FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper F

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. $$\mathrm{f}(x) = \frac{x^4 + x^3 - 13x^2 + 26x - 17}{x^2 - 3x + 3}.$$

Find the values of the constants A, B, C and D such that

$$\mathrm{f}(x) = x^2 + Ax + B + \frac{Cx + D}{x^2 - 3x + 3}.$$ [4]

2. Use the substitution $u = 1 - x^{\frac{1}{2}}$ to find

$$\int \frac{1}{1 - x^{\frac{1}{2}}}\, \mathrm{d}x.$$ [6]

3. A curve has the equation

$$4\cos x + 2\sin y = 3.$$

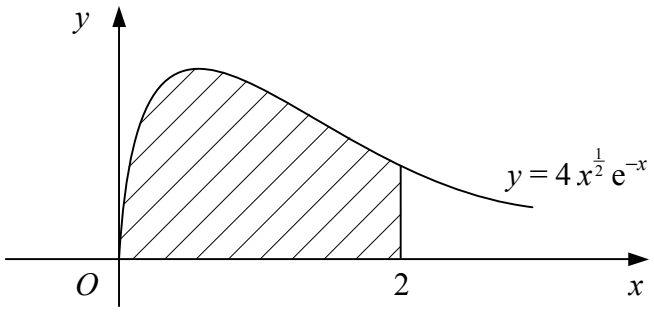
(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\sin x \sec y$. [4]

(ii) Find an equation for the tangent to the curve at the point $(\frac{\pi}{3}, \frac{\pi}{6})$, giving your answer in the form $ax + by = c$, where a and b are integers. [3]

4. *(i)* Express $\dfrac{3x + 6}{3x - x^2}$ in partial fractions. [3]

(ii) Evaluate $\displaystyle\int_1^2 \frac{3x + 6}{3x - x^2}\, \mathrm{d}x$. [4]

5.



The diagram shows the curve with equation $y = 4x^{\frac{1}{2}} \mathrm{e}^{-x}$.

The shaded region bounded by the curve, the x-axis and the line $x = 2$ is rotated through four right angles about the x-axis.

Find, in terms of π and e, the exact volume of the solid formed. [7]

6. $$f(x) = \frac{3}{\sqrt{1-x}}, \quad |x| < 1.$$

(i) Show that $f(\frac{1}{10}) = \sqrt{10}$. [2]

(ii) Expand $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [4]

(iii) Use your expansion to find an approximate value for $\sqrt{10}$, giving your answer to 8 significant figures. [1]

(iv) Find, to 1 significant figure, the percentage error in your answer to part *(c)*. [2]

7. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} + s \begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix}$$

and

$$\mathbf{r} = \begin{pmatrix} a \\ 6 \\ 3 \end{pmatrix} + t \begin{pmatrix} -5 \\ 14 \\ 2 \end{pmatrix},$$

where a is a constant and s and t are scalar parameters.

Given that the two lines intersect,

(i) find the position vector of their point of intersection, [4]

(ii) find the value of a. [2]

Given also that θ is the acute angle between the lines,

(iii) find the value of $\cos\theta$ in the form $k\sqrt{5}$ where k is rational. [4]

Turn over

8. A small town had a population of 9000 in the year 2001.

In a model, it is assumed that the population of the town, P, at time t years after 2001 satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.05P\mathrm{e}^{-0.05t}.$$

(i) Show that, according to the model, the population of the town in 2011 will be 13 300 to 3 significant figures. [7]

(ii) Find the value which the population of the town will approach in the long term, according to the model. [3]

9. A curve has parametric equations

$$x = t(t-1), \quad y = \frac{4t}{1-t}, \quad t \neq 1.$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [3]

The point P on the curve has parameter $t = -1$.

(ii) Show that the tangent to the curve at P has the equation

$$x + 3y + 4 = 0.$$ [3]

The tangent to the curve at P meets the curve again at the point Q.

(iii) Find the coordinates of Q. [6]