FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper D

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. *(i)* Show that

$$\sin (x + 30)° + \sin (x - 30)° \equiv a \sin x°,$$

where a is a constant to be found. [3]

(ii) Hence find the exact value of $\sin 75° + \sin 15°$, giving your answer in the form $b\sqrt{6}$. [3]

2. Solve each equation, giving your answers in exact form.

(i) $\ln (2x - 3) = 1$ [2]

(ii) $3e^{y} + 5e^{-y} = 16$ [5]

3. The curve C has the equation $y = 2e^{x} - 6 \ln x$ and passes through the point P with x-coordinate 1.

(i) Find an equation for the tangent to C at P. [4]

The tangent to C at P meets the coordinate axes at the points Q and R.

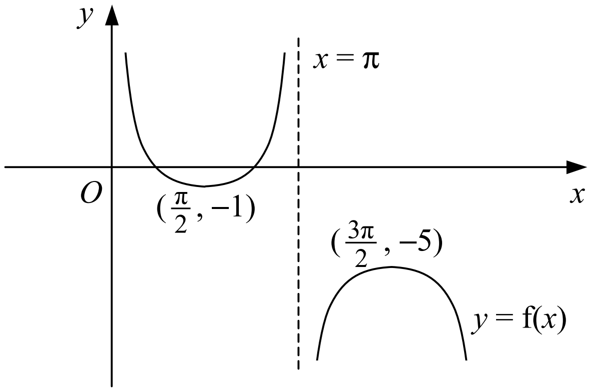
(ii) Show that the area of triangle OQR, where O is the origin, is $\dfrac{9}{3-e}$. [4]

4. The finite region R is bounded by the curve with equation $y = \dfrac{1}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 2$.

(i) Find the exact area of R. [4]

(ii) Show that the volume of the solid formed when R is rotated through four right angles about the x-axis is $\frac{1}{3}\pi$. [4]

5.



The diagram shows the graph of $y = \mathrm{f}(x)$. The graph has a minimum at $(\frac{\pi}{2}, -1)$, a maximum at $(\frac{3\pi}{2}, -5)$ and an asymptote with equation $x = \pi$.

(i) Showing the coordinates of any stationary points, sketch the graph of $y = |\mathrm{f}(x)|$. [2]

Given that

$$\mathrm{f} : x \to a + b \operatorname{cosec} x, \quad x \in \mathbb{R}, \quad 0 < x < 2\pi, \quad x \neq \pi,$$

(ii) find the values of the constants a and b, [3]

(iii) find, to 2 decimal places, the x-coordinates of the points where the graph of $y = \mathrm{f}(x)$ crosses the x-axis. [3]

6. *(i)* Prove the identity

$$2 \cot 2x + \tan x \equiv \cot x, \quad x \neq \tfrac{n}{2}\pi, \quad n \in \mathbb{Z}.$$ [5]

(ii) Solve, for $0 \leq x < \pi$, the equation

$$2 \cot 2x + \tan x = \operatorname{cosec}^2 x - 7,$$

giving your answers to 2 decimal places. [6]

Turn over

7. The function f is defined by

$$f : x \to 3e^{x-1}, \quad x \in \mathbb{R}.$$

(i) State the range of f. [1]

(ii) Find an expression for $f^{-1}(x)$ and state its domain. [3]

The function g is defined by

$$g : x \to 5x - 2, \quad x \in \mathbb{R}.$$

Find, in terms of e,

(iii) the value of gf(ln 2), [3]

(iv) the solution of the equation

$$f^{-1}g(x) = 4.$$ [4]

8. A curve has the equation $y = x^2 - \sqrt{4 + \ln x}$.

(i) Show that the tangent to the curve at the point where $x = 1$ has the equation

$$7x - 4y = 11.$$ [5]

The curve has a stationary point with x-coordinate α.

(ii) Show that $0.3 < \alpha < 0.4$ [3]

(iii) Show that α is a solution of the equation

$$x = \tfrac{1}{2}(4 + \ln x)^{-\frac{1}{4}}.$$ [2]

(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}(4 + \ln x_n)^{-\frac{1}{4}},$$

with $x_0 = 0.35$, to find α correct to 5 decimal places.

You should show the result of each iteration. [3]