FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper C – Marking Guide

1. $x(x-2)=0,\ x=0, 2\ \therefore$ crosses x-axis at (0, 0) and (2, 0)

volume $= \pi\int_0^2 (x^2-2x)^2\ dx$ M1

$= \pi\int_0^2 (x^4-4x^3+4x^2)\ dx$

$= \pi[\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3]_0^2$ M1 A1

$= \pi\{(\frac{32}{5} - 16 + \frac{32}{3}) - (0)\} = \frac{16}{15}\pi$ M1 A1 **(5)**

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2. *(i)* $3x+1=e^2$ M1

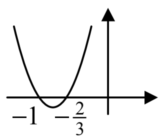
$x = \frac{1}{3}(e^2-1)$ A1

(ii) consider $\ln(3x^2+5x+3) \geq 0$

$\Rightarrow\ 3x^2+5x+3 \geq 1$ M1

$3x^2+5x+2 \geq 0$

$(3x+2)(x+1) \geq 0$ M1



$x \leq -1$ or $x \geq -\frac{2}{3}$ A1

$\therefore$ if (e.g.) $x = -\frac{3}{4}$, $\ln(3x^2+5x+3) = \ln\frac{15}{16} = -0.0645...$ M1

$\therefore$ if $x = -\frac{3}{4}$, $\ln(3x^2+5x+3) < 0$

$\therefore$ statement is false A1 **(7)**

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3. *(i)* $= \frac{1}{3x-2}\times 3 = \frac{3}{3x-2}$ M1 A1

(ii) $= \frac{2\times(1-x)-(2x+1)\times(-1)}{(1-x)^2} = \frac{3}{(1-x)^2}$ M1 A2

(iii) $= \frac{3}{2}x^{\frac{1}{2}}\times e^{2x} + x^{\frac{3}{2}}\times 2e^{2x} = \frac{1}{2}x^{\frac{1}{2}}e^{2x}(3+4x)$ M1 A2 **(8)**

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4. *(i)* $\cos^2 x = (\sqrt{3}-1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ M1

$\cos 2x = 2\cos^2 x - 1 = 2(4-2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$ M1 A1

(ii) $2(\cos y\cos 30 - \sin y\sin 30) = \sqrt{3}(\sin y\cos 30 - \cos y\sin 30)$ M1 A1

$\sqrt{3}\cos y - \sin y = \frac{3}{2}\sin y - \frac{1}{2}\sqrt{3}\cos y$ B1

$\frac{3}{2}\sqrt{3}\cos y = \frac{5}{2}\sin y$

$\tan y = \frac{3}{2}\sqrt{3} \div \frac{5}{2} = \frac{3}{5}\sqrt{3}$ M1 A1 **(8)**

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5. *(i)* $f(x) = (x-\frac{3}{2})^2 - \frac{9}{4} + 7 = (x-\frac{3}{2})^2 + \frac{19}{4}$ M1 A1

$\therefore\ f(x) \geq \frac{19}{4}$ A1

(ii) $= g(11) = 21$ M1 A1

(iii) $fg(x) = f(2x-1) = (2x-1)^2 - 3(2x-1) + 7$ M1

$\therefore\ 4x^2 - 4x + 1 - 6x + 3 + 7 = 17$

$2x^2 - 5x - 3 = 0$ A1

$(2x+1)(x-3) = 0$ M1

$x = -\frac{1}{2}, 3$ A1 **(9)**

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6. *(i)* $4\sin x + 3\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$R\cos\alpha = 4,\ R\sin\alpha = 3$ — M1

$\therefore\ R = \sqrt{4^2 + 3^2} = 5$ — A1

$\tan\alpha = \frac{3}{4},\ \alpha = 0.644$ (3sf) — A1

$\therefore\ 4\sin x + 3\cos x = 5\sin(x + 0.644)$

(ii) minimum $= -5$ — B1

occurs when $x + 0.6435 = \frac{3\pi}{2},\ x = 4.07$ (3sf) — M1 A1

(iii) $5\sin(2\theta + 0.6435) = 2$

$\sin(2\theta + 0.6435) = 0.4$ — M1

$2\theta + 0.6435 = \pi - 0.4115,\ 2\pi + 0.4115$

$2\theta = 2.087,\ 6.051$ — M1

$\theta = 1.04,\ 3.03$ (2dp) — A2 **(10)**

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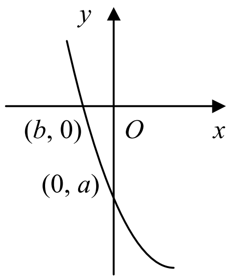
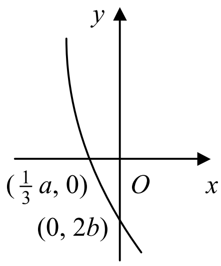
7. *(a)* *(i)*



(ii)



(i) M1 A1

(ii) M1 A2

(b) $x = 0 \Rightarrow y = -1\ \therefore b = -1$ — B1

$y = 0 \Rightarrow 2 - \sqrt{x+9} = 0$

$x = 2^2 - 9 = -5\ \therefore a = -5$ — M1 A1

(c) $y = 2 - \sqrt{x+9},\quad \sqrt{x+9} = 2 - y,\quad x + 9 = (2-y)^2$

$x = (2-y)^2 - 9$ — M1

$\therefore\ f^{-1}(x) = (2-x)^2 - 9$ — A1

$f(-9) = 2\ \therefore$ domain of $f^{-1}(x)$ is $x \in \mathbb{R},\ x \le 2$ — M1 A1 **(12)**

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8. *(i)* $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$ — M1 A1

grad $= -3$, grad of normal $= \frac{1}{3}$ — M1

$\therefore\ y - \frac{3}{2} = \frac{1}{3}(x - \frac{1}{4})$ $\quad [\ 4x - 12y + 17 = 0\ ]$ — A1

(ii) SP: $\frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x} = 0$

$\frac{1}{2\sqrt{x}} = 4e^{1-4x}$

$\frac{1}{8\sqrt{x}} = e^{1-4x}$ — M1

$8\sqrt{x} = e^{4x-1}$

$4x - 1 = \ln 8\sqrt{x}$ — M1

$x = \frac{1}{4}(1 + \ln 8\sqrt{x})$ — A1

(iii) $x_1 = 0.7699,\ x_2 = 0.7372,\ x_3 = 0.7317,\ x_4 = 0.7308 = 0.731$ (3dp) — M1 A1

(iv) let $f(x) = \frac{1}{2}x^{-\frac{1}{2}} - 4e^{1-4x}$

$f(0.7305) = -0.00025,\ f(0.7315) = 0.0017$ — M1

sign change, $f(x)$ continuous $\therefore$ root — A1

(v) $x_1 = 6.304,\ x_2 = 1.683 \times 10^{19}$

diverges rapidly away from root — B2 **(13)**

Total **(72)**

Performance Record – C3 Paper C

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	integration	exponentials and logarithms	differentiation	trigonometry	functions	trigonometry	functions	differentiation, numerical methods	
Marks	5	7	8	8	9	10	12	13	72
Student									