FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper I

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper I – Marking Guide

1. $\frac{dV}{dt} = 80$ — B1

$V = \frac{4}{3}\pi r^3 \therefore \frac{dV}{dr} = 4\pi r^2, \quad r = 6 \therefore \frac{dV}{dr} = 144\pi$ — M1 A1

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \quad \therefore 80 = 144\pi \times \frac{dr}{dt}$ — M1

$\frac{dr}{dt} = \frac{80}{144\pi} = \frac{5}{9\pi} = 0.177$ (3sf)

radius is increasing at rate of 0.177 cm per second — A1 **(5)**

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2. $\frac{3}{\sin\theta} = -8\cos\theta$ — M1

$3 = -8\sin\theta\cos\theta = -4\sin 2\theta$ — M1

$\sin 2\theta = -\frac{3}{4}$ — A1

$2\theta = 180 + 48.590,\ 360 - 48.590 = 228.590,\ 311.410$ — M1

$\theta = 114.3,\ 155.7$ (1dp) — A2 **(6)**

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3. *(a)* *(i)* $\ln\frac{x^2}{e} = \ln x^2 - \ln e = 2\ln x - 1 = 2y - 1$ — M1 A1

(ii) let $t = \log_2 x \Rightarrow x = 2^t$ — M1

$\ln x = t\ln 2$ — M1

$t = \frac{\ln x}{\ln 2} \quad \therefore \log_2 x = \frac{y}{\ln 2}$ — A1

(b) $\frac{y}{\ln 2} = 4 - (2y - 1), \quad y = (5 - 2y)\ln 2$

$y(2\ln 2 + 1) = 5\ln 2$ — M1

$y = \frac{5\ln 2}{2\ln 2 + 1}$ — M1

$x = e^y = 4.27$ (2dp) — A1 **(8)**

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4. *(i)* when $x = 1,\ (x-1)^2 = 0$ and $2 - \frac{2}{x} = 0 \therefore$ intersect — B1

when $x = 2,\ (x-1)^2 = 1$ and $2 - \frac{2}{x} = 1 \therefore$ intersect — B1

(ii) $= \pi\int_1^2 (2 - \frac{2}{x})^2\,dx - \pi\int_1^2 (x-1)^4\,dx$ — M1

$= \pi\int_1^2 (4 - 8x^{-1} + 4x^{-2})\,dx - \pi\int_1^2 (x-1)^4\,dx$ — M1

$= \pi[4x - 8\ln|x| - 4x^{-1}]_1^2 - \pi[\frac{1}{5}(x-1)^5]_1^2$ — M1 A2

$= \pi[(8 - 8\ln 2 - 2) - (4 - 0 - 4)] - \pi[\frac{1}{5} - 0]$ — M1

$= \pi(5\frac{4}{5} - 8\ln 2)$ — A1 **(9)**

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5. *(i)* $f(x) > 5$ — B1

(ii) $y = 5 + e^{2x-3}$

$2x - 3 = \ln(y - 5)$ — M1

$x = \frac{1}{2}[3 + \ln(y - 5)]$

$\therefore f^{-1}(x) = \frac{1}{2}[3 + \ln(x - 5)],\ x \in \mathbb{R},\ x > 5$ — A2

(iii) $x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$ — M1 A1

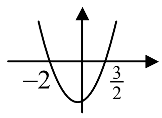
(iv) $f'(x) = 2e^{2x-3}$ — M1

grad = 4 — A1

$\therefore y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)] \quad [\,y = 4x + 1 - 2\ln 2\,]$ — M1 A1 **(10)**

6.	*(i)*	$\sqrt{3}\sin\theta + \cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
		$R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$	M1	
		$\therefore\ R = \sqrt{3+1} = 2$	A1	
		$\tan\alpha = \frac{1}{\sqrt{3}}$, $\alpha = \frac{\pi}{6}$	A1	
		$\therefore\ \sqrt{3}\sin\theta + \cos\theta = 2\sin(\theta + \frac{\pi}{6})$		
	(ii)	maximum = 2	B1	
		occurs when $\theta + \frac{\pi}{6} = \frac{\pi}{2}$, $\theta = \frac{\pi}{3}$	M1 A1	
	(iii)	$2\sin(\theta + \frac{\pi}{6}) + \sqrt{3} = 0$		
		$\sin(\theta + \frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$	M1	
		$\theta + \frac{\pi}{6} = -\frac{\pi}{3}, -\pi + \frac{\pi}{3} = -\frac{\pi}{3}, -\frac{2\pi}{3}$	M1	
		$\theta = -\frac{5\pi}{6}, -\frac{\pi}{2}$	A2	**(10)**

7.	*(i)*	$f'(x) = \frac{2x\times(4x+1)-(x^2+3)\times 4}{(4x+1)^2}$	M1 A1	
		$= \frac{4x^2+2x-12}{(4x+1)^2}$	A1	
	(ii)	$\frac{4x^2+2x-12}{(4x+1)^2} \geq 0$		
		for $x \neq -\frac{1}{4}$, $(4x+1)^2 > 0$ $\therefore$ $4x^2 + 2x - 12 \geq 0$	M1 A1	
		$2(2x-3)(x+2) \geq 0$	M1	
		$x \leq -2$ or $x \geq \frac{3}{2}$	A1	



	(iii)	x: 0, 1, 2, 3, 4, 5, 6		
		$f(x)$: 3, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{12}{13}$, $\frac{19}{17}$, $\frac{28}{21}$, $\frac{39}{25}$	M1	
		$I \approx \frac{1}{3}\times 1\times[3 + \frac{39}{25} + 4(\frac{4}{5} + \frac{12}{13} + \frac{28}{21}) + 2(\frac{7}{9} + \frac{19}{17})]$	M1	
		= 6.86 (3sf)	A1	**(10)**

8.	*(i)*	$f(x) \geq 0$	B1	
	(ii)	$= f(0) = 5$	M1 A1	
	(iii)	$fg(x) = f[\ln(x+3)] = \lvert 2\ln(x+3) - 5\rvert$	M1	
		$\therefore\ \lvert 2\ln(x+3) - 5\rvert = 3$		
		$2\ln(x+3) = 2, 8$	M1	
		$\ln(x+3) = 1, 4$	A1	
		$x = e - 3, e^4 - 3$	M1 A1	
	(iv)	let $h(x) = f(x) - g(x)$		
		$h(3) = -0.79$, $f(4) = 1.1$	M1	
		sign change, $h(x)$ continuous $\therefore$ root	A1	
	(v)	$x_1 = 3.396$, $x_2 = 3.428$, $x_3 = 3.430$, $x_4 = 3.431$	M1 A1	
	(vi)	$h(3.4305) = -0.000052$, $f(3.4315) = 0.0018$	M1	
		sign change, $h(x)$ continuous $\therefore$ root $\therefore$ $\alpha = x_4$ to 4sf	A1	**(14)**

Total **(72)**

Performance Record – C3 Paper I

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	connected rates	trigonometry	exponentials and logarithms	integration	functions	trigonometry	differentiation, Simpson’s rule	functions, numerical methods	
Marks	5	6	8	9	10	10	10	14	72
Student									