FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper J

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. Use Simpson's rule with four strips to estimate the value of the integral

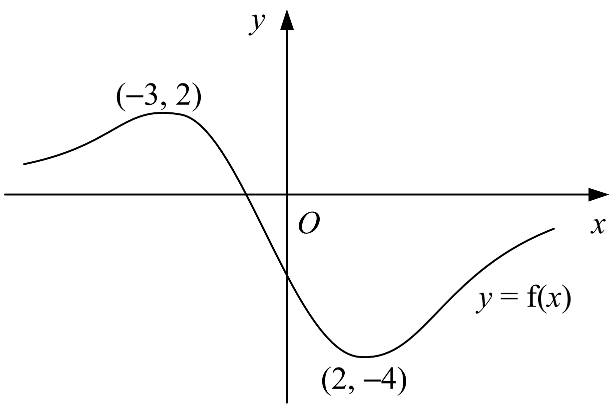
$$\int_0^3 e^{\cos x}\ dx. \qquad [4]$$

2. Giving your answers to 1 decimal place, solve the equation

$$5\tan^2 2\theta - 13\sec 2\theta = 1,$$

for θ in the interval $0 \leq \theta \leq 360°$. [7]

3.



The diagram shows the curve $y = f(x)$ which has a maximum point at $(-3, 2)$ and a minimum point at $(2, -4)$.

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

(i) $y = |f(x)|$, [2]

(ii) $y = 3f(2x)$. [3]

(b) Write down the values of the constants a and b such that the curve with equation $y = a + f(x + b)$ has a minimum point at the origin O. [2]

4. Find the values of x in the interval $-180 < x < 180$ for which

$$\tan(x + 45)° - \tan x° = 4,$$

giving your answers to 1 decimal place. [7]

5. The finite region R is bounded by the curve with equation $y = \sqrt[3]{3x-1}$, the x-axis and the lines $x = \frac{2}{3}$ and $x = 3$.

(i) Find the area of R. [4]

(ii) Find, in terms of π, the volume of the solid formed when R is rotated through four right angles about the x-axis. [4]

6. The functions f and g are defined by

$$\mathrm{f} : x \to 1 - ax, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \to x^2 + 2ax + 2, \quad x \in \mathbb{R},$$

where a is a constant.

Find, in terms of a,

(i) an expression for $\mathrm{f}^{-1}(x)$, [2]

(ii) the range of g. [3]

Given that $\mathrm{gf}(3) = 7$,

(iii) find the two possible values of a. [4]

7. The curve with equation $y = x^{\frac{5}{2}} \ln \frac{x}{4}$, $x > 0$ crosses the x-axis at the point P.

(i) Write down the coordinates of P. [1]

The normal to the curve at P crosses the y-axis at the point Q.

(ii) Find the area of triangle OPQ where O is the origin. [6]

The curve has a stationary point at R.

(iii) Find the x-coordinate of R in exact form. [2]

Turn over

8. *(i)* Solve the equation

$$\pi - 3\cos^{-1}\theta = 0. \qquad [2]$$

(ii) Sketch on the same diagram the curves $y = \cos^{-1}(x - 1)$, $0 \le x \le 2$ and $y = \sqrt{x+2}$, $x \ge -2$. [3]

Given that α is the root of the equation

$$\cos^{-1}(x - 1) = \sqrt{x+2},$$

(iii) show that $0 < \alpha < 1$, [2]

(iv) use the iterative formula

$$x_{n+1} = 1 + \cos\sqrt{x_n + 2}$$

with $x_0 = 1$ to find α correct to 3 decimal places.

You should show the result of each iteration. [3]

9. The number of bacteria present in a culture at time t hours is modelled by the continuous variable N and the relationship

$$N = 2000e^{kt},$$

where k is a constant.

Given that when $t = 3$, $N = 18\,000$, find

(i) the value of k to 3 significant figures, [3]

(ii) how long it takes for the number of bacteria present to double, giving your answer to the nearest minute, [4]

(iii) the rate at which the number of bacteria is increasing when $t = 3$. [4]