FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper C – Marking Guide

1.		$2x(2+y) + x^2\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M2 A1	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x(2+y)}{2y-x^2}$	M1 A1	**(5)**
2.		$u = \ln x,\ u' = \frac{1}{x},\ v' = x,\ v = \frac{1}{2}x^2$	M1	
		$I = [\frac{1}{2}x^2 \ln x]_1^2 - \int_1^2 \frac{1}{2}x\ \mathrm{d}x$	A1	
		$= [\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^2$	M1	
		$= (2 \ln 2 - 1) - (0 - \frac{1}{4}) = 2 \ln 2 - \frac{3}{4}$	M1 A1	**(5)**
3.		$= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 \sin x + \operatorname{cosec} x)^2\ \mathrm{d}x = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \sin^2 x + 4 + \operatorname{cosec}^2 x)\ \mathrm{d}x$	M1 A1	
		$= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 - 2 \cos 2x + 4 + \operatorname{cosec}^2 x)\ \mathrm{d}x$	M1	
		$= \pi[6x - \sin 2x - \cot x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	M1 A1	
		$= \pi\{(3\pi + 0 + 0) - (\pi - \frac{\sqrt{3}}{2} - \sqrt{3})\}$	M1	
		$= \pi(2\pi + \frac{3}{2}\sqrt{3}) = \frac{1}{2}\pi(4\pi + 3\sqrt{3})$	A1	**(7)**
4.	*(i)*	$= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3} = \frac{4x-2(x-3)}{(x+3)(x-3)}$	M1	
		$= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$	M1 A1	
	(ii)	$2^3 - 8 = 0\ \therefore\ (x - 2)$ is a factor of $(x^3 - 8)$	B1	
		$\begin{array}{rl} & x^2 + 2x + 4 \\ x-2 \big) & x^3 + 0x^2 + 0x - 8 \\ & x^3 - 2x^2 \\ & 2x^2 + 0x \\ & 2x^2 - 4x \\ & 4x - 8 \\ & 4x - 8 \end{array}$	M1 A1	
		$\therefore\ x^3 - 8 = (x - 2)(x^2 + 2x + 4)$		
		$\therefore\ \frac{x^3-8}{3x^2-8x+4} = \frac{(x-2)(x^2+2x+4)}{(3x-2)(x-2)} = \frac{x^2+2x+4}{3x-2}$	M1 A1	**(8)**
5.	*(i)*	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - 20)$	B1	
	(ii)	$\int \frac{1}{\theta-20}\ \mathrm{d}\theta = \int -k\ \mathrm{d}t$	M1	
		$\ln\vert\theta - 20\vert = -kt + c$	M1 A1	
		$t = 0,\ \theta = 37 \Rightarrow c = \ln 17$	M1	
		$\ln\vert\frac{\theta-20}{17}\vert = -kt,\quad \theta = 20 + 17\mathrm{e}^{-kt}$	A1	
		$t = 4,\ \theta = 36 \Rightarrow\quad 36 = 20 + 17\mathrm{e}^{-4k}$	M1	
		$k = -\frac{1}{4}\ln\frac{16}{17} = 0.01516$	A1	
		$t = 10,\ \theta = 20 + 17\mathrm{e}^{-0.01516 \times 10} = 34.6$°C (3sf)	A1	
	(iii)	$33 = 20 + 17\mathrm{e}^{-0.01516t}$		
		$t = -\frac{1}{0.01516}\ln\frac{13}{17} = 17.70$ minutes = 17 mins 42 secs	M1 A1	**(11)**

6.	*(i)*	$\frac{dx}{dt} = 6\cos t \times (-\sin t), \quad \frac{dy}{dt} = 2\cos 2t$	M1	
		$\frac{dy}{dx} = \frac{2\cos 2t}{-6\cos t \sin t} = \frac{2\cos 2t}{-3\sin 2t} = -\frac{2}{3}\cot 2t$	M1 A1	
	(ii)	$-\frac{2}{3}\cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$	M1 A1	
		$\therefore (\frac{3}{2}, 1), (\frac{3}{2}, -1)$	A1	
	(iii)	$t = \frac{\pi}{6},\ x = \frac{9}{4},\ y = \frac{\sqrt{3}}{2},\ \text{grad} = -\frac{2}{3\sqrt{3}}$	B1	
		$\therefore y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$	M1	
		$6\sqrt{3}\,y - 9 = -4x + 9$		
		$2x + 3\sqrt{3}\,y = 9$	A1	
	(iv)	$y^2 = \sin^2 2t = 4\sin^2 t\cos^2 t = 4(1 - \cos^2 t)\cos^2 t$	M1	
		$\cos^2 t = \frac{x}{3} \quad \therefore y^2 = 4(1 - \frac{x}{3})\frac{x}{3},\quad y^2 = \frac{4}{9}x(3 - x)$	M1 A1	**(12)**
7.	*(i)*	$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} \quad \therefore \mathbf{r} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix}$	M1 A1	
	(ii)	$-4 + s = 3 + 2t \quad (1)$		
		$1 + 5s = -7 - 3t \quad (2)$		
		$3 - 2s = 9 + t \quad (3)$	B1	
		$2 \times (1) + (3)$: $-5 = 15 + 5t,\ t = -4,\ s = -1$	M1 A1	
		sub. (2): $1 - 5 = -7 + 12$, not true $\therefore$ do not intersect	A1	
	(iii)	$\overrightarrow{OC} = \begin{pmatrix} 3+2t \\ -7-3t \\ 9+t \end{pmatrix},\ \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} 6+2t \\ -13-3t \\ 8+t \end{pmatrix}$	M1 A1	
		$\therefore \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 6+2t \\ -13-3t \\ 8+t \end{pmatrix} = 0, \quad 6 + 2t - 65 - 15t - 16 - 2t = 0$	M1 A1	
		$t = -5 \quad \therefore \overrightarrow{OC} = \begin{pmatrix} -7 \\ 8 \\ 4 \end{pmatrix}$	M1 A1	**(12)**
8.	*(i)*	$\frac{5-8x}{(1+2x)(1-x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$		
		$5 - 8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$	M1	
		$x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$	A1	
		$x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$	A1	
		coeffs $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$	M1 A1	
		$\mathrm{f}(x) = \frac{4}{1+2x} + \frac{2}{1-x} - \frac{1}{(1-x)^2}$		
	(ii)	$\mathrm{f}(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$		
		$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(2x)^3 + \ldots$	M1	
		$= 1 - 2x + 4x^2 - 8x^3 + \ldots$	A1	
		$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	B1	
		$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(-x)^3 + \ldots$		
		$= 1 + 2x + 3x^2 + 4x^3 + \ldots$	A1	
		$\mathrm{f}(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$	M1	
		$= 5 - 8x + 15x^2 - 34x^3 + \ldots$	A1	
	(iii)	$\lvert x \rvert < \frac{1}{2}$	A1	**(12)**
			Total	**(72)**

Performance Record – C4 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	differentiation	integration	integration	rational expressions	differential equation	parametric equations	vectors	partial fractions, binomial series	
Marks	5	5	7	8	11	12	12	12	72
Student									