FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper I

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong

C4 Paper I – Marking Guide

1.	*(i)*	$= \frac{1}{\cos x} \times (-\sin x) = -\tan x$	M1 A1	
	(ii)	$= 2x \times \sin 3x + x^2 \times 3\cos 3x = 2x\sin 3x + 3x^2\cos 3x$	M1 A1	**(4)**
2.	*(i)*	$2x + 3y + 3x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	M1 A1	
		$\frac{dy}{dx} = \frac{2x+3y}{4y-3x}$	M1 A1	
	(ii)	grad $= \frac{6-6}{-8-9} = 0$	M1	
		$\therefore$ normal parallel to y-axis $\therefore x = 3$	M1 A1	**(7)**
3.	*(i)*	$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{(2x+1)(x-3)}$		
		$= \frac{3(2x^2-5x-3)-(x-1)(2x+1)+(x+11)}{(2x+1)(x-3)}$	M1	
		$= \frac{4x^2-13x+3}{(2x+1)(x-3)}$	A1	
		$= \frac{(4x-1)(x-3)}{(2x+1)(x-3)}$	M1	
		$= \frac{4x-1}{2x+1}$	A1	
	(ii)	$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \ldots$	M1	
		$= 1 - 2x + 4x^2 - 8x^3 + \ldots$	A2	
		$\therefore f(x) = (4x-1)(1 - 2x + 4x^2 - 8x^3 + \ldots)$		
		$= 4x - 8x^2 + 16x^3 - 1 + 2x - 4x^2 + 8x^3 + \ldots$	M1	
		$= -1 + 6x - 12x^2 + 24x^3 + \ldots$	A1	**(9)**
4.	*(i)*	$\frac{dx}{dt} = 3t^2, \quad \frac{dy}{dt} = -2t^{-2}$	M1	
		$\frac{dy}{dx} = -\frac{2}{3}t^{-4}$	M1 A1	
		$t = 1,\ x = 2,\ y = 2,$ grad $= -\frac{2}{3}$, grad of normal $= \frac{3}{2}$	M1	
		$\therefore y - 2 = \frac{3}{2}(x-2)$	M1	
		$y = \frac{3}{2}x - 1$	A1	
	(ii)	$t = \frac{2}{y} \quad \therefore x = (\frac{2}{y})^3 + 1 = \frac{8}{y^3} + 1$	M1	
		$\therefore y^3 = \frac{8}{x-1}, \quad y = \frac{2}{\sqrt[3]{x-1}}$	M1 A1	**(9)**
5.	*(i)*	$15 - 17x \equiv A(1-3x)^2 + B(2+x)(1-3x) + C(2+x)$	M1	
		$x = -2 \quad \Rightarrow \quad 49 = 49A \quad \Rightarrow \quad A = 1$	B1	
		$x = \frac{1}{3} \quad \Rightarrow \quad \frac{28}{3} = \frac{7}{3}C \quad \Rightarrow \quad C = 4$	B1	
		coeffs $x^2 \quad \Rightarrow \quad 0 = 9A - 3B \quad \Rightarrow \quad B = 3$	M1 A1	
	(ii)	$= \int_{-1}^{0} (\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2})\ dx$		
		$= [\ln\vert 2+x\vert - \ln\vert 1-3x\vert + \frac{4}{3}(1-3x)^{-1}]_{-1}^{0}$	M1 A2	
		$= (\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$	M1	
		$= 1 + \ln 8$	A1	**(10)**

6.	*(i)*	$1 + 3\lambda = -5 \quad \therefore\ \lambda = -2$	M1	
		$p - \lambda = 9 \quad \therefore\ p = 7$	A1	
		$-5 + q\lambda = -9 \quad \therefore\ q = 2$	A1	
	(ii)	$1 + 3\lambda = 25 \quad \therefore\ \lambda = 8$	M1	
		when $\lambda = 8$, $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix} + 8\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 25 \\ -1 \\ 11 \end{pmatrix}$		
		$\therefore$ (25, −1, 11) lies on l	A1	
	(iii)	$\overrightarrow{OC} = \begin{pmatrix} 1+3\lambda \\ 7-\lambda \\ -5+2\lambda \end{pmatrix} \quad \therefore \begin{pmatrix} 1+3\lambda \\ 7-\lambda \\ -5+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 0$	M1	
		$3 + 9\lambda - 7 + \lambda - 10 + 4\lambda = 0$		
		$\lambda = 1 \quad \therefore\ \overrightarrow{OC} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}$, $C\,(4, 6, -3)$	M1 A1	
	(iv)	$A : \lambda = -2,\ B : \lambda = 8,\ C : \lambda = 1 \quad \therefore\ AC : CB = 3 : 7$	M1 A1	**(10)**
7.	*(i)*	$x = 2\sin u \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = 2\cos u$	M1	
		$x = 0 \Rightarrow u = 0,\ x = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$	B1	
		$I = \int_0^{\frac{\pi}{3}} \dfrac{1}{2\cos u} \times 2\cos u \ \mathrm{d}u = \int_0^{\frac{\pi}{3}} 1 \ \mathrm{d}u$	M1 A1	
		$= [u]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$	M1 A1	
	(ii)	$u = x,\ u' = 1,\ v' = \cos x,\ v = \sin x$	M1	
		$I = [x\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \ \mathrm{d}x$	A1	
		$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$	M1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(11)**
8.	*(i)*	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = kN$	B1	
	(ii)	$\int \dfrac{1}{N} \ \mathrm{d}N = \int k \ \mathrm{d}t$	M1	
		$\ln\lvert N\rvert = kt + c$	M1 A1	
		$t = 0,\ N = N_0 \Rightarrow \ln\lvert N_0\rvert = c$	M1	
		$\ln\lvert N\rvert = kt + \ln\lvert N_0\rvert$		
		$\ln\left\lvert \dfrac{N}{N_0} \right\rvert = kt$	M1	
		$\dfrac{N}{N_0} = \mathrm{e}^{kt}, \quad N = N_0\mathrm{e}^{kt}$	A1	
	(iii)	$2N_0 = N_0\mathrm{e}^{6k}$	M1	
		$k = \frac{1}{6}\ln 2 = 0.116$ (3sf)	M1 A1	
	(iv)	$10N_0 = N_0\mathrm{e}^{0.1155t}$		
		$t = \frac{1}{0.1155}\ln 10 = 19.932$ hours = 19 hours 56 mins	M1 A1	**(12)**
			Total	**(72)**

Performance Record – C4 Paper I

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	differentiation	differentiation	rational expressions, binomial series	parametric equations	partial fractions, integration	vectors	integration	differential equation	
Marks	4	7	9	9	10	10	11	12	72
Student									