OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			Attempt use of quotient rule or, after adjustment, product rule	*M1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x-8$ in denominator; allow M1 for numerator the wrong way round	For product rule attempt, *M1 for $k_1(3x-8)^{-1}+k_2(5x+4)(3x-8)^{-2}$ form and A1 for correct constants 5 and –3;
			Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv	A1	Allow if missing brackets implied by subsequent simplification or calculation	
			Substitute 2 to obtain -13 or equiv	A1		
			Attempt to find equation of tangent	M1	Dep *M; equation of tangent not normal	
			Obtain $y=-13x+19$ or $13x+y-19=0$	A1	Or similarly simplified equiv with 3 non-zero terms	
				[5]		
2	**(i)**		State or imply $\tan\theta=\frac{1}{4}$	B1		Note that both parts are to be answered without calculator so sufficient detail is needed
			State or imply use of $\frac{\tan\theta+1}{1-\tan\theta}$	B1		
			Obtain $\frac{5}{3}$ or $1\frac{2}{3}$ or $\frac{20}{12}$ or exact equiv	B1	But not unsimplified equiv (such as $\frac{5}{4}/\frac{3}{4}$)	
				[3]		
	(ii)		Attempt use of correct relevant identity or of right-angled triangle	M1	Such as $\text{cosec}^2\theta=1+\cot^2\theta$, or $\text{cosec}\,\theta=\frac{1}{\sin\theta}$ with attempt at $\sin\theta$, or use of Pythagoras' theorem in right-angled triangle	
			Obtain $\sqrt{17}$	A1	Final answer $\pm\sqrt{17}$ earns A0	
				[2]		

Question			Answer	Marks	Guidance	
3			Differentiate to obtain $kh^n(2+\sqrt{h})^5$	M1	Any non-zero constants k, n; condone presence of -192 here	
			Obtain $9h^{-\frac{1}{2}}(2+\sqrt{h})^5$ or unsimplified equiv	A1	Without -192 now	
			Divide 150 by their derivative, algebraic or numerical	*M1	Using any recognisable attempt at first derivative	
			Substitute $h=1.4$ and evaluate	M1	Dep *M; assume appropriate substitution if calculation goes wrong	
			Obtain 0.06 or 0.060 or 0.0603	A1	But not greater accuracy in final answer; units not needed unless change made to metres and/or hours	
				[5]		
4			Obtain $2a$ as one value of x	B1		Allow solution leading to $a=\frac{1}{2}x$ (B1) and $a=-\frac{1}{8}x$ (M1A1)
			Attempt to find second value of x	M1	By solving equation with signs of x and $5a$ different, or by squaring both sides and attempting solution of quadratic equation with three terms	If using quadratic formula to solve equation, substitution must be accurate
			Obtain $-8a$	A1	And no other values of x	
			Substitute each of at most two values of x (involving a) leading to one final answer in each case and showing correct application of modulus signs in at least one case	M1		
			Obtain $4a$ as final answer	A1	Obtained correctly from $x=2a$	
			Obtain $-14a$ as final answer	A1	Obtained correctly from $x=-8a$	
				[6]		

Question			Answer	Marks	Guidance	
5	(i)		State first derivative is $3e^{3x} - 12e^{2x}$	B1	Or equiv	
			Equate first derivative to zero and attempt solution of equation of form $k_1e^{3x} - k_2e^{2x} = 0$	M1	At least as far as $e^x = c$; M0 for false method such as $\ln(3e^{3x}) - \ln(12e^{2x}) = 0$	
			Obtain $\ln 4$ or exact equiv and no other	A1	Obtained by legitimate method	
			Substitute $x = \ln 4$ or $e^x = 4$ to confirm $y = 0$	A1	AG; using exact working with all detail present: needs sight of $4^3 - 6 \times 4^2 + 32$ or similar equiv	
				[4]		
	(ii)		Integrate to obtain $k_3e^{3x} + k_4e^{2x} + 32x$	M1	For non-zero constants	
			Obtain $\frac{1}{3}e^{3x} - 3e^{2x} + 32x$ or equiv	A1		
			Apply limits correctly to expression of form $k_3e^{3x} + k_4e^{2x} + 32x$	M1	Using limits 0 and their answer from part (i)	
			Simplify to obtain $32\ln 4 - 24$ or $64\ln 2 - 24$	A1	Or suitably simplified equiv	
				[4]		
6	(i)		State or clearly imply $a = \frac{1}{2}$	B1	$a = \frac{5}{2}$ and $b = \frac{1}{2}$ earn B0 B0	
			State or clearly imply $b = \frac{5}{2}$	B1	$\sin(-\frac{1}{2}\pi) + \frac{3}{2}$ and $\sin(\frac{1}{2}\pi) + \frac{3}{2}$ earn B0 B0	
			(Implied by, for example, just $\frac{1}{2}$ and $\frac{5}{2}$ stated in that order)			
				[2]		

Question			Answer	Marks	Guidance	
	(ii)	(a)	Carry out relevant calculations using radians	M1	Involving $8\sin^{-1}(x-\frac{3}{2})$ or $8\sin^{-1}(x-\frac{3}{2})-x$ or equiv; needs two explicit calculations	May carry out calculations in, for example, $\frac{3}{2}+\sin(\frac{1}{8}x)-x$
			Obtain 1.6 and 2.4 or –0.1 and 0.6	A1	Or equivs	
			Conclude with reference to $1.6<1.7$ but $2.4>1.8$, or to sign change	A1 **[3]**	Or equiv	
		(b)	State or imply $p=\frac{3}{2}$ and $q=\frac{1}{8}$	B1	Implied by presence in iterative formula	
			Obtain correct first iterate	B1	Having started with value x_1 such that $1.7\le x_1\le 1.8$; given to at least 4 s.f.	
			Carry out iteration process	M1	Obtaining at least three iterates in all; having started with any non-negative value; implied by an apparently converging sequence of plausible values; all values to at least 4 s.f.	Answer only can earn no more than the first B1 for values of *p* and *q*; working in degrees can earn no more than the first B1 (for *p* and *q*) and M1
			Obtain at least three correct iterates	A1	Allowing recovery after error	
			Conclude with clear statement that root is 1.712	A1 **[5]**	Final answer required to exactly 4 significant figures	
7	(i)		Integrate to obtain integral of form $k(7x+1)^{\frac{4}{3}}$	*M1	Any non-zero constant *k*	
			Obtain $\frac{3}{28}(7x+1)^{\frac{4}{3}}$	A1	Or unsimplified equiv	
			Apply limits correctly and attempt exact evaluation	M1	Dep *M; substitution of limits to be seen	
			Obtain $\frac{180}{7}$	A1 **[4]**	Or exact equiv such as $\frac{720}{28}$ or $25\frac{5}{7}$	
	(ii)		Attempt expression of form $k(y_0+4y_1+y_2)$	M1	Any constant *k*; attempting exact *y* values corresponding to *x* values 1, 5, 9	Missing brackets which are not implied by subsequent calculation and which lead to $ky_0+4y_1+y_2$ earn M0
			Obtain $\frac{4}{3}(\sqrt[3]{8}+4\times\sqrt[3]{36}+\sqrt[3]{64})$	A1		
			Obtain $8+\frac{16}{3}\sqrt[3]{36}$	A1 **[3]**	No need for *m* and *n* to be stated separately	

Question			Answer	Marks	Guidance	
	(iii)		Equate answers to parts (i) and (ii) and carry out complete correct relevant rearrangement	M1	Provided $\sqrt[3]{36}$ is involved	Correct answer only seen: M1A1 answer only seen: if follows correctly from their parts (i) and (ii): M1A0
			Obtain $\frac{93}{28}$ or $\frac{372}{112}$	A1 **[2]**	Or equiv of requested form	
8	**(i)**		Obtain 6 or $2+4$ at any stage for application of f	B1		
			Attempt composition of functions the right way round	M1		
			Obtain $a=\frac{1}{4}$ or $\frac{9}{36}$ or equiv	A1 **[3]**		
	(ii)		Obtain expression involving e^{y-2} or e^{x-2}	M1		
			Obtain $e^{x-2}-3$	A1		
			State $x \geq 2+\ln 3$ or equiv	B1 **[3]**	Not for >; not for decimal equiv ; using x	
	(iii)		Either: Apply f once to obtain $2+N$	B1		
			Apply f to their expression involving N	M1		
			Obtain $2+\ln(N+5)$ or $2+\ln(2+N+3)$	A1		
			Attempt solution of equation of form $2+\ln(pN+q)=\ln(53e^2)$	M1	Involving manipulation so that value of N is apparent	
			Obtain 48 from correct work	A1		

Question			Answer	Marks	Guidance	
			Or 1:			
			Obtain $\mathrm{ff}(x)$ of form $k_1 + \ln[k_2 + \ln(x+3)]$	M1	Or equiv with immediate substitution for x; missing bracket(s) may be implied by subsequent work	
			Obtain correct $2 + \ln[5 + \ln(x+3)]$	A1		
			Substitute for x to obtain $2 + \ln(N+5)$	A1		
			Attempt solution of equation of form $2 + \ln(pN+q) = \ln(53\mathrm{e}^2)$	M1	Involving manipulation so that value of N is apparent	
			Obtain 48 from correct work	A1		
			Or 2:			
			Apply f^{-1} to obtain $\mathrm{e}^{\ln(53\mathrm{e}^2)-2} - 3$	B1		
			Attempt simplification of expression involving ln and e	M1		
			Obtain $\mathrm{f}(\mathrm{e}^N - 3) = 50$	A1		
			Apply f, or apply f^{-1} to right-hand side	M1		
			Obtain 48	A1		
				[5]		
9	**(i)**		Use at least one addition formula accurately	M1	Without substituting values for $\cos 30°$, etc. yet	
			Obtain $\cos\theta$	A1	AG; necessary detail needed	
			State $\cos 4\theta = 2\cos^2 2\theta - 1$	B1	Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$	
			Attempt correct use of relevant formulae to express in terms of $\cos\theta$	M1	Or in terms of $\cos\theta$ and $\sin\theta$	
			Obtain correct unsimplified expression in terms of $\cos\theta$ only	A1	e.g. $2(2c^2-1)^2 - 1 + 4(2c^2-1)$	
			Simplify to confirm $8\cos^4\theta - 3$	A1	AG; necessary detail needed	
				[6]		

Question			Answer	Marks	Guidance	
	(ii)	(a)	Obtain $\frac{1}{12}$	B1		
			Substitute 0 for $\cos\theta$ in correct expression	M1	No need to specify greatest and least	
			Obtain $\frac{1}{4}$	A1		
				[3]		
		(b)	State or imply $8\cos^4(3\alpha)-3=1$	B1	Or $2\cos^2 6\alpha+4\cos 6\alpha-2=0$	
			Attempt correct method to obtain at least one value of α	M1	Allow for equation of form $\cos^4(3\alpha)=k$ where $0<k<1$ or for three-term quadratic equation in $\cos 6\alpha$	
			Obtain 10.9	A1	Or greater accuracy 10.921…	Answer(s) only: 0/4
			Obtain 49.1	A1	Or greater accuracy 49.078…; and no others between 0 and 60	
				[4]		

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