FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper E – Marking Guide

1. $f(x) = 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)}$

$= \frac{2x^2 - 7x + 5 + 4x(x-1) - 15}{(2x-5)(x-1)}$ M1

$= \frac{6x^2 - 11x - 10}{(2x-5)(x-1)}$ A1

$= \frac{(3x+2)(2x-5)}{(2x-5)(x-1)}$ M1

$= \frac{3x+2}{x-1}$ A1 **(4)**

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2. *(i)* $2x - 3y - 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ M1 A1

$\frac{dy}{dx} = \frac{2x-3y}{3x+2y}$ M1 A1

(ii) grad = 5 M1

$\therefore\ y + 2 = 5(x - 2)$ $[\ y = 5x - 12\ ]$ M1 A1 **(7)**

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3. *(i)* $= -\frac{1}{2}\ln|2 - x^2| + c$ M1 A2

(ii) $u = x^2,\ u' = 2x,\ v' = e^{-x},\ v = -e^{-x}$ M1

$I = -x^2 e^{-x} - \int -2x\,e^{-x}\,dx = -x^2 e^{-x} + \int 2x\,e^{-x}\,dx$ A1

$u = 2x,\ u' = 2,\ v' = e^{-x},\ v = -e^{-x}$ M1

$I = -x^2 e^{-x} - 2x\,e^{-x} - \int -2e^{-x}\,dx$ A1

$= -x^2 e^{-x} - 2x\,e^{-x} - 2e^{-x} + c$ A1 **(8)**

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4. *(i)* $y = \cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{1+\tan^2\theta}$ $\therefore\ y = \frac{1}{1+x^2}$ M2 A1

(ii) area $= \int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$x = \tan u,\ \frac{dx}{du} = \sec^2 u$ M1

$x = -1 \Rightarrow u = -\frac{\pi}{4},\ x = 1 \Rightarrow u = \frac{\pi}{4}$ B1

area $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\tan^2 u} \times \sec^2 u\,du$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\sec^2 u} \times \sec^2 u\,du = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du$ A1

$= [u]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4} - (-\frac{\pi}{4}) = \frac{\pi}{2}$ M1 A1 **(8)**

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5. *(i)* $= 4^{\frac{1}{2}}(1 - \frac{1}{4}x)^{\frac{1}{2}} = 2(1 - \frac{1}{4}x)^{\frac{1}{2}}$ B1

$= 2[1 + (\frac{1}{2})(-\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-\frac{1}{4}x)^2 + \ldots] = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \ldots$ M1 A2

(ii) $|x| < 4$ B1

(iii) $x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} = \sqrt{3.99} = \sqrt{\frac{399}{100}} = \frac{1}{10}\sqrt{399}$ M1

$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} \approx 2 - \frac{1}{400} - \frac{1}{640000} = 1.997498438$ M1

$\therefore\ \sqrt{399} \approx 10 \times 1.997498438 = 19.9749844$ (9sf) M1 A1 **(9)**

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6.	*(i)*	$\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$		
		$= -(\cos x)^{-2} \times (-\sin x)$	M1 A1	
		$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$	M1	
		$= \sec x \tan x$	A1	
	(ii)	$\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x\,(2 + \tan x)$	M1 A1	
		$x = 0,\ y = 1,\ \text{grad} = 2$	M1	
		$\therefore\ y = 2x + 1$	A1	
	(iii)	SP: $e^{2x} \sec x\,(2 + \tan x) = 0$	M1	
		$\tan x = -2$	M1	
		$x = -1.11$ (2dp)	A1	**(11)**
7.	*(i)*	$\overrightarrow{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	A1	
	(ii)	$3 - 3\lambda = -2 + 7\mu$ (1)		
		$6 + 2\lambda = 10 - 4\mu$ (2)		
		$-8 + 2\lambda = 6 + 6\mu$ (3)	B1	
		(3) – (2): $-14 = -4 + 10\mu,\ \mu = -1,\ \lambda = 4$	M1 A1	
		check (1) $3 - 12 = -2 - 7$, true $\therefore$ intersect	B1	
	(iii)	$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ $\therefore (-9, 14, 0)$	B1	
	(iv)	$\overrightarrow{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$		
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$	M1 A1	
		$\therefore\ [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}].(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$	M1	
		$15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$		
		$\mu = 3\ \therefore\ \overrightarrow{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$	M1 A1	**(12)**
8.	*(i)*	when $x = \frac{1}{4},\ \frac{dx}{dt} = \frac{3}{4} \div 6 = \frac{1}{8}$	M1	
		$\frac{dx}{dt} = kx(1 - x)\ \therefore\ \frac{1}{8} = k \times \frac{1}{4} \times \frac{3}{4},\ k = \frac{2}{3}\ \therefore\ \frac{dx}{dt} = \frac{2}{3}x(1 - x)$	M1 A1	
	(ii)	$\int \frac{1}{x(1-x)}\,dx = \int \frac{2}{3}\,dt$	M1	
		$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x},\quad 1 \equiv A(1 - x) + Bx$	M1	
		$x = 0 \Rightarrow A = 1$	A1	
		$x = 1 \Rightarrow B = 1$	A1	
		$\therefore\ \int (\frac{1}{x} + \frac{1}{1-x})\,dx = \int \frac{2}{3}\,dt$		
		$\ln\lvert x\rvert - \ln\lvert 1 - x\rvert = \frac{2}{3}t + c$	M1 A1	
		$t = 0,\ x = \frac{1}{4} \Rightarrow \ln\frac{1}{4} - \ln\frac{3}{4} = c,\quad c = \ln\frac{1}{3}$	M1	
		$t = 3 \Rightarrow \ln\lvert x\rvert - \ln\lvert 1 - x\rvert = 2 + \ln\frac{1}{3}$		
		$\ln\left\lvert\frac{3x}{1-x}\right\rvert = 2,\quad \frac{3x}{1-x} = e^2$	M1	
		$3x = e^2(1 - x),\quad x(e^2 + 3) = e^2$	M1	
		$x = \frac{e^2}{e^2+3}\ \therefore$ % destroyed $= \frac{e^2}{e^2+3} \times 100\% = 71.1\%$ (3sf)	A1	**(13)**
			Total	**(72)**

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Performance Record – C4 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	rational expressions	differentiation	integration	parametric equations	binomial series	differentiation	vectors	differential equation, partial fractions	
Marks	4	7	8	8	9	11	12	13	72
Student									

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