FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper J

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper J – Marking Guide

1.

x	0	0.75	1.5	2.25	3	
$e^{\cos x}$	2.7183	2.0786	1.0733	0.5336	0.3716	M1 A1

$I \approx \frac{1}{3} \times 0.75 \times [2.7183 + 0.3716 + 4(2.0786 + 0.5336) + 2(1.0733)]$ M1

$= 3.92$ (3sf) A1 **(4)**

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2. $5(\sec^2 2\theta - 1) - 13 \sec 2\theta = 1$ M1

$5 \sec^2 2\theta - 13 \sec 2\theta - 6 = 0$

$(5 \sec 2\theta + 2)(\sec 2\theta - 3) = 0$ M1

$\sec 2\theta = -\frac{2}{5}$ or 3 A1

$\cos 2\theta = -\frac{5}{2}$ (no solutions) or $\frac{1}{3}$

$2\theta = 70.529, 360 - 70.529, 360 + 70.529, 720 - 70.529$ M1

$= 70.529, 289.471, 430.529, 649.471$

$\theta = 35.3°, 144.7°, 215.3°, 324.7°$ (1dp) A3 **(7)**

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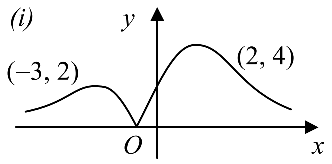
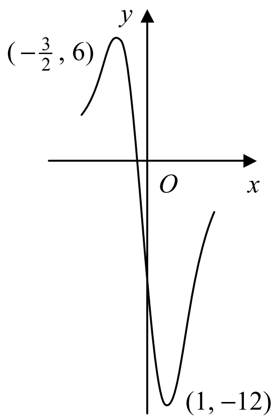
3. *(a)* *(i)*



(ii)



M1 A1

M2 A1

(b) $a = 4, b = 2$ B2 **(7)**

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4. $\frac{\tan x + \tan 45}{1 - \tan x \tan 45} - \tan x = 4$ M1

$\frac{\tan x + 1}{1 - \tan x} = 4 + \tan x$

$\tan x + 1 = (4 + \tan x)(1 - \tan x)$ M1

$\tan x + 1 = 4 - 3 \tan x - \tan^2 x$

$\tan^2 x + 4 \tan x - 3 = 0$ A1

$\tan x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$ M1

$x = 180 - 77.9, -77.9$ or $32.9, -180 + 32.9$

$x = -147.1, -77.9, 32.9, 102.1$ (1dp) A3 **(7)**

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5. *(i)* $= \int_{\frac{2}{3}}^{3} \sqrt[3]{3x-1}\ dx$

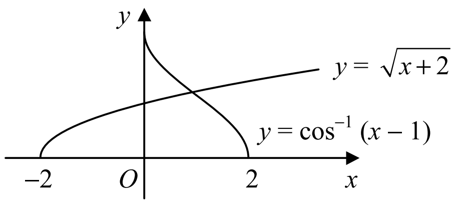
$= [\frac{1}{4}(3x-1)^{\frac{4}{3}}]_{\frac{2}{3}}^{3}$ M1 A1

$= \frac{1}{4}(16 - 1) = \frac{15}{4}$ M1 A1

(ii) $= \pi \int_{\frac{2}{3}}^{3} (3x-1)^{\frac{2}{3}}\ dx$ M1

$= \pi[\frac{1}{5}(3x-1)^{\frac{5}{3}}]_{\frac{2}{3}}^{3}$ A1

$= \frac{1}{5}\pi(32 - 1) = \frac{31}{5}\pi$ M1 A1 **(8)**

6.	*(i)*	$y = 1 - ax, \quad x = \frac{1-y}{a}$	M1	
		$\text{f}^{-1}(x) = \frac{1-x}{a}$	A1	
	(ii)	$\text{g}(x) = (x + a)^2 - a^2 + 2$	M1 A1	
		$\therefore \;\; \text{g}(x) \geq 2 - a^2$	A1	
	(iii)	$\text{gf}(3) = \text{g}(1 - 3a) = (1 - 3a)^2 + 2a(1 - 3a) + 2$	M1	
		$\therefore \;\; 1 - 6a + 9a^2 + 2a - 6a^2 + 2 = 7$		
		$3a^2 - 4a - 4 = 0$	A1	
		$(3a + 2)(a - 2) = 0$	M1	
		$a = -\frac{2}{3}, 2$	A1	**(9)**
7.	*(i)*	$(4, 0)$	B1	
	(ii)	$\frac{\text{d}y}{\text{d}x} = \frac{5}{2}x^{\frac{3}{2}} \times \ln \frac{x}{4} + x^{\frac{5}{2}} \times \frac{1}{x} = \frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2)$	M1 A1	
		grad $= 8$, grad of normal $= -\frac{1}{8}$	A1	
		$\therefore \;\; y - 0 = -\frac{1}{8}(x - 4)$	M1	
		at Q, $x = 0$, $y = \frac{1}{2}$	M1	
		area $= \frac{1}{2} \times \frac{1}{2} \times 4 = 1$	A1	
	(iii)	$\frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2) = 0$		
		$x > 0 \;\; \therefore \ln \frac{x}{4} = -\frac{2}{5}$	M1	
		$x = 4\text{e}^{-\frac{2}{5}}$	A1	**(9)**
8.	*(i)*	$\cos^{-1}\theta = \frac{\pi}{3}, \quad \theta = \cos \frac{\pi}{3} = \frac{1}{2}$	M1 A1	
	(ii)		B3	
	(iii)	let $\text{f}(x) = \cos^{-1}(x - 1) - \sqrt{x+2}$		
		$\text{f}(0) = 1.7, \;\; \text{f}(1) = -0.16$	M1	
		sign change, f(x) continuous $\therefore$ root	A1	
	(iv)	$x_1 = 0.83944, x_2 = 0.88598, x_3 = 0.87233,$		
		$x_4 = 0.87632, x_5 = 0.87515, x_6 = 0.87549$	M1 A1	
		$\therefore \;\; \alpha = 0.875$ (3dp)	A1	**(10)**
9.	*(i)*	$t = 3, \; N = 18\,000 \;\; \Rightarrow \;\; 18\,000 = 2000\text{e}^{3k}$		
		$\text{e}^{3k} = 9$	M1	
		$k = \frac{1}{3} \ln 9 = 0.732$ (3sf)	M1 A1	
	(ii)	$4000 = 2000\text{e}^{0.7324t}$	B1	
		$t = \frac{1}{0.7324} \ln 2 = 0.9464$ hours	M2	
		$\therefore$ doubles in 57 minutes (nearest minute)	A1	
	(iii)	$N = 2000\text{e}^{0.7324t}, \quad \frac{\text{d}N}{\text{d}t} = 0.7324 \times 2000\text{e}^{0.7324t} = 1465\text{e}^{0.7324t}$	M1 A1	
		when $t = 3$, $\frac{\text{d}N}{\text{d}t} = 13\,200$ $\therefore$ increasing at rate of 13 200 per hour (3sf)	M1 A1	**(11)**

Total **(72)**

Performance Record – C3 Paper J

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	Simpson's rule	trigonometry	functions	trigonometry	integration	functions	differentiation	trigonometry, numerical methods	exponentials and logarithms	
Marks	4	7	7	7	8	9	9	10	11	72
Student										