OCR
RECOGNISING ACHIEVEMENT

Monday 10 June 2013 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

4733360613

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The plane Π passes through the points with coordinates (1, 6, 2), (5, 2, 1) and (1, 0, –2).

(i) Find a vector equation of Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. **[2]**

(ii) Find a cartesian equation of Π. **[4]**

2 G consists of the set {1, 3, 5, 7} with the operation of multiplication modulo 8.

(i) Write down the operation table and, assuming associativity, show that G is a group. **[5]**

(ii) State the order of each element. **[1]**

(iii) Find all the proper subgroups of G. **[1]**

The group H consists of the set {1, 3, 7, 9} with the operation of multiplication modulo 10.

(iv) Explaining your reasoning, determine whether H is isomorphic to G. **[2]**

3 The differential equation

$$3xy^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y^3 = \frac{\cos x}{x}$$

is to be solved for $x > 0$. Use the substitution $u = y^3$ to find the general solution for y in terms of x. **[8]**

4 The complex numbers 0, 3 and $3\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ are represented in an Argand diagram by the points O, A and B respectively.

(i) Sketch the triangle OAB and show that it is equilateral. **[3]**

(ii) Hence express $3 - 3\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$ in polar form. **[2]**

(iii) Hence find $(3 - 3\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}})^5$, giving your answer in the form $a + b\sqrt{3}\,\mathrm{i}$ where a and b are rational numbers. **[3]**

5 Find the solution of the differential equation $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 5y = \mathrm{e}^{-x}$ for which $y = \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. **[11]**

6 The plane Π has equation $x + 2y - 2z = 5$. The line l has equation $\dfrac{x-1}{2} = \dfrac{y+1}{5} = \dfrac{z-2}{1}$.

(i) Find the coordinates of the point of intersection of l with the plane Π. **[3]**

(ii) Calculate the acute angle between l and Π. **[3]**

(iii) Find the coordinates of the two points on the line l such that the distance of each point from the plane Π is 2. **[5]**

7 A commutative group G has order 18. The elements a, b and c have orders 2, 3 and 9 respectively.

(i) Prove that ab has order 6. **[4]**

(ii) Show that G is cyclic. **[3]**

8 **(i)** Use de Moivre's theorem to show that $\cos 5\theta \equiv 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. **[5]**

(ii) Hence find the roots of $16x^4 - 20x^2 + 5 = 0$ in the form $\cos\alpha$ where $0 \leqslant \alpha \leqslant \pi$. **[4]**

(iii) Hence find the exact value of $\cos\frac{1}{10}\pi$. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.