FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper K

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. Show that

$$\int_1^7 \frac{2}{4x-1}\, dx = \ln 3.$$ [4]

2. Find the set of values of x such that

$$|3x+1| \leq |x-2|.$$ [5]

3. Find all values of θ in the interval $-180 < \theta < 180$ for which

$$\tan^2 \theta° + \sec \theta° = 1.$$ [6]

4. Solve each equation, giving your answers in exact form.

(i) $e^{4x-3} = 2$ [2]

(ii) $\ln (2y - 1) = 1 + \ln (3 - y)$ [4]

5. *(i)* Prove, by counter-example, that the statement

"$\operatorname{cosec} \theta - \sin \theta > 0$ for all values of θ in the interval $0 < \theta < \pi$"

is false. [2]

(ii) Find the values of θ in the interval $0 < \theta < \pi$ such that

$$\operatorname{cosec} \theta - \sin \theta = 2,$$

giving your answers to 2 decimal places. [5]

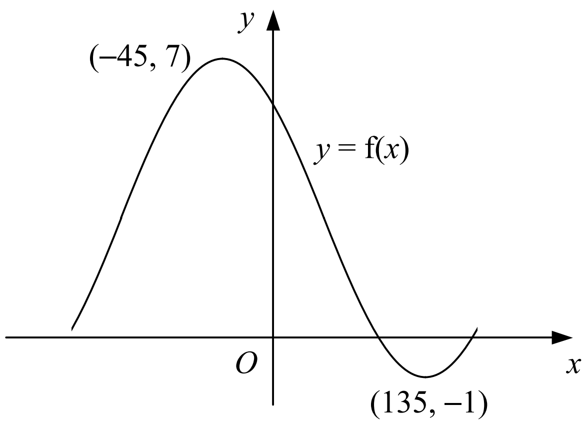
6. The curve C has the equation $y = x^2 - 5x + 2 \ln \frac{x}{3}$, $x > 0$.

(i) Show that the normal to C at the point where $x = 3$ has the equation

$$3x + 5y + 21 = 0.$$ [5]

(ii) Find the x-coordinates of the stationary points of C. [3]

7.



The diagram shows the curve $y = f(x)$ which has a maximum point at (–45, 7) and a minimum point at (135, –1).

(i) Showing the coordinates of any stationary points, sketch the curve with equation $y = 1 + 2f(x)$. [3]

Given that

$$f(x) = A + 2\sqrt{2}\cos x° - 2\sqrt{2}\sin x°, \quad x \in \mathbb{R}, \quad -180 \leq x \leq 180,$$

where A is a constant,

(ii) show that f(x) can be expressed in the form

$$f(x) = A + R\cos(x + \alpha)°,$$

where $R > 0$ and $0 < \alpha < 90$, [3]

(iii) state the value of A, [1]

(iv) find, to 1 decimal place, the x-coordinates of the points where the curve $y = f(x)$ crosses the x-axis. [4]

Turn over

8. The function f is defined by

$$f(x) \equiv 3 - x^2, \quad x \in \mathbb{R}, \quad x \geq 0.$$

(i) State the range of f. [1]

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. [3]

(iii) Find an expression for $f^{-1}(x)$ and state its domain. [3]

The function g is defined by

$$g(x) \equiv \frac{8}{3-x}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

(iv) Evaluate fg(–3). [2]

(v) Solve the equation

$$f^{-1}(x) = g(x).$$ [3]

9. A curve has the equation $y = (2x + 3)\,e^{-x}$.

(i) Find the exact coordinates of the stationary point of the curve. [4]

The curve crosses the y-axis at the point P.

(ii) Find an equation for the normal to the curve at P. [2]

The normal to the curve at P meets the curve again at Q.

(iii) Show that the x-coordinate of Q lies between –2 and –1. [3]

(iv) Use the iterative formula

$$x_{n+1} = \frac{3 - 3e^{x_n}}{e^{x_n} - 2},$$

with $x_0 = -1$, to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 2 decimal places. [2]

(v) Show that your value for x_4 is the x-coordinate of Q correct to 2 decimal places. [2]