FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper J

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. Show that

$$\int_2^4 x(x^2-4)^{\frac{1}{2}}\ \mathrm{d}x = 8\sqrt{3}.$$ [5]

2. *(i)* Simplify

$$\frac{2x^2+3x-9}{2x^2-7x+6}.$$ [2]

(ii) Find the quotient and remainder when $(2x^4 - 1)$ is divided by $(x^2 - 2)$. [4]

3. A curve has the equation

$$2\sin 2x - \tan y = 0.$$

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of x and y. [4]

(ii) Show that the tangent to the curve at the point $(\frac{\pi}{6}, \frac{\pi}{3})$ has the equation

$$y = \tfrac{1}{2}x + \tfrac{\pi}{4}.$$ [3]

4. The gradient at any point (x, y) on a curve is proportional to $\sqrt{y}$.

Given that the curve passes through the point with coordinates (0, 4),

(i) show that the equation of the curve can be written in the form

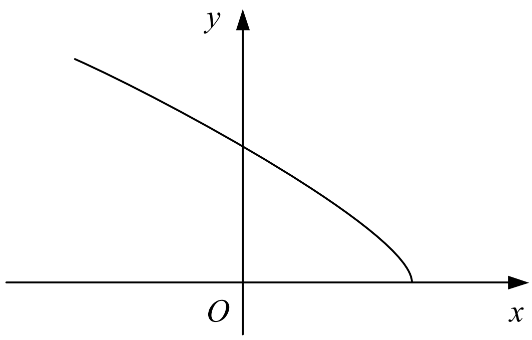
$$2\sqrt{y} = kx + 4,$$

where k is a positive constant. [5]

Given also that the curve passes through the point with coordinates (2, 9),

(ii) find the equation of the curve in the form $y = \mathrm{f}(x)$. [3]

5.



The diagram shows the curve with parametric equations

$$x = 2 - t^2, \quad y = t(t + 1), \quad t \geq 0.$$

(i) Find the coordinates of the points where the curve meets the coordinate axes. [3]

(ii) Find an equation for the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$. [6]

6.

$$\text{f}(x) = \frac{1+3x}{(1-x)(1-3x)}, \quad |x| < \tfrac{1}{3}.$$

(i) Find the values of the constants A and B such that

$$\text{f}(x) = \frac{A}{1-x} + \frac{B}{1-3x}.$$ [3]

(ii) Evaluate

$$\int_0^{\frac{1}{4}} \text{f}(x)\ \text{d}x,$$

giving your answer as a single logarithm. [4]

(iii) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

Turn over

7. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

and
$$\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + t \begin{pmatrix} 3 \\ a \\ b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(i) find a linear relationship between a and b. [2]

Given also that the two lines intersect,

(ii) find the values of a and b, [8]

(iii) find the coordinates of the point where they intersect. [2]

8. *(i)* Find

$$\int x^2 e^{\frac{1}{2}x} \; dx.$$ [6]

(ii) Using the substitution $u = \sin t$, evaluate

$$\int_0^{\frac{\pi}{2}} \sin^2 2t \cos t \; dt.$$ [7]