FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper D – Marking Guide

1.	$= [-\frac{1}{2}(1+\cos x)^2]_0^{\pi}$	M1 A1	
	$= -\frac{1}{2}(0-4) = 2$	M1 A1	**(4)**
2.	*(i)* $= \frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$	M1 A1	
	(ii) $= \frac{x+4}{(2x+1)(x+1)} - \frac{2}{2x+1}$	M1	
	$= \frac{(x+4)-2(x+1)}{(2x+1)(x+1)}$	M1	
	$= \frac{2-x}{(2x+1)(x+1)}$	A1	**(5)**
3.	$u = \ln x,\ u' = \frac{1}{x},\ v' = x^2,\ v = \frac{1}{3}x^3$	M1	
	$I = [\frac{1}{3}x^3 \ln x]_1^3 - \int_1^3 \frac{1}{3}x^2\ \mathrm{d}x$	A1	
	$= [\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$	A1	
	$= (9\ln 3 - 3) - (0 - \frac{1}{9})$	M1	
	$= 9\ln 3 - \frac{26}{9}$	A1	**(5)**
4.	*(i)* $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 + \cos t, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \cos t$	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos t}{1+\cos t}$	M1 A1	
	(ii) $\frac{\cos t}{1+\cos t} = 0,\ \cos t = 0,\ t = \frac{\pi}{2}$	M1 A1	
	$\therefore\ (\frac{\pi}{2} + 1, 1)$	A1	**(6)**
5.	$\int \frac{1}{y^2}\ \mathrm{d}y = \int \sqrt{x}\ \mathrm{d}x$	M1	
	$-y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + c$	M1 A1	
	$x = 1,\ y = -2 \Rightarrow \frac{1}{2} = \frac{2}{3} + c, \quad c = -\frac{1}{6}$	M1 A1	
	$-\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}, \quad \frac{1}{y} = \frac{1}{6} - \frac{2}{3}x^{\frac{3}{2}} = \frac{1}{6}(1 - 4x^{\frac{3}{2}})$	M1	
	$y = \frac{6}{1-4x^{\frac{3}{2}}}$	A1	**(7)**
6.	*(i)* $= \int (\sec^2 3x - 1)\ \mathrm{d}x$	M1	
	$= \frac{1}{3}\tan 3x - x + c$	M1 A1	
	(ii) $u = x^2 + 4 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	M1	
	$x = 0 \Rightarrow u = 4,\ x = 2 \Rightarrow u = 8$	B1	
	$I = \int_4^8 \frac{5}{2}u^{-2}\ \mathrm{d}u$	A1	
	$= [-\frac{5}{2}u^{-1}]_4^8$	M1	
	$= -\frac{5}{16} - (-\frac{5}{8}) = \frac{5}{16}$	M1 A1	**(9)**

7.	*(i)*	$6x - 2 + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$	M1 A1	
		$(-1, 3) \Rightarrow -6 - 2 + 3 - \frac{dy}{dx} + 6\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = 1$	M1 A1	
		grad of normal = -1		
		$\therefore\ y - 3 = -(x + 1)$	M1	
		$y = 2 - x$	A1	
	(ii)	sub. $\Rightarrow\ 3x^2 - 2x + x(2 - x) + (2 - x)^2 - 11 = 0$	M1	
		$3x^2 - 4x - 7 = 0$	A1	
		$(3x - 7)(x + 1) = 0$	M1	
		$x = -1$ (at P) or $\frac{7}{3}$ $\therefore (\frac{7}{3}, -\frac{1}{3})$	A1	**(10)**

8.	*(i)*	$\overrightarrow{AB} = (7\mathbf{i} - \mathbf{j} + 12\mathbf{k}) - (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = (10\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \lambda(5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$	A1	
	(ii)	$\overrightarrow{OC} = [\mu\,\mathbf{i} + (5 - 2\mu)\mathbf{j} + (-7 + 7\mu)\mathbf{k}]$		
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(3 + \mu)\mathbf{i} + (2 - 2\mu)\mathbf{j} + (-9 + 7\mu)\mathbf{k}]$	M1 A1	
		$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [(-7 + \mu)\mathbf{i} + (6 - 2\mu)\mathbf{j} + (-19 + 7\mu)\mathbf{k}]$	A1	
		$\overrightarrow{AC} \cdot \overrightarrow{BC} = (3 + \mu)(-7 + \mu) + (2 - 2\mu)(6 - 2\mu) + (-9 + 7\mu)(-19 + 7\mu) = 0$	M1	
		$\mu^2 - 4\mu + 3 = 0$	A1	
		$(\mu - 1)(\mu - 3) = 0$	M1	
		$\mu = 1, 3 \quad \therefore\ \overrightarrow{OC} = (\mathbf{i} + 3\mathbf{j})$ or $(3\mathbf{i} - \mathbf{j} + 14\mathbf{k})$	A2	
	(iii)	$AC = \sqrt{16+0+4} = 2\sqrt{5}$, $BC = \sqrt{36+16+144} = 14$	M1	
		area = $\frac{1}{2} \times 2\sqrt{5} \times 14 = 14\sqrt{5}$	M1 A1	**(13)**

9.	*(i)*	$\frac{8 - x}{(1+x)(2-x)} \equiv \frac{A}{1+x} + \frac{B}{2-x}$		
		$8 - x \equiv A(2 - x) + B(1 + x)$	M1	
		$x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$	A1	
		$x = 2 \Rightarrow 6 = 3B \Rightarrow B = 2 \ \therefore\ f(x) = \frac{3}{1+x} + \frac{2}{2-x}$	A1	
	(ii)	$= \int_0^{\frac{1}{2}} (\frac{3}{1+x} + \frac{2}{2-x})\ dx = [3 \ln\lvert 1 + x\rvert - 2 \ln\lvert 2 - x\rvert]_0^{\frac{1}{2}}$	M1 A1	
		$= (3 \ln \frac{3}{2} - 2 \ln \frac{3}{2}) - (0 - 2 \ln 2)$	M1	
		$= \ln \frac{3}{2} + \ln 4 = \ln 6$	A1	
	(iii)	$f(x) = 3(1 + x)^{-1} + 2(2 - x)^{-1}$		
		$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots$	B1	
		$(2 - x)^{-1} = 2^{-1}(1 - \frac{1}{2}x)^{-1}$	M1	
		$= \frac{1}{2}[1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2}(-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-\frac{1}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{2}(1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	A1	
		$\therefore\ f(x) = 3(1 - x + x^2 - x^3 + \ldots) + (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	M1	
		$= 4 - \frac{5}{2}x + \frac{13}{4}x^2 - \frac{23}{8}x^3 + \ldots$	A1	**(13)**

Total **(72)**

Performance Record – C4 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	integration	rational expressions	integration	parametric equations	differential equation	integration	differentiation	vectors	partial fractions, integration, binomial series	
Marks	4	5	5	6	7	9	10	13	13	72
Student										