OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Monday 22 JANUARY 2001 Afternoon 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

You are permitted to use a graphical calculator in this paper.

Answer **all** questions.

There is an **INSERT** provided for Question **4** part **(a) (ii)** and Question **5** parts **(a) (i)** and **(b) (i)**.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

This question paper consists of 5 printed pages, 3 blank pages and an insert.

(SB) HN/3 QA16188

[Turn over

Section A

1 (a) Give two properties that a graph must possess for it to be a tree. [2]

(b) Draw three *different* trees each containing 5 vertices and 4 edges. [2]

[Total: 4]

2 Six items with weights given in the table are to be packed into boxes each of which has a capacity of 10 kg.

Item	A	B	C	D	E	F
Weight (kg)	2	1	6	3	3	5

(i) Use the first-fit algorithm to pack the items, saying how many boxes are needed. [2]

(ii) Give an optimal solution. [2]

[Total: 4]

3 Consider the following linear programming problem.

$$\begin{array}{ll} \text{maximise} & 5x + 2y \\ \text{subject to} & 2x + 3y \leqslant 9 \\ & 3x + y \leqslant 13 \\ & x \geqslant 0 \\ & y \geqslant 0 \end{array}$$

(i) Use a graphical method to solve the problem. [6]

(ii) Give the optimum integer solution. [1]

[Total: 7]

Section B

4 An insert is provided for part (a) (ii) of this question.

(a) Andrea scores on average 6 marks out of 15 when she attempts past A level mathematics questions. Her recent scores have the probability distribution shown in the table.

Score	4	5	6	7	8	9
Probability	0.2	0.2	0.3	0.1	0.1	0.1

(i) Give a rule for using two-digit random numbers, beginning with 00, to simulate Andrea's scoring on an A level mathematics question. [2]

(ii) Use the two-digit random numbers given in the table provided on the insert to simulate Andrea's marks on examination papers consisting of 4 questions. Write your simulated question scores and the total score for each paper in the spaces provided in the table. [4]

(iii) Use your simulations to estimate the probability of Andrea scoring a total of 24 marks or more on a paper. [1]

(b) Bart also scores on average 6 marks per question. His scores on twelve past questions are shown in the frequency table below.

Score	4	5	6	7	8	9	10
Frequency	3	2	4	0	2	0	1

(i) Give an efficient rule for using two-digit random numbers, beginning with 00, to simulate Bart's scoring on an A level mathematics question. [2]

(ii) Use the following two-digit random numbers to simulate Bart's scores on **five** examination papers, each containing 4 questions. (Read the numbers from left to right on successive rows.)

20	09	99	12	65	76	48
96	05	00	21	55	42	41
38	16	57	81	72	45	92
78	03	47	17	23	09	19

Write down the total score for each paper. [4]

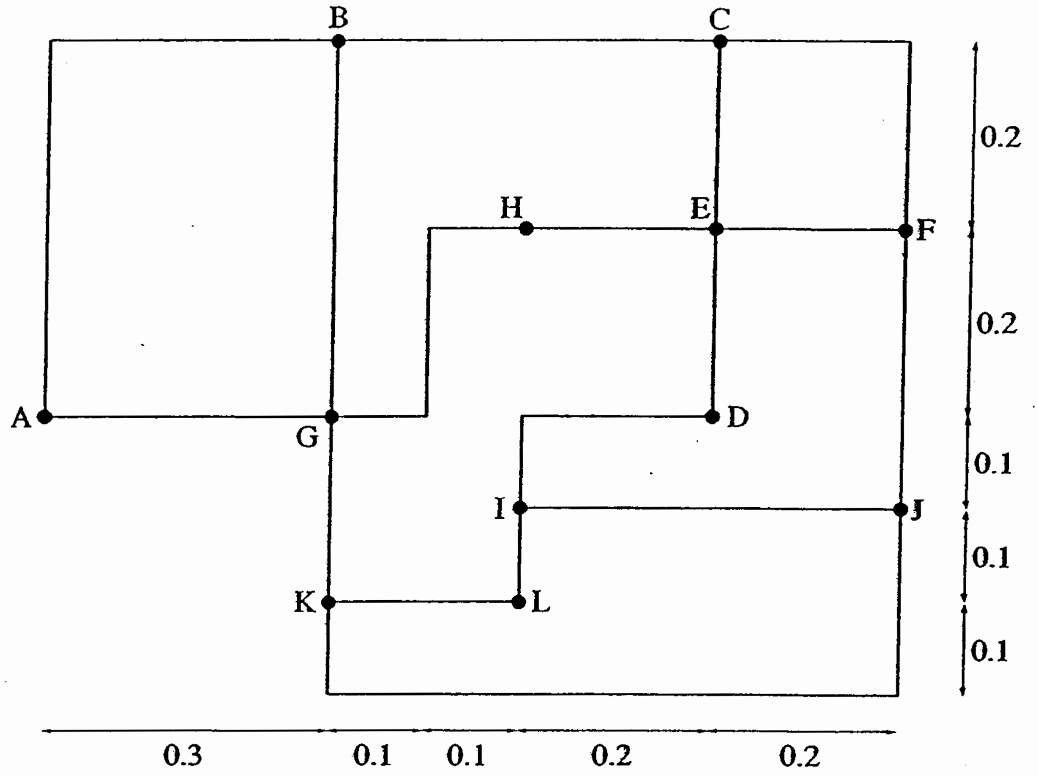
(iii) Use your simulations to estimate the probability of Bart scoring a total of 24 marks or more on a paper. [1]

(iv) State how you could improve the simulation. [1]

[Total: 15]

5 **An insert is provided for this question.**

A new microchip is being designed. It includes 11 components, labelled B to L, all of which need to be connected to A, either directly or via another component. The positioning of A and the components, and the routes along which connections can be made, are shown in the diagram. The marked distances are measured in mm.



(a) The cost of manufacturing the microchip is proportional to the total length of connections.

(i) Use an appropriate algorithm to find a set of connections of minimum total length. Draw your connections on Fig. 5.1 on the insert, and give their total length. [5]

(ii) Give the name of the algorithm you have used, and describe it briefly. [2]

(b) The time taken to transfer information from A to each other component is proportional to the length of the connection from A to that component. Each of these times, rather than the total length of the connections, is to be minimised.

(i) Using Fig. 5.2 on the insert, apply Dijkstra's algorithm to find the shortest distances from A to each of the other components. Show your working in the boxes on the diagram. [4]

(ii) List the connections that are needed, and give their total length. [4]

[Total: 15]

6 The table shows the activities involved in building a short length of road to bypass a village. The table gives their durations and their immediate predecessors.

Activity		Duration (weeks)	Immediate predecessor(s)
A	Survey sites	8	–
B	Purchase land	22	A
C	Supply materials	10	–
D	Supply machinery	4	–
E	Excavate cuttings	9	B, D
F	Build bridges and embankments	11	B, C, D
G	Lay drains	9	E, F
H	Lay hardcore	5	G
I	Lay bitumen	3	H
J	Install road furniture	10	E, F

(a) Draw an activity on arc network for these activities. [4]

(b) Mark on your diagram the early time and the late time for each event. Give the minimum completion time and the critical activities. [6]

(c) Each of the tasks E, F, G and J can be speeded up at extra cost. The maximum number of weeks by which each task can be shortened, and the extra cost per week saved, are shown in the table below.

Task	E	F	G	J
Maximum number of weeks by which task may be shortened	3	3	1	3
Cost per week of shortening task (in thousands of pounds)	30	15	6	20

(i) Find the new shortest time for the bypass to be completed. [2]

(ii) List the activities which will need to be speeded up to achieve the shortest time found in part (i), and the time by which each must be shortened. [2]

(iii) Find the total extra cost needed to achieve the new shortest time. [1]

[Total: 15]

Candidate Name	Centre Number	Candidate Number

OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

INSERT 1 hour 20 minutes

Insert for Question 4(a)(ii)

Paper	Question number	1	2	3	4	Total score
1	Random number	20	09	99	12	
	Score					
2	Random number	65	76	48	96	
	Score					
3	Random number	05	00	21	55	
	Score					
4	Random number	57	81	72	45	
	Score					
5	Random number	92	78	03	47	
	Score					

This insert consists of 2 printed pages.

[Turn over

Insert for Question 5

(a) (i)

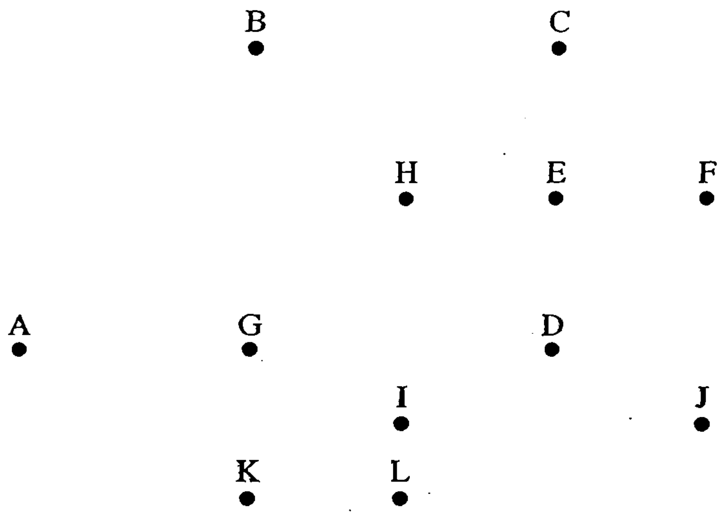


Fig. 5.1

Total length:..........................mm

(b) (i)

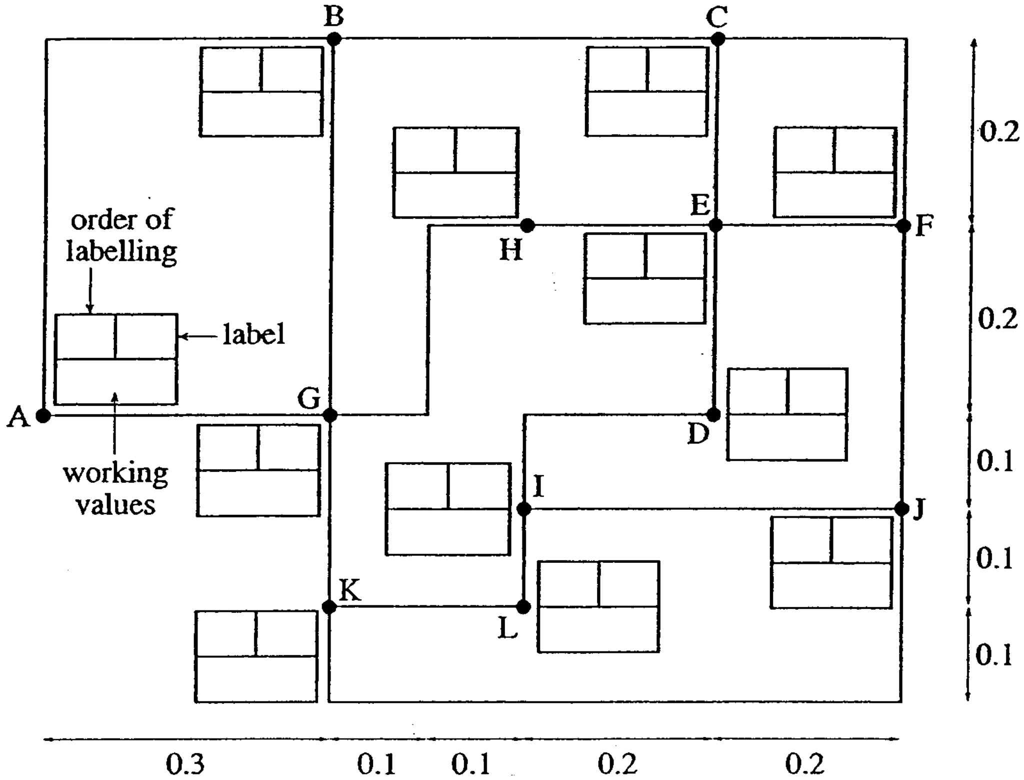


Fig. 5.2

Mark Scheme

OCR

General Certificate Examination

Advanced Level

MEI STRUCTURED MATHEMATICS

2620: Decision and Discrete Mathematics I

Instructions to markers

M marks are for method and are dependent on correct numerical substitution/correct application. Method marks can only be awarded if the method used would have led to the correct answer had not an arithmetic error occurred.
M marks may be awarded following evidence of an **sca** (substantially correct attempt).

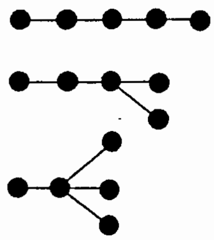
M marks can be implied by correct answers.

A marks are for accuracy, and are dependent upon the immediately preceding **M** mark. They cannot be awarded unless the **M** mark is awarded.

B marks are for specific results or statements, and are independent of method.

√ marks are for follow-through. This applies to **A** marks for answers which follow correctly from a previous incorrect result. Whilst mark schemes will occasionally emphasise a follow-through requirement, the default will be to apply follow-through whenever possible. The exception to this are **A** marks which are labelled **cao** (correct answer only).

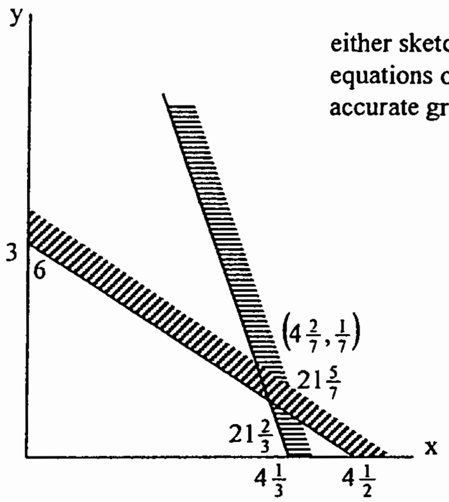
MR Where a candidate misreads all or part of a question, and where the integrity/difficulty of the question is not affected, a penalty (of –1, -2 or –3) can be applied (according to the extent of the work affected), and the question marked as read.
Note that it is **not** a misread if a candidate makes an error in copying his own work.

1 (a) connected	B1
no cycles (or n−1 arcs)	B1
(b) e.g. 	M1 A1

2 (a) box: 1 / 2 / 3; items: A, B, C / D, E / F — 3 boxes are needed.	M1 A1
(b) box: 1 / 2; items: A, D, F / B, C, E — or — box: 1 / 2; items: A, E, F / B, C, D	M1 A1

3 (i)

either sketch + simultaneous equations or reading from an accurate graph

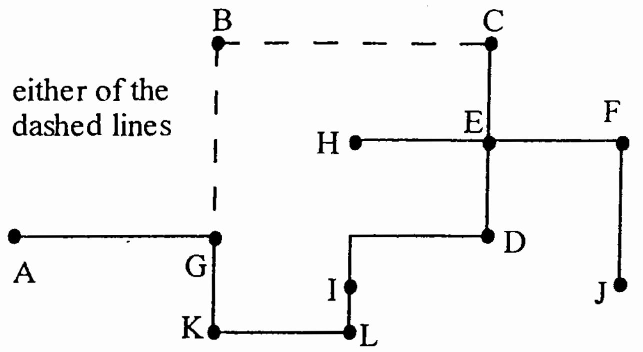


B1	labels and scale
B1	line
B1	line
B1	shading
M1	values or obj line gradient
A1	best point + value

(ii) 20 at (4, 0) — B1 cao

4(a) (i)	4	5	6	7	8	9	
	00-19	20-39	40-69	70-79	80-89	90-99	M1 A1
(ii)	5	4	9	4	22		M1 sca
	6	7	6	9	28		A2 simulated values
	4	4	5	6	19		(-1 each error)
	6	8	7	6	27		A1√ totals
	9	7	4	6	26		
(iii)	0.6						B1
(b) (i)	4	5	6	8	10		
	00-23	24-39	40-71	72-87	88-95		M1 not all used
							A1
(ii)	4	4	4	6	18		M1 sca
	8	6	4	4	22		A2 simulated values
	4	6	6	6	22		(-1 each error)
	5	4	6	8	23		A1√ totals
	8	6	10	8	32		
(iii)	0.2						B1√
(iv)	more repetitions						B1

5(a)(i)

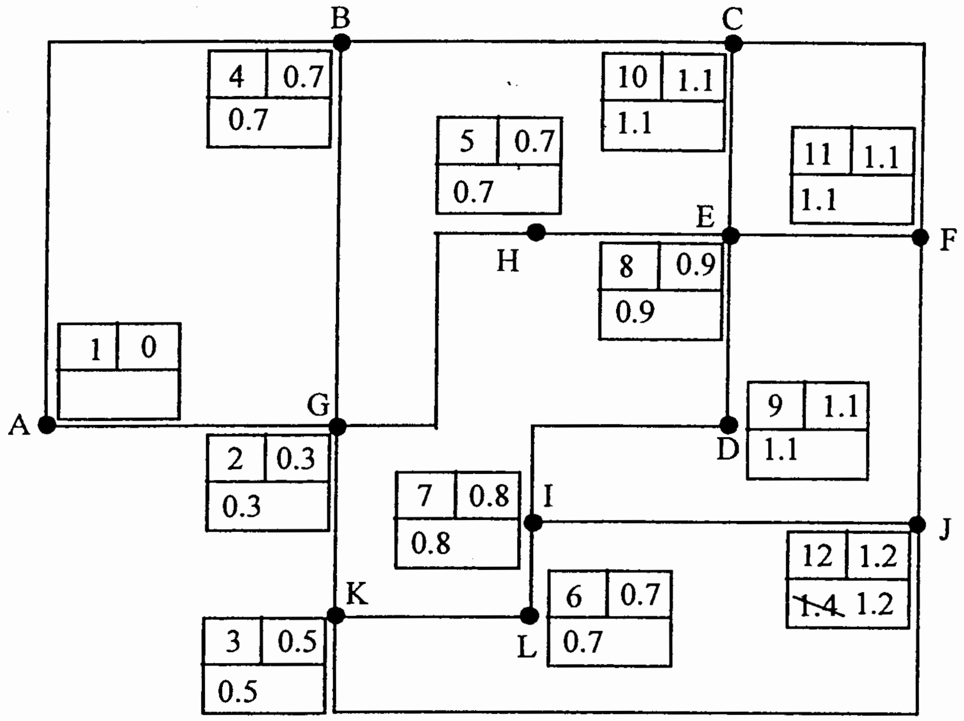


B1	AGKLIDE
B1	EC and EH
B1	EFJ
B1	GB or CB

Total length = 2.6 mm — B1 cao

(ii) Prim: connect in nearest to connected set
Kruskal: Shortest arc s.t. no cycles — B1 B1

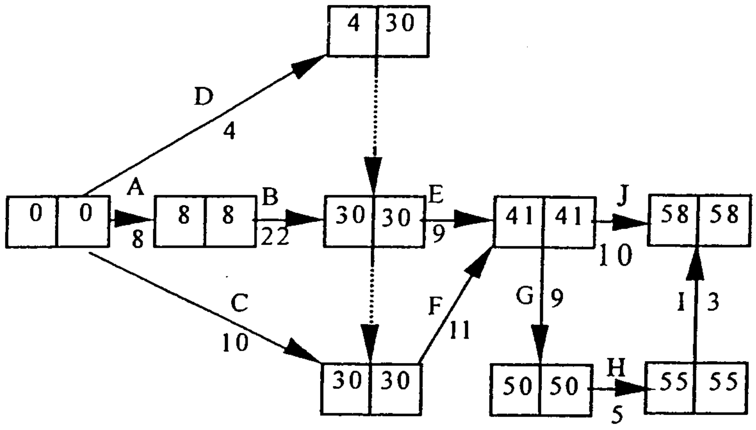
(b)(i)



M1	Dijkstra
A1	labels
A1	order of labelling
A1	working values

(ii) AG; GK; KL; LI; IJ; — B1
GH; HE; EF;
EC; — B1
ED;
GB — B1

Total length = 2.8 mm — B1 cao

6.(a)&(b) 	M1 sca (activity on arc) A1 dummy activities + E and F A1 A, B, C, D A1 G, H, I, J M1 forward pass A1 M1 backward pass A1
time – 58 weeks critical – A; B; F; G; H; I	B1 cao B1 cao
(c) (i) 54 weeks	M1 not J A1
(ii) E – 1 week F – 3 weeks G – 1 week	A1 E - 1 week A1 other 2
(iii) £81000	A1 cao

Examiner’s Report

Decision and Discrete Mathematics 1 (2620)

General Comments

Candidates were well prepared for this paper, particularly for the simulation question.

Comments on Individual Questions

Question 1 (Graphs)

Most candidates were able to gain both marks in part (a). In part (b) many produced isomorphic graphs, believing them to be different.

Question 2 (Algorithms)

This was very straightforward, and few candidates had any difficulty with it.

Question 3 (Linear programming)

Most candidates drew the graph sufficiently well to score the first 4 marks from this question. Few were able to proceed further. The question was designed to test candidates' abilities to identify the optimal point with an appropriate degree of accuracy. Not only could most not do this, but they also failed to display any method in making the selection of optimal point. Few appreciated that anything different was being asked in part (ii).

(i) $21\frac{5}{7}$ at $\left(4\frac{2}{7}, \frac{1}{7}\right)$, (ii) 20 at (4, 0).

Question 4 (Simulation)

Part (a) was almost uniformly correct, with the exception of the odd arithmetic mistake. Part (b)(i) was divided between the majority with the correct solution (or a common error in which the numbers 00-96 were used), and those who tried to divide the entire range to fit the required proportions. A very small number provided odd (but workable) rules using 60, 72 or 84 random numbers rather than 96. In part (b)(ii) there were many possible misreadings of the instructions, and most were seen at some point. Some candidates read the random numbers in columns rather than rows; some read columns and omitted columns with an out of range random number; some read in rows, but simulated five questions instead of four; some used all seven numbers in a row; on one script there was a seemingly random selection of the random numbers which had been provided. Follow-through was applied as far as was possible. Part (b)(iii) was straightforward, and found so by candidates. Part (b)(iv), was often the part that really tested the understanding of the process of simulation.

(a)(iii) 0.6; (b)(iii) 0.2, using the obvious rule.

Question 5 (Networks)

Part (a)(i) was generally done well. Part (a)(ii) was less well done. More candidates gained credit here than didn't, but full marks were achieved by few. In contrast to work on the rest of the paper, Dijkstra was badly done by many candidates. A large minority were clearly doing Nearest Neighbour or Prims or Kruskals, or were using a trial and error approach (the order of labelling giving clear indication of this). This was quite surprising. Many candidates failed to answer the question in (b)(ii), giving routes instead of arcs. For those who did try to do what was asked, the substitution of ID for ED was the most common error.

(a)(i) length = 2.6 mm; (b)(ii) length = 2.8 mm.

Question 6 (Networks)

A sizeable minority scored 15 out of 15 for this question. They knew what they were doing, and went about it with admirable efficiency. For the remainder a mark of 3 or 4 was usual in part (a). By far the most common error was in the precedences for E and F, where no distinction was made with respect to C. A far too common problem was that posed by candidates using activity on node. This is not in the specification, and solutions using it were able to gain little credit in parts (a) and (b). A few other candidates tried to construct a cascade chart instead of an activity network. For part (c) the most common mark was 2 out of five. Candidates did not realise that it becomes necessary to reduce E as well as F.

(b) 58 weeks; A, B, F, G, H, I;
(c)(i) 54 weeks, (ii) E (1 week); F (3); G (1), (iii) £81000.

Decision and Discrete Mathematics (5519/2)

General Comments

Some of the points below refer equally to 2620/2, and are duplicated in the report for that unit.

Most of the projects were on the networks, CPA or simulation syllabus areas, with more on networks than any other area. Many of them were good or very good; others were rather routine (and generally they were assessed so to be). Most were well written; some would have benefited from the use of a grammar checker or spell checker (or both).

Candidates who had been given very prescriptive advice or even fully specified problems tended to produce much less successful projects. Such candidates cannot score much for problem identification.

Except in simulations, candidates too often do not explain how they obtained their data (e.g. network arc lengths or times, or activity durations and/or precedences). They also often fail to consider the quality of this information and the implications of its inaccuracy.

There were fewer cases this time of unnecessary out-of-context theory being quoted, but it did happen. The candidate can assume that the reader is familiar with the workings of the standard algorithms. What the reader needs to see is evidence of the candidate's correct application of the algorithm in the context of the problem identified. What is not required is a textbook-type description of, say, Prim's algorithm, followed by a completed minimum spanning tree - with absolutely no evidence that Prim has actually been used. What is wanted is something that shows the order in which the vertices were added.

Candidates aiming for high marks need to consider their extension ideas sensibly. Changes of the "there is an accident on road X" or "the bricks can't be delivered before Tuesday" type often result in no new skills being demonstrated. Often what happens is that exactly the same techniques are demonstrated on a slightly different (and sometimes easier) problem.

Algorithms

Very few centres had candidates submitting work on this section of the syllabus. Coursework on algorithms is not an option in the new specification (2620 or 2621).

Networks

Most of these were TSP problems.

Some extremely good work was seen, but some candidates chose problems which were either too simple (i.e. too few vertices) or totally artificial.

A surprising number of candidates claiming to be using Prim's algorithm were in fact using a nearest neighbour approach - which was not always spotted by the marker.

Candidates should be encouraged to produce complete tables of shortest distances between nodes, so that the methods being applied are valid. Examples were seen where removal of a particular node from an incomplete network disconnected the network.

Nearest neighbour approaches usually give a better upper bound than does 2 x MST, so there is little point in using the latter.

Repeated calculations, such as those seen in obtaining a lower bound, are better put in an appendix so that they do not interfere with the flow of the report.

Centres are reminded that in the new specifications, TSP is on 2621 and not 2620. TSP projects cannot be accepted for 2620.

Decision Analysis

Too few examples were seen to make any constructive comment.

Simulation

Much of the work in this area was very good indeed, with some very original ideas. Many simulations were done using Excel or programs written by the candidate, and some of these were very creative. This use of technology has the obvious advantage of allowing repeated runs and longer runs. Moderators are always concerned about the time that has been used when they see lengthy hand calculations. It should be noted, however, that computer procedures need to be explained.

Two problems with simulations merit mention here. One is the totally artificial simulation, with timings and probabilities which have been invented. These usually score very poorly. A simulation needs to be based upon observation. The other problem is lack of validation of the model. Before exploring whether a different queuing discipline would work better, it is essential to check that the current situation has been modelled reasonably accurately.

CPA

Most of the projects in this area were suitable. Some of the networks were too small and/or too linear to offer much more scope than an examination question. In some cases tasks were broken down unnecessarily into a sequence of linear sub-tasks.

Some previous points bear repetition here. Too many precedence networks and precedence tables did not agree with each other. It is important that the network is derived from the precedence table. Often, too much familiarity with the real problem resulted in the network being the actual starting point, with old habits rather than *necessary* precedences being incorporated. More care is needed with dummy activities. It is worth repeating here that it is required that no two activities should start and end with the same pair of nodes.

Most CPA was carried through to resource levelling as a matter of course. Some candidates dealt well with specialist workers, but others allocated unsuitable people to tasks.

Numerical Analysis

A relatively small number of candidates entered for this syllabus. The vast majority of them were able to demonstrate proficiency in using numerical techniques relevant to their chosen task. Most were able to implement the chosen method successfully using spreadsheets, and some excellent work was seen.

However, a small minority either did not submit computer output or did not use a computer at all, preferring to perform (what must have been in some cases long-winded) calculations by hand. Centres are urged to encourage their candidates to use technology where appropriate. A small number of candidates wasted time and effort by replicating bookwork, for which it is expected that no credit will be given. This is particularly the case for 'error analysis' - candidates are expected to apply theoretical formulae to the problem in hand rather than reproduce their derivation. Alternatively extrapolation, by considering the differences in successive approximations, or 'iteration' by, for example, Romberg's method, may be the approach used. In general the standard of internal assessment was very high, with only a small number of minor changes being made in the moderation process.