1+t)\mathbf{i}+(0.5+2t)\mathbf{j}+2t\mathbf{k}]$ $= (4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}$ perpendicular when $\vec{HM}.\vec{GH}=0$ $\Rightarrow [(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}].[\mathbf{i}+2\mathbf{j}+2\mathbf{k}] = 0$ $\Rightarrow 4-t+8-4t+6-4t = 0$ $\Rightarrow 18-9t = 0, \Rightarrow t = 2$ at this time $\vec{HM} = 2\mathbf{i}+\mathbf{k}$, HM = $\sqrt{5}$ km	M1 M1 A1 cao A1 f.t. A1 cao [5]	$\vec{HM} = (4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)k$ $\vec{HM}.\vec{GH} = 0$ Allow this (M1) for $\vec{HM}$.(their $\vec{GH}$) $18-9t$ $t = 2$ f.t. their equation $\sqrt{5} = 2.24$ km
	14	

Examiner’s Report

Pure Mathematics 3 (5503)

General Comments

There was some excellent work from stronger candidates and a very pleasing number scored 60+ marks. At the other end of the scale very few candidates scored 10 marks or less; even the weakest candidates could find some part of each question which they could answer. There was a very satisfactory distribution of marks between these two extremes. The presentation of work was generally good although occasionally untidy work was difficult to follow. In some questions candidates failed to show sufficient working to enable marks to be given for incorrect results, or when deriving a result given in the question. An example of the former was in question 3 part (ii) where candidates often gave only the briefest of explanations, sin $2t = 0$, or perhaps only the answers, $x = 1, x = 2, x = -1$. If one of these was incorrect doubt was thrown on the derivation of the others. In Q.1(b)(ii) full marks could only be obtained for the result $\frac{P}{2-P} = e^{\frac{t}{2}}$ if an arbitrary constant was introduced into the solution of the differential equation and the initial condition used to find its value.

Comments on Individual Questions

Question 1 (Partial fractions and differential equations)

This question was generally answered well and a good number of candidates obtained full marks. Part (a) (i) was nearly always done by integration, in which case common errors were the omission of an arbitrary constant or $\ln P = kt + C \Rightarrow P = e^{kt} + e^{c}$. Those few candidates who differentiated $P = Ae^{kt}$ often failed to point out that the result was equal to kP.

Almost all candidates obtained the correct partial fractions in part (b)(i) but many then went straight on to integrate them without any reference to the differential equation, often introducing a t simply to obtain the given result. In the integration a common error was the omission of the negative sign from $-\ln(2 - P)$. Those candidates who, correctly, separated the variables of the differential equation before integrating both sides, quite often omitted an arbitrary constant or, having included one, failed to find its value. Perhaps they did not realise that the initial condition in the first line of the question still applied in the alternative model.

Part(b)(iii) was usually answered well, although errors in algebra prevented some candidates from obtaining the correct result. There was a consolation mark for those candidates who, having failed to express P in terms of t, substituted the value of t in the given result to verify the approximate value of P.

There were some good answers to parts (a)(ii) and (b)(iv) but some comments were rather too general, missing the main points, in the first model that the growth was unlimited, and in the case of the second model failing to state the limiting value of P.

(a)(i) $k = 0.215$; (b)(i) $\frac{4}{P(2-P)} = \frac{2}{P} + \frac{2}{2-P}$, (iii) $P = \frac{2e^{\frac{1}{2}t}}{1+e^{\frac{1}{2}t}}$, (iv) $P \rightarrow 2$.

Question 2 (Integration by parts and binomial series)

This question was very well answered indeed, some twenty per cent of all candidates scoring full or almost full marks.

Many candidates integrated sin $2x$ correctly. Errors included the wrong sign, 2 instead of ½, and just occasionally cos x instead of cos $2x$. More common was the omission of an arbitrary constant.

Most candidates were able to apply the method of integration by parts confidently and most also used the first part of the question correctly. The majority of candidates who obtained the correct indefinite integral went on to substitute the given limits and to obtain the given result. Perhaps the most common error in the algebra was to lose one of the several $\frac{1}{2}$s which came in at various stages, but errors also occurred in the sign of the second integration.

Most candidates made a good attempt at the first part of the question on the binomial series. Just a few wrote $(1 - 4x^2)^{-1}$ or $(1 - 2x)^{-1}$ instead of $(1 - 4x^2)^{-\frac{1}{2}}$ and some forgot the – sign in front of $4x^2$ when carrying out the expansion. However a good proportion of the candidates achieved the correct series. This was not true for the range of validity which very many either omitted or stated incorrectly. Some simply failed to simplify $\sqrt{\frac{1}{4}}$ in the correct range but many gave $x < \frac{1}{2}$, or $x < \sqrt{\frac{1}{4}}$ or $x^2 < \frac{1}{4}$.

Not many candidates were able to prove the identity in part (b)(ii). The most able used the method of multiplying the top and the bottom lines of the left hand side by $\sqrt{1+2x}$ to obtain the right hand side but others attempted in a variety of ways to show that both sides were equal to some common fraction or expression. This often involved squaring both sides and then either factorising $(1 - 4x^2)$ and cancelling on the right hand side, or cross multiplying and expanding both sides. These methods were not always explained well. It was pleasing, however, that many candidates using any of the above methods were familiar with, and could put to good use, the factors of a 'square minus a square'.

Whether they could do this part, or not, most candidates went on to deduce the required binomial expansion using their result from part (i). There was a good number of correct expansions or at least correct products following errors in part (i).

(a)(i) $-\frac{1}{2}\cos 2x + C$; (b)(i) $1 + 2x^2 + 6x^4 \ldots, -\frac{1}{2} < x < \frac{1}{2}$,
(ii) $1 + 2x + 2x^2 + 4x^3 + 6x^4 + 12x^5 + \ldots$).

Question 3 (Trigonometric functions, parametric coordinates and equations)

More candidates scored full marks on this question than on any other but there were also more candidates with no marks. Many candidates did not understand what was meant by exact values and answered part (i) by finding the value of α and then writing down the values of sin α etc. as decimals.

Most candidates appeared to understand what was required in part (ii) and, if they showed any working at all, wrote down $y = 0$ and then, usually, $\sin 2t = 0$. Solutions then were rarely clear. $t = 0$

was often seen and sometimes $t = \pi$ but rarely $t = \frac{\pi}{2}$, although the three correct values of x often followed.

The presentation of part (iii) was also often incomplete and confused. Very many candidates were clearly using the standard results $R = \sqrt{(a^2 + b^2)}$ and, for example, $\cos\alpha = \frac{a}{R}$, and then substituting $a = 1$ and $b = 2$ from $\cos t + 2\sin t$. However, these candidates only very rarely stated that this gave the angle α of part (i).

The final part of the question in part (iii) was not well answered and only the most able candidates obtained both the coordinates of C correctly. A small number attempted to do this by differentiating $x = \cos t + 2\sin t$ and equating the result to zero. Just a few succeeded.

Most candidates were able to obtain an expression for $\frac{dy}{dx}$ from the parametric equations. A fairly common error was $\frac{2\cos 2t}{-\sin t + 2\cos 2t} = \frac{-2\cos 2t}{\sin t + 2\cos t}$.

The final substitution was often not correct, perhaps because many candidates did not know what value of t to substitute.

(i) $\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{4}{5}$, (ii) 1, 2, −1, (iii) $R = \sqrt{5}$, $\left(\sqrt{5}, \frac{4}{5}\right)$, (iv) $\frac{2\cos 2t}{2\cos t - \sin t}$, −1.

Question 4 (Vectors)

This question was not done nearly so well as the previous questions, perhaps because some candidates were running short of time, but more likely because they ran into difficulties. Almost all candidates were able to score the first mark, obtaining the coordinates of the take-off point, G, in some form or other; the actual coordinates (1, 0.5, 0), the position vector $\mathbf{i} + 0.5\,\mathbf{j}$ or as a column vector, or either coordinates or column vector in two dimensions (benefit of doubt given that $z = 0$ was implied, if not stated). Unfortunately very many candidates then failed to use this starting point in their subsequent work, in parts (ii) and (iii). Many candidates were good at visualising triangles and others were well able to use the dot product to find the angle between two vectors, but very often these methods were applied with the vectors **OG, OH** and **OF** instead of **GH** and **GF**.

This difficulty did not apply to part (iv), where the coordinates of the point of entry to the cloud were required, and the majority of candidates secured another three marks here.

There were some remarkably good solutions to part (v) from the best candidates. The problem of the origin remained however, many candidates using **OH** when they should have been using **GH** in their dot product. This error was even less excusable in this part because the two directions which were perpendicular were clearly stated in the question.

(i) (1, 0.5, 0), (ii) 42°, (iii) 027°, (iv) (2, 2.5, 2), (v) $t = 2$, 2.24km.

Section B (Comprehension)

This question proved to be a good test of the candidates' comprehension and there was a good spread of marks. Some ten per cent gained full or nearly full marks and a further ten per cent failed to score or only obtained one or two marks.

Many candidates lost marks by merely asserting statements which they were asked to justify. Thus in Q.2, it was not sufficient to substitute the values of d, f and T and state the given result, some of the intermediate working was needed to gain the final mark. Similarly, in Q.3, it was necessary to demonstrate the algebraic steps needed to transform one expression to the other. Quite a large number of candidates assumed that $kd = f$ in this question. In question 4 many candidates wrote down the geometric progression correctly – $1 + 1.02 + 1.02^2 + 1.02^3 + \ldots + 1.02^{10,000}$ but then instead of summing the series they merely asserted that this was clearly greater than 10^{11}.

Q.5 also required some precise statements rather than simply, "because the space ships will travel faster the speed of colonisation will be greater". Good candidates here calculated the new value of $v = \frac{9}{910}$ and, by comparing this with the original value $\frac{1}{110}$ deduced that the conclusions would not be altered.

Again, in the final question, the derivation of the number 10 000 from the value of d and the diameter of the galaxy was the best way of showing that it was independent of T, f, and N.

Indeed it was not easy to establish this independence directly, although some candidates made good attempts to do so.

1. 9.46×10^{20}m.