Friday 16 May 2014 – Afternoon

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find the determinant of the matrix $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$. **[3]**

2 The complex number 7 + 3i is denoted by z. Find

(i) $|z|$ and arg z, **[2]**

(ii) $\frac{z}{4-\mathrm{i}}$, showing clearly how you obtain your answer. **[3]**

3 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix. Find

(i) $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$, **[2]**

(ii) $\mathbf{A}^{-1}$, **[2]**

(iii) $(\mathbf{AB}^{-1})^{-1}$. **[3]**

4 **(a)** Find the matrix that represents a shear with the y-axis invariant, the image of the point (1, 0) being the point (1, 4). **[2]**

(b) The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by **X**. **[2]**

(ii) Find the value of the determinant of **X** and describe briefly how this value relates to the transformation represented by **X**. **[2]**

5 The cubic equation $2x^3 + 3x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u + 2$ to find a cubic equation in u. **[3]**

(ii) Hence find the value of $\frac{1}{\alpha-2} + \frac{1}{\beta-2} + \frac{1}{\gamma-2}$. **[4]**

6 **(i)** Show that $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{4(r+1)}{r^2(r+2)^2}$. **[6]**

(iii) Find $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. **[2]**

7 The loci C_1 and C_2 are given by $\arg(z-2-2\mathrm{i}) = \frac{1}{4}\pi$ and $|z| = |z-10|$ respectively.

(i) Sketch on a single Argand diagram the loci C_1 and C_2. **[4]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$0 \leqslant \arg(z-2-2\mathrm{i}) \leqslant \tfrac{1}{4}\pi \text{ and } |z| \geqslant |z-10|.$$ **[3]**

8 **(i)** Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. **[4]**

(ii) Hence find $\sum_{r=n}^{2n} r(r^2-2)$, giving your answer in a fully factorised form. **[5]**

9 The roots of the equation $x^3 - kx^2 - 2 = 0$ are α, β and γ, where α is real and β and γ are complex.

(i) Show that $k = \alpha - \dfrac{2}{\alpha^2}$. **[2]**

(ii) Given that $\beta = u + \mathrm{i}v$, where u and v are real, find u in terms of α. **[4]**

(iii) Find v^2 in terms of α. **[4]**

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. **[2]**

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. **[1]**

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part **(ii)** is correct. **[5]**

END OF QUESTION PAPER