FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong

C4 Paper H – Marking Guide

1. $= \dfrac{(x-10)(2x-1)-(x-8)(x+4)}{(x-3)(x+4)(2x-1)}$ M1

$= \dfrac{x^2-17x+42}{(x-3)(x+4)(2x-1)}$ A1

$= \dfrac{(x-14)(x-3)}{(x-3)(x+4)(2x-1)}$ M1

$= \dfrac{x-14}{(x+4)(2x-1)}$ A1 **(4)**

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2. *(i)* $= 1 + (\tfrac{3}{2})(4x) + \dfrac{(\frac{3}{2})(\frac{1}{2})}{2}(4x)^2 + \dfrac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{3\times 2}(4x)^3 + \ldots$ M1

$= 1 + 6x + 6x^2 - 4x^3 + \ldots$ A3

(ii) $|x| < \tfrac{1}{4}$ B1 **(5)**

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3. $6x + y + x\dfrac{dy}{dx} - 4y\dfrac{dy}{dx} = 0$ M1 A1

$(1, 4) \Rightarrow 6 + 4 + \dfrac{dy}{dx} - 16\dfrac{dy}{dx} = 0, \quad \dfrac{dy}{dx} = \tfrac{2}{3}$ M1 A1

grad of normal $= -\tfrac{3}{2}$ M1

$\therefore\ y - 4 = -\tfrac{3}{2}(x - 1)$ M1

$2y - 8 = -3x + 3$

$3x + 2y - 11 = 0$ A1 **(7)**

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4. *(i)* $\overrightarrow{PQ} = (2\mathbf{i} - 9\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) = (3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ M1

$\therefore\ \mathbf{r} = (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) + s(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ A1

(ii) $6 + t = 2 \quad \therefore t = -4$ M1

$a + 4t = -9 \quad \therefore a = 7$ A1

$b - t = 1 \quad \therefore b = -3$ A1

(iii) $= \cos^{-1}\left|\dfrac{3\times1+(-1)\times4+(-2)\times(-1)}{\sqrt{9+1+4}\times\sqrt{1+16+1}}\right|$ M1 A1

$= \cos^{-1}\dfrac{1}{\sqrt{14}\times\sqrt{18}} = 86.4°$ (1dp) M1 A1 **(9)**

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5. *(i)* $\dfrac{dx}{dy} = \tfrac{1}{2}\sec\tfrac{y}{2}\tan\tfrac{y}{2}$ M1

$0 \le y < \pi \ \therefore \tan\tfrac{y}{2} \ge 0 \ \therefore\ \dfrac{dx}{dy} = \tfrac{1}{2}\sec\tfrac{y}{2}\sqrt{\sec^2\tfrac{y}{2} - 1} = \tfrac{1}{2}x\sqrt{x^2-1}$ M1

$\dfrac{dy}{dx} = 1 \div \dfrac{dx}{dy} = \dfrac{2}{x\sqrt{x^2-1}}$ M1 A1

(ii) $\dfrac{dy}{dx} = \tfrac{1}{2}(3+2\cos x)^{-\frac{1}{2}} \times (-2\sin x) = -\dfrac{\sin x}{\sqrt{3+2\cos x}}$ M1 A1

$x = \tfrac{\pi}{3},\ y = 2,\ \text{grad} = -\tfrac{1}{4}\sqrt{3}$ B1

$\therefore\ y - 2 = -\tfrac{1}{4}\sqrt{3}(x - \tfrac{\pi}{3}) \quad [\ 3\sqrt{3}x + 12y - 24 - \pi\sqrt{3} = 0\ ]$ M1 A1 **(9)**

6.	(i)	$\frac{dx}{dt} = \frac{1\times(2-t)-t\times(-1)}{(2-t)^2} = \frac{2}{(2-t)^2}$, $\quad \frac{dy}{dt} = -(1+t)^{-2}$	M1 B1	
		$\frac{dy}{dx} = -\frac{1}{(1+t)^2} \div \frac{2}{(2-t)^2} = -\frac{(2-t)^2}{2(1+t)^2} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$	M1 A1	
	(ii)	$t = 1,\ x = 1,\ y = \frac{1}{2}$, grad $= -\frac{1}{8}$	B1	
		grad of normal $= 8$		
		$\therefore\ y - \frac{1}{2} = 8(x-1) \qquad [\, y = 8x - \frac{15}{2}\,]$	M1 A1	
	(iii)	$x(2-t) = t$	M1	
		$2x = t(1+x),\ t = \frac{2x}{1+x}$	A1	
		$y = \frac{1}{1+\frac{2x}{1+x}} = \frac{1+x}{(1+x)+2x} \quad \therefore\ y = \frac{1+x}{1+3x}$	M1 A1	**(11)**
7.	(i)	$u = x^2,\ u' = 2x,\ v' = \sin x,\ v = -\cos x$	M1	
		$I = -x^2\cos x - \int -2x\cos x\ dx = -x^2\cos x + \int 2x\cos x\ dx$	A1	
		$u = 2x,\ u' = 2,\ v' = \cos x,\ v = \sin x$	M1	
		$I = -x^2\cos x + 2x\sin x - \int 2\sin x\ dx$	A1	
		$= -x^2\cos x + 2x\sin x + 2\cos x + c$	A1	
	(ii)	$u = 1 + \sin x \Rightarrow \frac{du}{dx} = \cos x$	M1	
		$x = 0 \Rightarrow u = 1,\ x = \frac{\pi}{2} \Rightarrow u = 2$	B1	
		$I = \int_1^2 u^3\ du$	M1 A1	
		$= [\frac{1}{4}u^4]_1^2$	M1	
		$= 4 - \frac{1}{4} = \frac{15}{4}$	A1	**(11)**
8.	(i)	$\frac{dV}{dt} = -kV,\quad \frac{dV}{dh} = 10\pi h - \pi h^2$	B2	
		$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \quad \therefore\ -kV = (10\pi h - \pi h^2)\frac{dh}{dt}$	M1	
		$-\frac{1}{3}k\pi h^2(15-h) = \pi h(10-h)\frac{dh}{dt}$		
		$-kh(15-h) = 3(10-h)\frac{dh}{dt} \quad \therefore\ \frac{dh}{dt} = -\frac{kh(15-h)}{3(10-h)}$	M1 A1	
	(ii)	$\frac{3(10-h)}{h(15-h)} \equiv \frac{A}{h} + \frac{B}{15-h}$, $\quad 3(10-h) \equiv A(15-h) + Bh$	M1	
		$h = 0 \Rightarrow A = 2,\ h = 15 \Rightarrow B = -1 \quad \therefore\ \frac{3(10-h)}{h(15-h)} \equiv \frac{2}{h} - \frac{1}{15-h}$	A2	
	(iii)	$\int \frac{3(10-h)}{h(15-h)}\ dh = \int -k\ dt,\quad \int (\frac{2}{h} - \frac{1}{15-h})\ dh = \int -k\ dt$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert = -kt + c$	M1 A1	
		$t = 0,\ h = 5 \Rightarrow\ 2\ln 5 + \ln 10 = c,\quad c = \ln 250$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert - \ln 250 = -kt$		
		$\ln\frac{h^2(15-h)}{250} = -kt,\quad \frac{h^2(15-h)}{250} = e^{-kt},\quad h^2(15-h) = 250e^{-kt}$	M1 A1	
	(iv)	$t = 2,\ h = 4 \Rightarrow\ 176 = 250e^{-2k}$		
		$k = -\frac{1}{2}\ln\frac{176}{250} = 0.175$ (3sf)	M1 A1	**(16)**
			Total	**(72)**

Performance Record – C4 Paper H

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	rational expressions	binomial series	differentiation	vectors	differentiation	parametric equations	integration	differential equation, partial fractions	
Marks	4	5	7	9	9	11	11	16	72
Student									