

GCE Examinations  
Advanced / Advanced Subsidiary

## **Core Mathematics C3**

Paper K

### MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

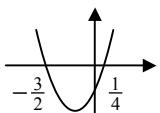


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### C3 Paper K – Marking Guide

1.	$I = \left[ \frac{1}{2} \ln  4x - 1  \right]_1^7$ $= \frac{1}{2} (\ln 27 - \ln 3)$ $= \frac{1}{2} \ln 9 = \ln 9^{\frac{1}{2}} = \ln 3$	M1 A1 M1 A1	(4)	
<hr/>				
2.	$(3x + 1)^2 \leq (x - 2)^2$ $8x^2 + 10x - 3 \leq 0$ $(4x - 1)(2x + 3) \leq 0$ $-\frac{3}{2} \leq x \leq \frac{1}{4}$		M1 A1 M1 A2	(5)
<hr/>				
3.	$\sec^2 \theta - 1 + \sec \theta = 1$ $\sec^2 \theta + \sec \theta - 2 = 0$ $(\sec \theta + 2)(\sec \theta - 1) = 0$ $\sec \theta = -2 \text{ or } 1$ $\cos \theta = -\frac{1}{2} \text{ or } 1$ $\theta = 180 - 60, -180 + 60 \text{ or } 0$ $\theta = -120, 0, 120$	M1 M1 A1 M1 A2	(6)	
<hr/>				
4.	<p>(i) <math>4x - 3 = \ln 2</math> <math>x = \frac{1}{4}(3 + \ln 2)</math></p> <p>(ii) <math>\ln(2y - 1) - \ln(3 - y) = \ln \frac{2y - 1}{3 - y} = 1</math> <math>\frac{2y - 1}{3 - y} = e</math> <math>2y - 1 = e(3 - y), \quad y(e + 2) = 3e + 1</math> <math>y = \frac{3e + 1}{e + 2}</math></p>	M1 A1 M1 A1 M1 A1	(6)	
<hr/>				
5.	<p>(i) if <math>\theta = \frac{\pi}{2}</math>, <math>\sin \theta = 1</math>, <math>\operatorname{cosec} \theta = 1</math> <math>\therefore \operatorname{cosec} \theta - \sin \theta = 1 - 1 = 0</math> <math>\therefore</math> statement is false</p> <p>(ii) <math>1 - \sin^2 \theta = 2 \sin \theta</math> <math>\sin^2 \theta + 2 \sin \theta - 1 = 0</math> <math>\sin \theta = \frac{-2 \pm \sqrt{4 + 4}}{2} = -1 - \sqrt{2}</math> (no solutions) or <math>-1 + \sqrt{2}</math> <math>\theta = 0.4271, \pi - 0.4271</math> <math>\theta = 0.43, 2.71</math> (2dp)</p>	M1 A1 M1 M1 A1 A2	(7)	
<hr/>				
6.	<p>(i) <math>\frac{dy}{dx} = 2x - 5 + \frac{2}{x}</math> <math>x = 3, y = -6, \text{ grad} = \frac{5}{3}</math> grad of normal = <math>-\frac{3}{5}</math> <math>\therefore y + 6 = -\frac{3}{5}(x - 3)</math> <math>5y + 30 = -3x + 9</math> <math>3x + 5y + 21 = 0</math></p> <p>(ii) SP: <math>2x - 5 + \frac{2}{x} = 0, \quad 2x^2 - 5x + 2 = 0</math> <math>(2x - 1)(x - 2) = 0</math> <math>x = \frac{1}{2}, 2</math></p>	M1 A1 M1 M1 A1 M1 M1 A1	(8)	

7.	(i)		M2 A1
	(ii)	$2\sqrt{2} \cos x - 2\sqrt{2} \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$ $R \cos \alpha = 2\sqrt{2}, R \sin \alpha = 2\sqrt{2}$ $\therefore R = \sqrt{8+8} = 4$ $\tan \alpha = 1, \alpha = 45$ $\therefore f(x) = A + 4 \cos(x + 45)^\circ$	M1 A1 A1
	(iii)	3	B1
	(iv)	$3 + 4 \cos(x + 45) = 0$ $\cos(x + 45) = -\frac{3}{4}$ $x + 45 = 180 - 41.4, 180 + 41.4 = 138.6, 221.4$ $x = 93.6, 176.4 \text{ (1dp)}$	M1 M1 A2 <b>(11)</b>

8.	(i)	$f(x) \leq 3$	B1
	(ii)		B3
	(iii)	$y = 3 - x^2, \quad x^2 = 3 - y, \quad x = \pm\sqrt{3-y}$ $f^{-1}(x) = \sqrt{3-x}, \quad x \in \mathbb{R}, \quad x \leq 3$	M1 A2
	(iv)	$= f\left(\frac{4}{3}\right) = \frac{11}{9}$	M1 A1
	(v)	$\sqrt{3-x} = \frac{8}{3-x}, \quad 3-x = \frac{64}{(3-x)^2}$ $(3-x)^3 = 64, \quad 3-x = 4$ $x = -1$	M1 A1 <b>(12)</b>

9.	(i)	$\frac{dy}{dx} = 2 \times e^{-x} + (2x + 3) \times (-e^{-x}) = -(2x + 1)e^{-x}$ $\text{SP: } -(2x + 1)e^{-x} = 0$ $x = -\frac{1}{2} \therefore \left(-\frac{1}{2}, 2e^{\frac{1}{2}}\right)$	M1 A1 M1 A1
	(ii)	$x = 0, y = 3, \text{ grad} = -1, \text{ grad of normal} = 1$ $\therefore y = x + 3$	M1 A1
	(iii)	$x + 3 = (2x + 3)e^{-x}, \quad x + 3 - (2x + 3)e^{-x} = 0$ $\text{let } f(x) = x + 3 - (2x + 3)e^{-x}$ $f(-2) = 8.4, \quad f(-1) = -0.72$ $\text{sign change, } f(x) \text{ continuous } \therefore \text{root}$	M1 A1
	(iv)	$x_1 = -1.1619, x_2 = -1.2218, x_3 = -1.2408, x_4 = -1.2465 = -1.25 \text{ (2dp)}$	M1 A1
	(v)	$f(-1.255) = 0.026, \quad f(-1.245) = -0.016$ $\text{sign change, } f(x) \text{ continuous } \therefore \text{root}$	M1 A1 <b>(13)</b>

Total **(72)**

