

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**  
**A2 GCE**  
**4727/01**

**MATHEMATICS**  
**Further Pure Mathematics 3**  
**QUESTION PAPER**

**TUESDAY 9 JUNE 2015: Morning**  
**DURATION: 1 hour 30 minutes**  
**plus your additional time allowance**

**MODIFIED ENLARGED**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4727/01**  
**List of Formulae (MF1)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

**Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**

**IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Answer ALL the questions.**

**Read each question carefully. Make sure you know what you have to do before starting your answer.**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

## **INFORMATION FOR CANDIDATES**

**This information is the same on the Printed Answer Book and the Question Paper.**

**The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**

**YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**

**The total number of marks for this paper is 72.**

**Any blank pages are indicated.**

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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**Answer ALL the questions.**

- 1 Find the general solution of the differential equation**

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x. \quad [8]$$

- 2 The elements of a group  $G$  are polynomials of the form  $a + bx + cx^2$ , where  $a, b, c \in \{0, 1, 2, 3, 4\}$ . The group operation is addition, where the coefficients are added modulo 5.**

**(i) State the identity element. [1]**

**(ii) State the inverse of  $3 + 2x + x^2$ . [2]**

**(iii) State the order of  $G$ . [1]**

**The proper subgroup  $H$  contains  $2 + x$  and  $1 + x$ .**

**(iv) Find the order of  $H$ , justifying your answer. [4]**

- 3 The plane  $\Pi$  passes through the points  $(1, 2, 1)$ ,  $(2, 3, 6)$  and  $(4, -1, 2)$ .**

**(i) Find a cartesian equation of the plane  $\Pi$ . [5]**

**The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$ .**

**(ii) Find the coordinates of the point of intersection of  $\Pi$  and  $l$ . [3]**

**(iii) Find the acute angle between  $\Pi$  and  $l$ . [3]**

- 4 In an Argand diagram, the complex numbers  $0$ ,  $z$  and  $ze^{\frac{1}{6}i\pi}$  are represented by the points  $O$ ,  $A$  and  $B$  respectively.

(i) Sketch a possible Argand diagram showing the triangle  $OAB$ . Show that the triangle is isosceles and state the size of angle  $AOB$ . [4]

The complex numbers  $1 + i$  and  $5 + 2i$  are represented by the points  $C$  and  $D$  respectively. The complex number  $w$  is represented by the point  $E$ , such that  $CD = CE$  and angle  $DCE = \frac{1}{6}\pi$ .

(ii) Calculate the possible values of  $w$ , giving your answers exactly in the form  $a + bi$ . [5]

- 5 Find the particular solution of the differential equation

$$x \frac{dy}{dx} + 3y = x^2 + x$$

for which  $y = 1$  when  $x = 1$ , giving  $y$  in terms of  $x$ . [8]

- 6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4} = \frac{y-1}{-2} = \frac{z+1}{3}. \quad [7]$$

- 7 (i) Use de Moivre's theorem to show that

$$\tan 4\theta \equiv \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}. \quad [4]$$

- (ii) Hence find the exact roots of

$$t^4 + 4\sqrt{3}t^3 - 6t^2 - 4\sqrt{3}t + 1 = 0. \quad [5]$$

- 8 Let  $G$  be any multiplicative group.  $H$  is a subset of  $G$ .  $H$  consists of all elements  $h$  such that  $hg = gh$  for every element  $g$  in  $G$ .

- (i) Prove that  $H$  is a subgroup of  $G$ . [8]

Now consider the case where  $G$  is given by the following table:

	$e$	$p$	$q$	$r$	$s$	$t$
$e$	$e$	$p$	$q$	$r$	$s$	$t$
$p$	$p$	$q$	$e$	$s$	$t$	$r$
$q$	$q$	$e$	$p$	$t$	$r$	$s$
$r$	$r$	$t$	$s$	$e$	$q$	$p$
$s$	$s$	$r$	$t$	$p$	$e$	$q$
$t$	$t$	$s$	$r$	$q$	$p$	$e$

- (ii) Show that  $H$  consists of just the identity element.

[4]

END OF QUESTION PAPER

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