FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper D – Marking Guide

1. *(i)* LHS = $\sin x \cos 30 + \cos x \sin 30 + \sin x \cos 30 - \cos x \sin 30$ — M1 A1
$= 2 \sin x \cos 30$
$= \sqrt{3} \sin x \qquad [\, a = \sqrt{3} \,]$ — A1

(ii) let $x = 45$, $\quad \sin 75 + \sin 15 = \sqrt{3} \sin 45$ — M1
$= \sqrt{3} \times \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$ — M1 A1 **(6)**

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2. *(i)* $2x - 3 = e$ — M1
$x = \frac{1}{2}(e + 3)$ — A1

(ii) $3e^{2y} - 16e^{y} + 5 = 0$ — M1
$(3e^{y} - 1)(e^{y} - 5) = 0$ — M1
$e^{y} = \frac{1}{3}, 5$ — A1
$y = \ln \frac{1}{3}, \ln 5$ — M1 A1 **(7)**

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3. *(i)* $\frac{dy}{dx} = 2e^{x} - \frac{6}{x}$ — M1
$x = 1, \; y = 2e, \;$ grad $= 2e - 6$ — A1
$\therefore \; y - 2e = (2e - 6)(x - 1)$ — M1 A1
$[\, y = (2e - 6)x + 6 \,]$

(ii) $x = 0 \Rightarrow y = 6$
$y = 0 \Rightarrow (2e - 6)x + 6 = 0$
$x = \frac{-6}{2e - 6} = \frac{3}{3 - e}$ — M1 A1
area $= \frac{1}{2} \times 6 \times \frac{3}{3 - e} = \frac{9}{3 - e}$ — M1 A1 **(8)**

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4. *(i)* $= \int_1^2 \frac{1}{2x - 1} \, dx$
$= [\frac{1}{2} \ln |2x - 1|]_1^2$ — M1 A1
$= \frac{1}{2}(\ln 3 - 0) = \frac{1}{2} \ln 3$ — M1 A1

(ii) $= \pi \int_1^2 \frac{1}{(2x - 1)^2} \, dx$
$= \pi[-\frac{1}{2}(2x - 1)^{-1}]_1^2$ — M1 A1
$= \pi[-\frac{1}{6} - (-\frac{1}{2})] = \frac{1}{3}\pi$ — M1 A1 **(8)**

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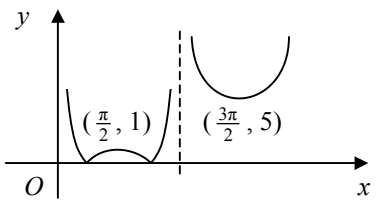
5. *(i)*



B2

(ii) $(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b$
$(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b$ — B1
adding, $\quad -6 = 2a \;\; \therefore \; a = -3, \; b = 2$ — M1 A1

(iii) $-3 + 2 \operatorname{cosec} x = 0$
$\operatorname{cosec} x = \frac{3}{2}, \qquad \sin x = \frac{2}{3}$ — M1
$x = 0.73, \pi - 0.7297$
$x = 0.73, 2.41$ (2dp) — A2 **(8)**

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6. *(i)* $\text{LHS} \equiv \frac{2\cos 2x}{\sin 2x} + \frac{\sin x}{\cos x}$ M1

$\equiv \frac{\cos 2x}{\sin x \cos x} + \frac{\sin x}{\cos x}$ M1

$\equiv \frac{\cos 2x + \sin^2 x}{\sin x \cos x}$ A1

$\equiv \frac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$ M1

$\equiv \frac{\cos^2 x}{\sin x \cos x} \equiv \frac{\cos x}{\sin x} \equiv \cot x \equiv \text{RHS}$ A1

(ii) $\cot x = \text{cosec}^2 x - 7, \quad \cot x = 1 + \cot^2 x - 7$ M1

$\cot^2 x - \cot x - 6 = 0, \quad (\cot x + 2)(\cot x - 3) = 0$ M1

$\cot x = -2$ or 3 A1

$\tan x = -\frac{1}{2}$ or $\frac{1}{3}$ M1

$x = \pi - 0.4636$ or 0.32

$x = 0.32, 2.68$ (2dp) A2 **(11)**

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7. *(i)* $f(x) > 0$ B1

(ii) $y = 3e^{x-1}$

$x - 1 = \ln \frac{y}{3}$ M1

$x = 1 + \ln \frac{y}{3}$

$f^{-1}(x) = 1 + \ln \frac{x}{3}, \ x \in \mathbb{R}, \ x > 0$ A2

(iii) $f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$ M1 A1

$gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$ A1

(iv) $f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln \frac{5x-2}{3}$ M1 A1

$\therefore \ 1 + \ln \frac{5x-2}{3} = 4, \quad \frac{5x-2}{3} = e^3$ M1

$x = \frac{1}{5}(3e^3 + 2)$ A1 **(11)**

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8. *(i)* $\frac{dy}{dx} = 2x - \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$ M1 A1

$x = 1, \ y = -1, \ \text{grad} = \frac{7}{4}$ A1

$\therefore \ y + 1 = \frac{7}{4}(x - 1)$ M1

$4y + 4 = 7x - 7$

$7x - 4y = 11$ A1

(ii) SP: $2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0$ M1

let $f(x) = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$

$f(0.3) = -0.40, \ f(0.4) = 0.088$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(iii) $2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0 \quad \Rightarrow \quad 2x = \frac{1}{2x\sqrt{4 + \ln x}}$

$x^2 = \frac{1}{4\sqrt{4 + \ln x}} = \frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}$ M1

$x = \sqrt{\frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4 + \ln x)^{-\frac{1}{4}}$ A1

(iv) $x_1 = 0.381512, \ x_2 = 0.378775, \ x_3 = 0.378999,$ M1 A1

$x_4 = 0.378981, \ x_5 = 0.378982, \quad \therefore \ \alpha = 0.37898$ (5dp) A1 **(13)**

Total **(72)**

Performance Record – C3 Paper D

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	trigonometry	exponentials and logarithms	differentiation	integration	functions, trigonometry	trigonometry	functions	differentiation, numerical methods	
Marks	6	7	8	8	8	11	11	13	72
Student									