FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper L – Marking Guide

1.	$= \frac{5x(x-2)+3(x-4)}{(x-4)(x+1)(x-2)} = \frac{5x^2-7x-12}{(x-4)(x+1)(x-2)}$	M1 A1
	$= \frac{(5x-12)(x+1)}{(x-4)(x+1)(x-2)} = \frac{5x-12}{(x-4)(x-2)}$	M1 A1 **(4)**
2.	$2x + 2y^2 + 2x \times 2y\frac{dy}{dx} + \frac{dy}{dx} = 0$	M1 A2
	$\frac{dy}{dx} = -\frac{2x+2y^2}{4xy+1}$	M1 A1 **(5)**
3.	$= \int_0^{\frac{\pi}{3}} 2 \sin x \cos^2 x \ dx$	M1
	$= [-\frac{2}{3}\cos^3 x]_0^{\frac{\pi}{3}}$	M1 A1
	$= -\frac{1}{12} - (-\frac{2}{3}) = \frac{7}{12}$	M1 A1 **(5)**
4.	*(i)* $\cos 2t = \frac{1}{2},\ 2t = \frac{\pi}{3},\ t = \frac{\pi}{6}$	M1 A1
	(ii) $\frac{dx}{dt} = -2\sin 2t,\ \frac{dy}{dt} = -\text{cosec } t \cot t$	M1
	$\frac{dy}{dx} = \frac{-\text{cosec } t \cot t}{-2\sin 2t}$	M1 A1
	$t = \frac{\pi}{6},\ y = 2,\ \text{grad} = 2$	
	$\therefore\ y - 2 = 2(x - \frac{1}{2})$	M1
	$y = 2x + 1$	A1 **(7)**
5.	*(i)* $\frac{2+20x}{1+2x-8x^2} = \frac{2+20x}{(1-2x)(1+4x)} \equiv \frac{A}{1-2x} + \frac{B}{1+4x}$	
	$2 + 20x \equiv A(1 + 4x) + B(1 - 2x)$	M1
	$x = \frac{1}{2} \Rightarrow 12 = 3A \Rightarrow A = 4$	A1
	$x = -\frac{1}{4} \Rightarrow -3 = \frac{3}{2}B \Rightarrow B = -2$	A1
	$\frac{2+20x}{1+2x-8x^2} \equiv \frac{4}{1-2x} - \frac{2}{1+4x}$	
	(ii) $\frac{2+20x}{1+2x-8x^2} = 4(1-2x)^{-1} - 2(1+4x)^{-1}$	
	$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-2x)^3 + \ldots$	M1
	$= 1 + 2x + 4x^2 + 8x^3 + \ldots$	A1
	$(1+4x)^{-1} = 1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(4x)^3 + \ldots$	
	$= 1 - 4x + 16x^2 - 64x^3 + \ldots$	A1
	$\frac{2+20x}{1+2x-8x^2} = 4(1 + 2x + 4x^2 + 8x^3 + \ldots) - 2(1 - 4x + 16x^2 - 64x^3 + \ldots)$	M1
	$= 2 + 16x - 16x^2 - 160x^3 + \ldots$	A1 **(8)**
6.	$x = 2\tan u \Rightarrow \frac{dx}{du} = 2\sec^2 u,\quad x = 0 \Rightarrow u = 0,\ x = 2 \Rightarrow u = \frac{\pi}{4}$	M1 B1
	$I = \int_0^{\frac{\pi}{4}} \frac{4\tan^2 u}{4\sec^2 u} \times 2\sec^2 u \ du = \int_0^{\frac{\pi}{4}} 2\tan^2 u \ du$	M1 A1
	$= \int_0^{\frac{\pi}{4}} (2\sec^2 u - 2) \ du$	M1
	$= [2\tan u - 2u]_0^{\frac{\pi}{4}}$	M1
	$= (2 - \frac{\pi}{2}) - (0) = \frac{1}{2}(4 - \pi)$	M1 A1 **(8)**

7.	*(i)*	$\overrightarrow{AB} = (4\mathbf{j} + \mathbf{k}) - (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) = (-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + s(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	A1	
		at C, $2 - s = -1,\ s = 3$	M1 A1	
		$\therefore\ \overrightarrow{OC} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}) = (-18\mathbf{i} + 28\mathbf{j} - \mathbf{k})$	A1	
	(ii)	$\overrightarrow{AC} = 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}),\ AC = 3\sqrt{81+144+1} = 45.10$	M1 A1	
		$\therefore$ distance $= 200 \times 45.10 = 9020$ m $= 9.02$ km (3sf)	M1 A1	**(9)**
8.	*(i)*	$= \int (\sec^2 x - 1)\ dx$	M1	
		$= \tan x - x + c$	M1 A1	
	(ii)	$= \int \frac{\sin x}{\cos x}\ dx$, let $u = \cos x$, $\frac{du}{dx} = -\sin x$	M1	
		$= \int \frac{1}{u} \times (-1)\ du = -\int \frac{1}{u}\ du$	A1	
		$= -\ln\lvert u \rvert + c = \ln\lvert u^{-1} \rvert + c = \ln\lvert \sec x \rvert + c$	M1 A1	
	(iii)	volume $= \pi \int_0^{\frac{\pi}{3}} x \tan^2 x\ dx$		
		$u = x,\ u' = 1,\ v' = \tan^2 x,\ v = \tan x - x$	M1	
		$I = x(\tan x - x) - \int (\tan x - x)\ dx$	A1	
		$= x \tan x - x^2 - \ln\lvert \sec x \rvert + \frac{1}{2}x^2 + c$	A1	
		volume $= \pi[x \tan x - \frac{1}{2}x^2 - \ln\lvert \sec x \rvert]_0^{\frac{\pi}{3}}$		
		$= \pi\{(\frac{1}{3}\sqrt{3}\,\pi - \frac{1}{18}\pi^2 - \ln 2) - (0)\}$	M1	
		$= \frac{1}{18}\pi^2(6\sqrt{3} - \pi) - \pi \ln 2$	A1	**(12)**
9.	*(i)*	$\int \frac{1}{P}\ dP = \int k\ dt$	M1	
		$\ln\lvert P \rvert = kt + c$	A1	
		$t = 0,\ P = 300 \Rightarrow \ln 300 = c$	M1	
		$\ln\lvert P \rvert = kt + \ln 300$		
		$\ln\lvert \frac{P}{300} \rvert = kt,\quad \frac{P}{300} = e^{kt},\quad P = 300e^{kt}$	M1 A1	
	(ii)	$t = 1,\ P = 360 \Rightarrow 360 = 300e^{k}$	M1	
		$k = \ln\frac{6}{5} = 0.182$ (3sf)	A1	
	(iii)	$P = 300e^{0.1823t}$		
		when $t = 2,\ P = 432$; when $t = 3,\ P = 518$	B1	
		model does not seem suitable as data diverges from predictions	B1	
	(iv)	$\int \frac{1}{P}\ dP = \int (0.4 - 0.25 \cos 0.5t)\ dt$		
		$\ln\lvert P \rvert = 0.4t - 0.5 \sin 0.5t + c$	M1	
		$t = 0,\ P = 300 \Rightarrow \ln 300 = c$	M1	
		$\ln\lvert \frac{P}{300} \rvert = 0.4t - 0.5 \sin 0.5t \quad [\,P = 300e^{0.4t - 0.5 \sin 0.5t}\,]$	A1	
	(v)	second model: $t = 1, 2, 3 \Rightarrow P = 352, 438, 605$	B1	
		the second model seems more suitable as it fits the data better	B1	**(14)**
			Total	**(72)**

Performance Record – C4 Paper L

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	rational expressions	differentiation	integration	parametric equations	partial fractions, binomial series	integration	vectors	integration	differential equation	
Marks	4	5	5	7	8	8	9	12	14	72
Student										