FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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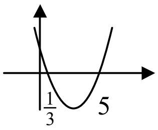
C3 Paper B – Marking Guide

1. $(2x-3)^2 > (x+2)^2$ M1

$3x^2 - 16x + 5 > 0$ A1

$(3x-1)(x-5) > 0$ M1

$x < \frac{1}{3}$ or $x > 5$ A2 **(5)**



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2. $3(\text{cosec}^2 x - 1) - 4 \text{ cosec } x + \text{cosec}^2 x = 0$ M1

$4 \text{ cosec}^2 x - 4 \text{ cosec } x - 3 = 0$

$(2 \text{ cosec } x + 1)(2 \text{ cosec } x - 3) = 0$ M1

$\text{cosec } x = -\frac{1}{2}$ or $\frac{3}{2}$ A1

$\sin x = -2$ (no solutions) or $\frac{2}{3}$ M1

$x = 0.73, \pi - 0.7297$

$x = 0.73, 2.41$ (2dp) A2 **(6)**

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3. *(i)* $\frac{dx}{dy} = 2y - \frac{3}{y} = \frac{2y^2-3}{y}$ M1 A1

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{y}{2y^2-3}$ A1

(ii) $y = \frac{1}{2},\ x = \frac{1}{4}$, grad $= -\frac{1}{5}$ B1

$\therefore\ y - \frac{1}{2} = -\frac{1}{5}(x - \frac{1}{4})$ M1

$20y - 10 = -4x + 1$

$4x + 20y - 11 = 0$ A1 **(6)**

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4. *(i)*

x	0	0.25	0.5	0.75	1	
xe^{2x}	0	0.4122	1.3591	3.3613	7.3891	M1

$I \approx \frac{1}{3} \times 0.25 \times [0 + 7.3891 + 4(0.4122 + 3.3613) + 2(1.3591)]$ M1

$= 2.10$ (3sf) A1

(ii) $= [-\frac{1}{2}e^{1-2x}]_{\frac{1}{2}}^{1}$ M1 A1

$= -\frac{1}{2}(e^{-1} - 1) = \frac{1}{2}(1 - e^{-1})$ M1 A1 **(7)**

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5. *(i)* $= \int_1^5 \frac{1}{\sqrt{3x+1}}\, dx = [\frac{2}{3}(3x+1)^{\frac{1}{2}}]_1^5$ M1 A1

$= \frac{2}{3}(4-2) = \frac{4}{3}$ M1 A1

(ii) $= \pi \int_1^5 \frac{1}{3x+1}\, dx$

$= \pi[\frac{1}{3}\ln|3x+1|]_1^5$ M1 A1

$= \frac{1}{3}\pi(\ln 16 - \ln 4) = \frac{1}{3}\pi \ln 4 = \frac{2}{3}\pi \ln 2$ $[\ k = \frac{2}{3}\ ]$ M1 A1 **(8)**

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6. (a)	let radius = r, $\therefore \tan 30° = \frac{1}{\sqrt{3}} = \frac{r}{h}$, $r = \frac{h}{\sqrt{3}}$	M1	
	$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h \times \frac{h^2}{3} = \frac{1}{9}\pi h^3$	A1	
(b) (i)	$\frac{dV}{dt} = 120$, $\frac{dV}{dh} = \frac{1}{3}\pi h^2$	B1	
	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$, $120 = \frac{1}{3}\pi h^2 \frac{dh}{dt}$, $\frac{dh}{dt} = \frac{360}{\pi h^2}$	M1 A1	
	when $h = 6$, $\frac{dh}{dt} = 3.18$ cm s^{-1} (2dp)	M1 A1	
(ii)	$V = 8 \times 120 = 960 = \frac{1}{9}\pi h^3$ $\therefore h = \sqrt[3]{\frac{9 \times 960}{\pi}} = 14.011$	M1	
	$\therefore \frac{dh}{dt} = 0.58$ cm s^{-1} (2dp)	A1	**(9)**
7. (i)	LHS $\equiv 2\sin x \cos x - \frac{\sin x}{\cos x}$	M1	
	$\equiv \frac{2\sin x\cos^2 x - \sin x}{\cos x}$	M1 A1	
	$\equiv \frac{\sin x(2\cos^2 x - 1)}{\cos x} \equiv \frac{\sin x}{\cos x} \times \cos 2x \equiv \tan x \cos 2x \equiv$ RHS	M1 A1	
(ii)	$\tan x \cos 2x = 2\cos 2x$		
	$\cos 2x(\tan x - 2) = 0$	M1	
	$\cos 2x = 0$ or $\tan x = 2$	A1	
	$2x = 90, 270$ or $x = 63.4$		
	$x = 45°, 63.4°$ (3sf), $135°$	A2	**(9)**
8. (i)	$t = 0$, $m = 480$	B1	
	$\therefore t = 10$, $m = 0.998 \times 480 = 479.04$	M1	
	$\therefore 479.04 = 400 + 80e^{-10k}$		
	$e^{-10k} = \frac{79.04}{80}$	A1	
	$k = -\frac{1}{10}\ln\frac{79.04}{80} = 0.00121$ (3sf)	M1 A1	
(ii)	$475 = 400 + 80e^{-kt}$, $e^{-kt} = \frac{75}{80}$	M1	
	$t = -\frac{1}{k}\ln\frac{75}{80} = 53.5$ (3sf)	A1	
(iii)	$\frac{dm}{dt} = -80ke^{-kt}$	M1 A1	
	$t = 100$, $\frac{dm}{dt} = -80ke^{-100k} = -0.0856$	M1	
	$\therefore$ decreasing at rate of 0.0856 g yr^{-1} (3sf)	A1	**(11)**
9. (i)	$f(x) < 3$	B1	
(ii)	$= f(2) = 3 - e^4$	M1 A1	
(iii)	$y = 3 - e^{2x}$, $e^{2x} = 3 - y$, $2x = \ln(3 - y)$, $x = \frac{1}{2}\ln(3 - y)$	M1	
	$\therefore f^{-1}(x) = \frac{1}{2}\ln(3 - x)$, $x \in \mathbb{R}$, $x < 3$	A2	
(iv)	e.g. $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$ so they intersect on the line $y = x$, hence $f^{-1}(x) = f(x) \Rightarrow f^{-1}(x) = x$	B2	
(v)	$x_1 = 0.4581$, $x_2 = 0.4664$, $x_3 = 0.4648$, $x_4 = 0.4651$, $x_5 = 0.4651$	M1 A1	
	$\therefore \alpha = 0.465$ (3sf)	A1	**(11)**
		Total	**(72)**

Performance Record – C3 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	functions	trigonometry	differentiation	Simpson's rule, integration	integration	connected rates	trigonometry	exponentials and logarithms, differentiation	functions, numerical methods	
Marks	5	6	6	7	8	9	9	11	11	72
Student										