

**Monday 10 June 2013 – Morning**

**A2 GCE MATHEMATICS**

**4727/01 Further Pure Mathematics 3**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 The plane  $\Pi$  passes through the points with coordinates  $(1, 6, 2)$ ,  $(5, 2, 1)$  and  $(1, 0, -2)$ .
- (i) Find a vector equation of  $\Pi$  in the form  $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ . [2]
- (ii) Find a cartesian equation of  $\Pi$ . [4]
- 2  $G$  consists of the set  $\{1, 3, 5, 7\}$  with the operation of multiplication modulo 8.
- (i) Write down the operation table and, assuming associativity, show that  $G$  is a group. [5]
- (ii) State the order of each element. [1]
- (iii) Find all the proper subgroups of  $G$ . [1]
- The group  $H$  consists of the set  $\{1, 3, 7, 9\}$  with the operation of multiplication modulo 10.
- (iv) Explaining your reasoning, determine whether  $H$  is isomorphic to  $G$ . [2]
- 3 The differential equation
- $$3xy^2 \frac{dy}{dx} + 2y^3 = \frac{\cos x}{x}$$
- is to be solved for  $x > 0$ . Use the substitution  $u = y^3$  to find the general solution for  $y$  in terms of  $x$ . [8]
- 4 The complex numbers  $0$ ,  $3$  and  $3e^{\frac{1}{3}\pi i}$  are represented in an Argand diagram by the points  $O$ ,  $A$  and  $B$  respectively.
- (i) Sketch the triangle  $OAB$  and show that it is equilateral. [3]
- (ii) Hence express  $3 - 3e^{\frac{1}{3}\pi i}$  in polar form. [2]
- (iii) Hence find  $(3 - 3e^{\frac{1}{3}\pi i})^5$ , giving your answer in the form  $a + b\sqrt{3}i$  where  $a$  and  $b$  are rational numbers. [3]
- 5 Find the solution of the differential equation  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}$  for which  $y = \frac{dy}{dx} = 0$  when  $x = 0$ . [11]
- 6 The plane  $\Pi$  has equation  $x + 2y - 2z = 5$ . The line  $l$  has equation  $\frac{x-1}{2} = \frac{y+1}{5} = \frac{z-2}{1}$ .
- (i) Find the coordinates of the point of intersection of  $l$  with the plane  $\Pi$ . [3]
- (ii) Calculate the acute angle between  $l$  and  $\Pi$ . [3]
- (iii) Find the coordinates of the two points on the line  $l$  such that the distance of each point from the plane  $\Pi$  is 2. [5]

- 7 A commutative group  $G$  has order 18. The elements  $a$ ,  $b$  and  $c$  have orders 2, 3 and 9 respectively.
- (i) Prove that  $ab$  has order 6. [4]
  - (ii) Show that  $G$  is cyclic. [3]
- 8
- (i) Use de Moivre's theorem to show that  $\cos 5\theta \equiv 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$ . [5]
  - (ii) Hence find the roots of  $16x^4 - 20x^2 + 5 = 0$  in the form  $\cos \alpha$  where  $0 \leq \alpha \leq \pi$ . [4]
  - (iii) Hence find the exact value of  $\cos \frac{1}{10}\pi$ . [3]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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