FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

# Core Mathematics C4

## Paper G

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Solomon Press

*Written by Shaun Armstrong*

# C4 Paper G – Marking Guide

| | | | |
|---|---|---|---|
| **1.** | $= \dfrac{x^2(2x+1)}{(x+2)(x-2)} \times \dfrac{x-2}{(2x+1)(x-3)}$ | M1 A1 | |
| | $= \dfrac{x^2}{(x+2)(x-3)}$ | M1 A1 | **(4)** |
| **2.** | $3x^2 + 2y + 2x\dfrac{dy}{dx} - 2y\dfrac{dy}{dx} = 0$ | M1 A1 | |
| | $(2, -4) \Rightarrow 12 - 8 + 4\dfrac{dy}{dx} + 8\dfrac{dy}{dx} = 0, \quad \dfrac{dy}{dx} = -\frac{1}{3}$ | M1 A1 | |
| | grad of normal = 3 | M1 | |
| | $\therefore\ y + 4 = 3(x - 2)$ | M1 | |
| | $y = 3x - 10$ | A1 | **(7)** |
| **3.** | $u = e^x - 1 \Rightarrow \dfrac{du}{dx} = e^x = u + 1$ | M1 | |
| | $x = \ln 2 \Rightarrow u = 1,\ x = \ln 5 \Rightarrow u = 4$ | B1 | |
| | $I = \displaystyle\int_1^4 \frac{(u+1)^2}{\sqrt{u}} \times \frac{1}{u+1}\ du$ | M1 | |
| | $= \displaystyle\int_1^4 (u^{\frac{1}{2}} + u^{-\frac{1}{2}})\ du$ | A1 | |
| | $= [\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}]_1^4$ | M1 A1 | |
| | $= (\frac{16}{3} + 4) - (\frac{2}{3} + 2) = \frac{20}{3}$ | M1 A1 | **(8)** |
| **4.** | *(i)* $= 1 + (-3)(ax) + \frac{(-3)(-4)}{2}(ax)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(ax)^3 + \ldots$ | M1 A1 | |
| | $= 1 - 3ax + 6a^2x^2 - 10a^3x^3 + \ldots$ | A1 | |
| | *(ii)* $\dfrac{6-x}{(1+ax)^3} = (6 - x)(1 - 3ax + 6a^2x^2 + \ldots)$ | | |
| | coeff. of $x^2 = 36a^2 + 3a = 3$ | M1 | |
| | $12a^2 + a - 1 = 0$ | A1 | |
| | $(4a - 1)(3a + 1) = 0$ | M1 | |
| | $a = -\frac{1}{3}, \frac{1}{4}$ | A1 | |
| | *(iii)* $a = -\frac{1}{3}\ \therefore\ \dfrac{6-x}{(1+ax)^3} = (6 - x)(\ldots + \frac{2}{3}x^2 + \frac{10}{27}x^3 + \ldots)$ | M1 | |
| | coeff. of $x^3 = (6 \times \frac{10}{27}) + (-1 \times \frac{2}{3}) = \frac{20}{9} - \frac{2}{3} = \frac{14}{9}$ | A1 | **(9)** |
| **5.** | *(i)* $\dfrac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \dfrac{A}{1-2x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$ | | |
| | $7 + 3x + 2x^2 \equiv A(1 + x)^2 + B(1 - 2x)(1 + x) + C(1 - 2x)$ | M1 | |
| | $x = \frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ | B1 | |
| | $x = -1 \Rightarrow 6 = 3C \Rightarrow C = 2$ | B1 | |
| | coeffs $x^2 \Rightarrow 2 = A - 2B \Rightarrow B = 1$ | M1 A1 | |
| | $\therefore\ f(x) = \dfrac{4}{1-2x} + \dfrac{1}{1+x} + \dfrac{2}{(1+x)^2}$ | | |
| | *(ii)* $= \displaystyle\int_1^2 \left(\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}\right) dx$ | | |
| | $= [-2\ln\lvert 1 - 2x\rvert + \ln\lvert 1 + x\rvert - 2(1 + x)^{-1}]_1^2$ | M1 A2 | |
| | $= (-2\ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1)$ | M1 | |
| | $= -\ln 3 - \ln 2 + \frac{1}{3} = \frac{1}{3} - \ln 6 \quad [\,p = \frac{1}{3}, q = 6\,]$ | A1 | **(10)** |

| | | | | |
|---|---|---|---|---|
| **6.** | *(i)* | $\overrightarrow{AB} = (5\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$ | B1 | |
| | | $\overrightarrow{AC} = (7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (5\mathbf{i} - 5\mathbf{j} - 10\mathbf{k}) = \frac{5}{3}\overrightarrow{AB}$ | M1 | |
| | | $\therefore$ $\overrightarrow{AC}$ is parallel to $\overrightarrow{AB}$, also common point $\therefore$ single straight line | A1 | |
| | *(ii)* | 3 : 2 | B1 | |
| | *(iii)* | $\overrightarrow{AD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ | B1 | |
| | | $\overrightarrow{BD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (5\mathbf{i} - 4\mathbf{j}) = (-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$ | B1 | |
| | | $\overrightarrow{AD} \cdot \overrightarrow{BD} = -2 + 10 - 8 = 0$ $\therefore$ perpendicular | M1 A1 | |
| | *(iv)* | $= \frac{1}{2} \times \sqrt{1+4+4} \times \sqrt{4+25+16} = \frac{1}{2} \times 3 \times 3\sqrt{5} = \frac{9}{2}\sqrt{5}$ | M2 A1 | **(11)** |
| **7.** | *(i)* | $\int x \ \mathrm{d}x = \int k(5-t) \ \mathrm{d}t$ | M1 | |
| | | $\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c$ | M1 A1 | |
| | | $t = 0, \ x = 0 \Rightarrow c = 0$ | B1 | |
| | | $t = 2, \ x = 96 \Rightarrow 4608 = 8k, \quad k = 576$ | M1 | |
| | | $t = 1 \Rightarrow \frac{1}{2}x^2 = 576 \times \frac{9}{2}, \quad x = \sqrt{5184} = 72$ | M1 A1 | |
| | *(ii)* | 3 hours 5 mins $\Rightarrow t = 3.0833, \ x = \sqrt{12284} = 110.83$ | M1 A1 | |
| | | $\therefore \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{576(5-3.0833)}{110.83} = 9.96, \quad \frac{\mathrm{d}x}{\mathrm{d}t} < 10$ so she should have left | M1 A1 | **(11)** |
| **8.** | *(i)* | $\mathrm{f}'(x) = \frac{(-\sin x)\times(2-\sin x) - \cos x \times(-\cos x)}{(2-\sin x)^2}$ | M1 A1 | |
| | | $= \frac{-2\sin x + \sin^2 x + \cos^2 x}{(2-\sin x)^2}$ | | |
| | | $= \frac{1 - 2\sin x}{(2-\sin x)^2}$ | A1 | |
| | *(ii)* | $x = \pi, \ y = -\frac{1}{2}$, grad $= \frac{1}{4}$ | B1 | |
| | | $\therefore \ y + \frac{1}{2} = \frac{1}{4}(x - \pi) \qquad [\, x - 4y - 2 - \pi = 0 \,]$ | M1 A1 | |
| | *(iii)* | from graph, min. and max. values at SP | | |
| | | SP: $\frac{1-2\sin x}{(2-\sin x)^2} = 0$ | | |
| | | $\sin x = \frac{1}{2}$ | M1 | |
| | | $x = \frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$ | A1 | |
| | | at SP, $y = \frac{\pm\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \pm\frac{1}{3}\sqrt{3}$ | M1 | |
| | | $\therefore$ min. $= -\frac{1}{3}\sqrt{3}$, max. $= \frac{1}{3}\sqrt{3}$ | A1 | |
| | *(iv)* | $\sin x$ and $\cos x$ both have period $2\pi$<br>$\mathrm{f}(x)$ is a function of $\sin x$ and $\cos x$ and $\therefore$ also has period $2\pi$<br>$\therefore$ values of $\mathrm{f}(x)$ in interval $0 \le x \le 2\pi$ are just repeated | B2 | **(12)** |
| | | | Total | **(72)** |

# Performance Record – C4 Paper G

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | Total |
|---|---|---|---|---|---|---|---|---|---|
| Topic(s) | rational expressions | differentiation | integration | binomial series | partial fractions, integration | vectors | differential equation | differentiation | |
| Marks | 4 | 7 | 8 | 9 | 10 | 11 | 11 | 12 | 72 |
| Student | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |