GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

OCR Report to Centres

January 2013

This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

Overview – Pure

Effective communication is an important aspect of taking an examination paper. Many candidates realise this, invariably setting out solutions clearly and often signalling the steps of longer solutions by adding a few words. Longer, unstructured questions do pose extra problems for candidates; rather than launching into solutions immediately, a moment's thought to plan the approach pays dividends. Candidates can proceed with greater clarity and, by announcing briefly what each step sets out to do, can also communicate their intentions. This helps the examiner chart a way through the solutions but must also be of immense help to the candidates.

One question in this series where poor communication was very evident occurred in unit 4724. As the report notes, many candidates provided solutions to Q4(i) which provided no help at all to anyone reading the script and trying to assess the procedure adopted. With all questions where the answer is given, examiners are expecting to be persuaded and convinced. Far too often, though, the attempt seems to consist of anything but a well-constructed argument; the candidate offers a few random calculations and hopes the examiner will sort it out.

Another type of request where improvement is possible in many cases concerns the occasional question where a brief comment or explanation is requested. Again, thought as to what is actually required would enable many candidates to provide more precise and telling comments. The words used should be the appropriate mathematical ones; it is surprising, for example, that in the early units the use of unacceptable terms such as 'flip' and 'move' for curve transformations still occur regularly. The development of accurate mathematical vocabulary is part of mathematics at this level.

4721 Core Mathematics 1

Most candidates were very well prepared for this paper. A large number of candidates scored very high marks, including some with full marks. Almost all candidates attempted the majority of the paper, with only questions 8 and 9(iii) not being attempted by a significant number of candidates. Nonetheless, there was also a significant minority of candidates, scoring fewer than 20 marks out of 72, for whom entry seemed inappropriate. Candidates appeared to have sufficient time to finish and for some candidates there was evidence of checked and corrected work. In cases where work is altered or restarted, centres should advise candidates to indicate which solution is the attempt they wish to be marked as this is not always clear and contradictory statements cannot earn credit.

Candidates now seem familiar and comfortable with the Printed Answer Book format. The use of additional sheets was very rare this series, as was the unnecessary use of graph paper for the sketch in Q3(i).

The best candidates presented clear and fluent solutions that showed clear understanding of the mathematics needed to answer the questions; they manipulated algebraic expressions efficiently and drew clear appropriate conclusions. A number of "informal" methods were used, particularly in question 6, that demonstrated clear geometrical insight. Indeed, candidates who relied too much on memorised (or incorrectly memorised) formulae were far more likely to make arithmetical errors, particularly with negative numbers. Indeed, the failure of a sizeable minority to deal effectively with negative numbers led in some cases to a significant loss of marks in several questions. The later questions in the paper discriminated particularly well with the more able scoring well in the last three questions. The majority of candidates were, however, able to demonstrate their acquired mathematical skills throughout the paper, performing well in differentiation and in solving quadratic equations where most candidates recognised the most appropriate solution method for the different types of equation.

Comments on individual questions

1) (i) Almost all candidates recognised from the wording of the question that factorisation was not appropriate and most opted to use the quadratic formula. Most were successful in the initial substitution but a significant number failed to deal accurately with the negative value of c. Many had difficulty simplifying the resulting surd. Some only divided the first term by 2; others erroneously divided $\sqrt{44}$ by 2 to get $\sqrt{22}$. Most of the candidates who attempted to complete the square were successful, although a number failed to find two roots. Overall, about 70% of candidates secured full marks.

(ii) This was generally more successful than part (i). Almost all candidates correctly differentiated the expression and most accurately substituted the given value of x to get –16; 85% of candidates gained both marks. The most common error was to use 5 instead of –5.

2) (i) Only a tiny number of candidates failed to secure the mark for this simple recall of index notation; $\frac{1}{3}$ and 1 were occasionally seen.

(ii) Most candidates knew how to deal with the negative index and rewrote the equation as $\frac{1}{t^3} = 64$ or equivalent. Thereafter, however, a significant number could not proceed further, with –4 being a common wrong answer.

(iii) Although a large majority of candidates realised the need to find a cube root, many applied this only to the p^6 term and not to the 8. Those that were successful often omitted the negative solution thus surrendering the final mark.

3) (i) Most candidates recognised that the cubic was negative and sketched an appropriate curve with correct y- and x- intercepts clearly labelled. Some candidates who multiplied out the full expression made an error in finding the y-intercept; others with an otherwise correct solution omitted this. Those who drew a positive cubic did not normally see that this contradicted the required value on one of their axes.

(ii) Although the majority of candidates recognised the transformation as being a reflection, their skills in describing this using correct mathematical language were inadequate. Many chose the wrong axis and even those who were correct used words like "flip", "mirror" and were reflecting "along" or "through" or "parallel to" the y-axis; these did not gain credit. Centres need to continue to develop candidates' use of vocabulary when describing transformations.

4) (i) Almost all candidates recognised the need to eliminate a variable and chose to eliminate y. There were errors in finding the quadratic, but most then went on to factorise correctly and find the values of both variables; forgetting to find y is now comparatively rare. A large number of candidates, however, found the substitution of $x = -\frac{1}{2}$ to find y difficult and many lost this mark.

(ii) One acceptable response was that one root implied that the line was a tangent to the curve. The question did not specify that a geometrical comment was required and so "meeting at one point" was another acceptable response. Candidates who made an error in part (i) were rewarded for a consistent conclusion relating to their roots. Use of the word "cross" is unhelpful; for example, in the case where there are no solutions saying "they do not cross" does not exclude the possibility that they touch. A number of candidates were using stock phrases irrespective of their answer to (i), such as "they are perpendicular" or "it just touches the x-axis" or stating the line was a tangent when they had found two different roots; these of course gained no credit.

5) (i) Most candidates secured two marks for this question by accurately multiplying out both pairs of brackets. Common errors included multiplying both expressions instead of subtracting them and errors in dealing with negative coefficients. Only about 60% secured all the marks for this relatively simple algebraic manipulation.

(ii) This question was generally approached very well, either by multiplying out all three brackets or more efficiently identifying only the x^2 terms. Again, errors with signs and errors in simplification, such as $2kx^2 + x^2 = 3kx^2$, were seen regularly. Several other candidates did not proceed at all after their expansions. Seemingly they were unaware of how to use the information regarding the coefficient given in the question; often they substituted –3 for x^2 . Around half of candidates presented fully correct solutions.

6) (i) There was a variety of approaches to this question, many of which worked well, with errors mostly being seen in the subtraction of negative numbers. The most successful method was to find the equation of the line through the given point and then substitute x for the other point. Also very successful was the informal method of counting up or down in 4s. The gradient method needed more care with the negative numbers and was by far the method most prone to error, both in substitution and subsequent calculation.

(ii) Again informal methods were often more successful than formal ones; use of the mid-point formula was more successful in finding m than q.

(iii) In this question candidates again used a variety of methods, with many spotting that the difference between d and –2 had to be $\sqrt{36}$; commonly the negative root was missed here leading to a score of 3 out of 4. Most candidates approached this more formally, using Pythagoras' theorem and again weak algebra and difficulties with negative values led to the loss of marks; some struggled to square $2\sqrt{13}$ accurately.

7) **(i)** Although a large number of candidates secured all three marks for this question, a lot of errors were seen in the initial stages. Most commonly, candidates forgot to square the 3 or divided both terms by *x* before simplifying. Candidates who obtained a single term gained a follow-through mark for correct differentiation, but those who differentiated "term by term" received no credit.

(ii) Most candidates realised that $\sqrt[3]{x}$ is the same as $x^{\frac{1}{3}}$ and the large majority went on to differentiate correctly, although the resulting negative power caused issues for some candidates.

(iii) Whereas most candidates were able to get the power of *x* correct, rewriting the question as $y = 2x^{-3}$ instead of $y = \frac{1}{2}x^{-3}$ was extremely common and as a result the modal mark for this question was 1 out of 2, achieved by almost half of candidates.

8) This unstructured question proved to be very demanding. Most candidates recognised the need to find the discriminant and the majority realised that this needed to be less than zero. Given that both terms involved algebraic manipulation, determining the discriminant proved challenging to a large number of candidates. Similarly, the solution of the resulting quadratic inequality proved challenging, with added difficulty seeming to result from the fact that both roots were negative; a significant number thought that $-\frac{1}{9}$ was less than –1, showing these roots in the wrong positions on the *x*-axis and getting the inequality the wrong way round when their intention was to choose the inside region. The best candidates handled all these obstacles well and produced short fluent solutions gaining all seven marks (as achieved by around one-third of candidates); some candidates were unable to start the question at all, instead trying to solve the equation using the quadratic formula.

9) **(i)** This standard piece of bookwork was generally done very well, with around three-quarters of candidates scoring all three marks. Only occasionally was the centre seen as (2, –10). The most common cause of errors was again dealing with negative numbers, particularly when squaring to find the radius, or not subtracting appropriately after completing the square.

(ii) This was managed well by most candidates, with substitution of the point into the original equation generally a more successful approach than using Pythagoras' theorem.

(iii) A large number of candidates secured full marks on this question and almost all managed to secure partial credit. Some candidates simplified $\frac{-3}{-6}$ to $-\frac{1}{2}$. The incorrect simplification of $\frac{3}{6}$ to $\frac{1}{3}$ was also common. The majority remembered to find the negative reciprocal of their gradient and then substituted this correctly to find an equation of a straight line. Some candidates still miss the detail of the question and do not give the correct answer in the required form, needlessly losing the final mark. Another not uncommon error was to attempt to differentiate implicitly when candidates clearly had either not yet met this technique or did not understand the process; successful solutions using this approach were extremely rare.

10) Many candidates realised what needed to be done in this unstructured question and a large proportion secured the first five marks by correctly differentiating the equation of the curve and equating this to 8, the gradient of the line. A relatively common error at this stage was to equate to the negative reciprocal of the gradient, showing confusion regarding parallel and perpendicular gradients. The resulting disguised quadratic proved far more difficult than usual as many candidates did not recognise this out of context, as it is more usually seen as a question in its own right. Of the candidates who did realise the need to make a substitution, many did not multiply by x^2 and incorrectly substituted *y* for x^{-2} and y^2 for x^4; they secured no more marks. Those who proceeded correctly usually factorised the simple resulting quadratic and remembered to take the square root to find x, although it was quite common to omit the –3. Thereafter, the vast majority of successful candidates found the corresponding value(s) of y correctly, although a number erroneously substituted into the line rather than the curve. This question proved appropriately discriminating with less than a quarter of candidates scoring full marks.

4722 Core Mathematics 2

General comments

Candidates seemed to find this paper accessible, and it gave them plenty of opportunity to display their skills. Most candidates were able to make an attempt at all of the questions, with only a few candidates omitting some questions. Candidates seemed to be well prepared, and familiar with all of the topic areas. There were a number of routine questions which allowed candidates to display their knowledge, but also some more challenging questions that required candidates to formulate effective strategies to solve the problem posed. In general, most candidates showed sufficient detail to make their method clear but this was not always the case. This was particularly apparent in Q4(ii) and Q7(iii), where examiners sometimes struggled to discern whether a valid method had actually been attempted. There were also some instances of candidates not being able to follow their own working, leading them to go wrong. A number of candidates lost marks through a lack of brackets when finding differences that involved several terms, or when evaluating a definite integral.

When one part of a question follows on from a previous part, it is important that candidates take care when transcribing these values. It was also noticeable that some candidates later went back and amended their solution to an early part of the question but then failed to follow these amendments through to other parts of the question that used these values. As in previous sessions, a number of candidates failed to show sufficient working when showing a given answer; examiners will expect all necessary steps to be shown explicitly if full marks are to be awarded.

It is also important that candidates give answers to the required degree of accuracy, including any exact answers, and also that they work to a sufficient degree of accuracy throughout the question. This should include efficient use of the calculator memory rather than using rounded or truncated values from previous parts of the question.

Comments on individual questions

1) (i) This proved to be a straightforward start to the paper, and most candidates used the sine rule accurately to gain full marks.

(ii) Most candidates were also successful in this part of the question, with either the sine rule or the cosine rule being used accurately. Some candidates lost the accuracy mark through using a rounded value from part (i), and others lost both marks through using the wrong angle.

2) (i) Most candidates gained both of the marks available. The most common approach was to use the formula for the *n*th term of an arithmetic progression and the majority did so successfully, showing enough detail to be convincing. Some candidates first derived $u_n = 4n + 3$ and then used this to show the given result, and a few resorted to writing out the sequence term by term.

(ii) Most candidates gained the first two marks with ease, using one of the two relevant formulae to sum the first thirty-five terms. The next two marks proved a little more problematical for many and they struggled to identify an appropriate strategy, with a mismatch between the values used for the number of terms and the first term. Those who attempted the sum of fifty terms from which they subtracted the sum of the first thirty-five terms were usually successful, and the working suggested that some were aided in their attempts by the given answer.
A number of candidates attempted to sum from the 36th to the 50th term, but many struggled to identify that they were summing fifteen terms, and errors were also made in

determining the first term. One of the more successful methods was to find values for the 36th and the 50th terms and then use $\frac{1}{2}n(a+l)$. Whilst most candidates gained at least two marks on this question, there were a number who seemed unfamiliar with sigma notation, or used incorrect values for *a* and/or *n* despite the hint given in part (i).

3) (i) Most candidates scored full marks on this question, with just a few using the *y*-coordinate rather than the gradient.

(ii) Most candidates also scored full marks on this part of the question, although some spoiled an otherwise correct solution by failing to write the final answer as an equation. Whilst the majority of candidates recognised the need to integrate and could attempt to do so, a surprising number then stopped at this point and made no attempt to evaluate *c*. There were a few candidates who, upon seeing the request to find an equation, immediately attempted to use $y = mx + c$ without first considering whether a linear function was involved. The majority of candidates appreciated the need to first expand the bracket, but it was disappointing that, at this level, some were unable to do so accurately.

4) (i) Virtually all candidates were able to carry out an efficient and accurate attempt at the binomial expansion with only the occasional slip resulting in the loss of a mark. A very small number of candidates omitted the binomial coefficients, or failed to write the terms as products. Some candidates attempted the C4 technique of first taking out a factor of 2^5, but this was often poorly executed and candidates would be well advised to select the method most appropriate to the question posed.

(ii) This proved to be one of the most challenging questions on the paper, and many candidates had little idea of how to formulate an appropriate strategy. A number of candidates felt unable to even make an attempt at the question, possibly because they had not seen anything similar on previous papers.

The most common method attempted was to replace *x* with $3y + y^2$ in the expansion found in part (i). This often resulted in partial success, but common errors were to use $3y^3$ rather than $(3y)^3$ and to not appreciate that $(3y + y^2)^2$ would also provide a relevant term. A surprisingly popular method involved attempting the expansion of $\{(1+y)(2+y)\}^5$ but a common error was to omit the power of 5 from one of the brackets. The third main method was to attempt an alternative expansion such as $\{(2+3y)+y^2\}^5$, and there were also some candidates who attempted to multiply out all five brackets but this was very rarely successful.

A number of solutions simply consisted of a jumble of numbers with no attempt to actually explain the working. If examiners cannot discern whether a valid method has been used then it is difficult to award credit.

5) (i) Most candidates could quote both of the required identities and then attempt to use them. Whilst $\sin^2 x = 1 - \cos^2 x$ was usually used correctly, the use of $\tan x$ caused more problems as candidates were expected to also deal with the fraction to gain the method mark and a number struggled to do so. Some candidates used poor notation, such as omitting the *x* from their trigonometric ratios, and others spoiled an otherwise correct solution by failing to give an equation as their final answer.

(ii) This part of the question was done very well by the majority of the candidates, who were able to identify the fact that the given equation was a quadratic in $\cos x$ and attempt an appropriate method to solve it. The four required roots then usually followed, though some candidates struggled to find the secondary angles with 318.2° being a common wrong answer. Others lost marks by discarding the negative root to the quadratic, failing to realise that this would also lead to valid solutions.

6) (i) Many candidates were successful in this part of the question, with the most popular approach being to first find $d = -3.5$ and then use a second equation to find *x*. This was usually successful, although sign errors proved a pitfall for some. However, a number of candidates made no further progress beyond finding *d*, often because they did not consider a third equation. The other common method was to find two expressions for *d* by considering the difference of consecutive terms which could then be equated and solved. This was an elegant and concise method, but a lack of brackets resulted in errors being made. Other, more creative, solutions were also seen including adding the sum of the three terms and equating this to an expression for S_3.

(ii)(a) Virtually all of the candidates gained the first mark for stating the three relevant terms, and most also gained the final two marks for finding the sum to infinity, though a few used $\frac{4}{3}$ as their ratio. It was the second mark that proved to be the most challenging.

Candidates had been asked to verify that the terms did form a geometric progression, and were expected to provide a convincing proof that considered the ratio between two pairs of terms, or an equivalent justification. Whilst some candidates did provide this explanation, far too many assumed that it was a geometric progression and simply found the ratio from a single pair of terms.

(ii)(b) This proved to be a challenging question for many candidates. Whilst most were able to make some attempt at it, it was often not enough to gain even the first mark. The most efficient solution was to equate two algebraic expressions for the ratio, and then rearrange them to get a quadratic which could then be solved. Some candidates were able to provide a concise and elegant solution in this way. Some candidates did embark on this method, but then attempted to first simplify their fractions which invariably went wrong. Others started with the generic equations for the *n*th term of a geometric progression so that when they eliminated *r* their equation involved the square or square root of a rational expression.

7) (i) The majority of candidates were able to find the angle correctly, providing enough detail of the method used to be convincing. Using the cosine rule was a more popular approach than using a right-angled triangle. A surprising number of candidates first gave the required angle in degrees and then showed their conversion from degrees to radians, rather than simply setting their calculator into the appropriate mode.

(ii) Nearly all of the candidates could attempt to find the length of an arc, but not enough thought was given to deciding which angle to use. Most candidates did gain full marks, either by first doubling the given angle to get 1.0822 radians and then using this in the appropriate formula, or by using the original angle of 0.5411 radians and then multiplying the associated arc length by four.

(iii) Most candidates were able to gain the first two marks on this question, one for attempting the area of a sector and one for attempting the area of a triangle, though this was not always relevant. Many candidates then made no further progress, usually because they had used the angle of 0.5411 incorrectly in $\frac{1}{2}r^2(q - \sin q)$, or an equivalent method.

Some candidates gave a little more thought to the method required, and a number of correct solutions were seen. The most common method was to find the area of a segment and then double it, and a rather more long-winded method was to first find the unshaded area in triangle *ABC*. An efficient and elegant solution was to find the difference between the total area of two sectors and the area of the rhombus, though some candidates mistakenly believed it to be a square. Some, otherwise correct, solutions were spoiled by a loss of accuracy in the working and hence in the final answer.

8) (i) The majority of candidates could identify the relevant transformation, but many then lost marks through a lack of precision when describing it. Examiners expected to see the word translation used, rather than more colloquial descriptions such as move or shift. Equally, the description of the translation had to indicate three units in the positive *x*-direction, with no ambiguity. The most successful candidates made effective use of vector notation.

(ii) The vast majority of candidates were able to state the correct value, with 3^2 and $\log_2 3$ being the most common errors.

(iii) Most candidates were also able to find the required value in this part as well, though it was not quite so well done. Candidates seemed familiar with the method to remove the logarithm, though in some cases this was spoiled by first attempting to split $\log_2(x - 3)$ into two terms. The other common error was to use 1.8^2 rather than $2^{1.8}$.

(iv) This final part of the question proved to be somewhat more challenging. Most candidates could gain the first mark for attempting a relevant equation, although some simply equated the two *y*-coordinates. A second method mark was then available for correctly combining two logarithm terms, and a reasonable number gained this mark, including some who had not gained the first mark. Successful candidates were then able to complete the question to gain full marks. Some candidates failed to get more than the first two marks as they subtracted the functions in the incorrect order when equating the difference to 4. A few of the more astute candidates considered both possible differences and then justified which to select as their final answer.

9) (i) The quality of responses to this question varied considerably. Not knowing how to deal with the rational expression proved to be a stumbling block for many candidates, resulting in flawed integration attempts. These candidates could still gain a mark for attempting to use limits correctly in their integral, and many did gain this mark though some went back to using the original function. Candidates who appreciated the need to first rewrite the integrand usually did so successfully. Dividing each term by x^2 tended to be the more popular and successful approach. Some attempted to multiply through by x^{-2} which gave the correct final term, but errors in applying rules of indices sometimes led to the first two terms being wrong.

The integration was usually carried out correctly on whatever function they had at this stage, as was the attempted use of limits. Of the candidates who had been successful up to this point, fewer than half were then able to show the given equation correctly. Giving the first term as $2a^2$ rather than $(2a)^2$ was a common error, and substituting values into $4x^{-1}$ caused difficulties for many; those who first rewrote this term in fractional form were usually more successful.

(ii) The majority of candidates chose to use the factor theorem to confirm that $a = 1$ was a root, and this was invariably correct. Other methods were also used, but some candidates lost an easy mark by failing to address this part of the question. Finding the quotient was done well, although the absence of a linear term in the function caused problems for some, especially those using long division. There was a variety of methods used to find the quotient, including inspection. This method is fine if done successfully but it makes gaining partial credit extremely unlikely as no working is shown. Candidates attempting this method would be well advised to expand their brackets to check for accuracy. Those who had the correct quotient could usually solve the resulting equation correctly, although completing the square was not as successful as using the formula. The surd was invariably simplified correctly, but only a small minority of candidates referred back to the question and appreciated the need to reject the negative solution.

4723 Core Mathematics 3

General comments

The first six questions on this paper provided plenty of accessible requests and the vast majority of candidates were able to show their competence with the topics and techniques being assessed. Q8(ii) was also answered well with many candidates coping easily with the algebraic skills needed. Questions 7 and 9 presented more problems although most candidates were able to earn at least a few marks here. The only questions not attempted at all by significant numbers were Q9(ii) and Q9(iii); this was due more to the difficulty of these two parts than to any time pressure.

Presentation of their work by many candidates was good and it is always pleasing to note that candidates recognise that effective communication is important, not only out of courtesy to the examiner but because a candidate taking care with the presentation is less likely to make careless mistakes. However, there were also many candidates who showed a lack of facility with algebra and this cost candidates a number of marks.

The mean mark for the paper was 46.1 out of 72 and the standard deviation was 14.0. It is pleasing to report that 2% of the candidates recorded 70 marks or more, with 43 very capable candidates obtaining full marks. Despite the accessibility and familiarity of some of the requests, there were a few candidates who struggled to obtain marks; 1% of the candidates recorded 10 marks or fewer.

Comments on individual questions

1) **(i)** This question was answered well with 75% of candidates earning all three marks. Use of the quotient rule was the usual approach; a few candidates had the terms in the numerator the wrong way round but a more common error, and an avoidable one, was the simplification of $3(2x+1) - 6x$ in the numerator to give 1. Some candidates opted for the product rule and were not always successful, failure to apply the chain rule being the principal cause of error.

(ii) This was also answered well, again with 75% of candidates earning three marks. Failure to include a factor *x* in the derivative was the most common error. In this part, and in part (i), a number of candidates omitted to substitute 2 to find the gradient as requested.

2) **(i)** There were three approaches taken in attempting to find the value of $\operatorname{cosec} A$. One was to consider a right-angled triangle with sides 1, 2 and $\sqrt{5}$. Candidates then had little difficulty in writing down the correct answer. A second approach involved trying to use an appropriate identity and a successful outcome was not so common. Some candidates evidently knew the relevant identity or obtained it by manipulating $\sin^2 A + \cos^2 A = 1$. On some scripts, $\cot^2 A + 1 = \operatorname{cosec}^2 A$ immediately became $\cot A + 1 = \operatorname{cosec} A$. Other candidates proposed an incorrect identity linking $\operatorname{cosec} A$ and $\tan A$. A number of candidates ignored the information about *A* being acute and concluded with $\operatorname{cosec} A = \pm\frac{1}{2}\sqrt{5}$, an answer that did not earn the second mark. The third approach involved resorting to calculators and giving an approximate value; no credit was allowed.

(ii) This was answered very well with 80% of candidates earning all three marks. The appropriate identity was quoted and, in most cases, the steps to find the value of $\tan B$ were carried out accurately.

3) **(a)** This question revealed widespread misunderstanding about the basic definition of the modulus function. 54% of the candidates earned all three marks, usually having no difficulty in identifying the two values of *t* as –3 and 3 leading immediately to 7 and 5 respectively as the required answers. But 28% of candidates earned no marks. The modulus signs prompted the use of $\pm$ all over the place so that the four answers –5, 5, –7 and 7 were commonly offered; for others, the presence of modulus signs prompted squaring, again seldom with any satisfactory outcome.

(b) It is disappointing to record the fact that only 44% of candidates earned all four marks on this inequality. The more popular approach involved squaring both sides of the inequality. There were some errors, usually involving the square of $3\sqrt{2}$, but most did square both sides accurately. There were then errors involving signs and the manipulation of the surds.

Other candidates dealt with either an equation or inequality (or occasionally four such) where each side was linear in *x*. Often the critical value $x = -\sqrt{2}$ was reached but it was then a rather haphazard process to reach a conclusion.

A neat approach involves careful sketches of $y = \left|x - \sqrt{2}\right|$ and $y = \left|x + 3\sqrt{2}\right|$; the critical value and the answer are immediately apparent. Such an approach was not seen very often.

4) **(i)** This was generally answered well and 75% of candidates recorded all three marks. A few were unable to recognise 250 grams as the initial value and solutions then tended to have *m* and 2*m* creating difficulties. Most candidates carried out a second calculation involving 2000 grams to find the second answer but examiners were encouraged to note a pleasing number of candidates showing awareness of the properties of exponential growth and realising that the first answer needed to be tripled. Since this was a question in context, a final answer $\frac{\ln 2}{0.021}$ to the first request did not earn the accuracy mark.

(ii) Most candidates had no difficulty in differentiating accurately although an extra factor *t* appeared on a few scripts. Candidates then calculated the value of *t* for which the mass is 400 grams to use in their derivative. Only very few candidates realised that it was not necessary to do this. Differentiation gives $\frac{\mathrm{d}m}{\mathrm{d}t} = 250 \times 0.021 \times \mathrm{e}^{0.021t}$ and this can be written as $0.021m$, meaning that 0.021×400 gives the required rate immediately.

The fact that the rate was requested for a particular value of the mass led some candidates to rewrite the equation for *t* in terms of *m*. Differentiation to find $\frac{\mathrm{d}t}{\mathrm{d}m}$ was not always done correctly and, even when it was, a failure to find the reciprocal of the result often meant that the final answer was wrong.

5) **(i)** 50% of the candidates recorded all four marks and another 34% recorded three marks out of four. These figures reflect the fact that most of the candidates had no difficulty with the integration although a few did rewrite *y* as $(3x+1)^{\frac{1}{2}}$. The main problem concerned the fact that the answer was given in the question and that, as a result, candidates were expected to provide a convincing conclusion. Candidates going immediately from $4\sqrt{28} - 4\sqrt{7}$ to the given $4\sqrt{7}$ did not earn the final mark. An extra step or two to justify the conclusion was needed. Use of a calculator to compare decimal approximations of $4\sqrt{28} - 4\sqrt{7}$ and $4\sqrt{7}$ also did not earn the final mark.

(ii) Candidates knew in general terms what was required in this question but the details were often incorrect. There were a few slips in squaring y and π was sometimes missing but the vast majority started with $\int \frac{36\pi}{3x+1}\,\mathrm{d}x$. Integrating to obtain $36\pi\ln(3x+1)$ was a common mistake. Some careful work involving logarithm properties was then needed to present the answer in the required form and many candidates struggled at this point. Most were able to reach an expression involving $\ln 4$ but concluding with an answer in the form $k\ln 2$ unexpectedly proved to be challenging for some. Some stopped as soon as they reached $12\pi\ln 4$ and, for others, division throughout by 2 was their means of moving from $12\pi\ln 4$ to $6\pi\ln 2$. For a routine request, it is a little disappointing to note that only 41% of the candidates recorded full marks.

6) **(i)** Examiners were reasonably tolerant in assessing the two sketches but, even so, many attempts were not as assured as they should have been. Many attempts at the sketch of $y = 8 - 2x^2$ were far from being symmetrical about the y-axis and a few were even straight lines. Many attempts at the sketch of $y = \ln x$ clearly touched the negative y-axis; others passed through the origin, existed for negative values of x or had the wrong curvature. For full credit, the parabola had to be shown in each of the four quadrants and the logarithm graph had to be shown in the first and fourth quadrants. The third mark was earned if the curves were correct in the first quadrant and if the one point of intersection was indicated in some way. Some candidates did not earn this final mark because they failed to draw attention to the point of intersection.

(ii) Candidates were required here merely to mention the fact that the curves cross the x-axis at 1 and 2 and that the x-coordinate of the point of intersection lies between these two values. Many managed this but there were some lengthy and convoluted attempts as well. Candidates embarking on a sign change routine did not earn the mark.

(iii) Candidates had no difficulty with this part and 88% of them duly earned all four marks.

(iv) This request proved much more challenging and 45% of candidates earned no marks. There were many attempts that involved finding the equations of the transformed curves; candidates seemed to hope that equating these would somehow reveal the coordinates of the new point of intersection. Candidates adopting the appropriate course of tracking the transformation of the point of intersection did not always succeed. Many were correct in stating the x-coordinate as 3.92 but it was common then for the y-coordinate to be incorrectly given as 5.46, resulting from $4\ln 3.917$.

7) **(i)** A few candidates did not recognise the need to use the product rule here but most did and, indeed, 58% of candidates duly earned all three marks. A common error was the differentiation of $\ln(2y+3)$ to produce $\frac{1}{2y+3}$ and, in many cases, one of the terms was $y\ln(2y+3)$. Full credit was not given when necessary brackets were absent; $\ln 2y + 3$ appeared in many answers. As soon as a correct expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ was produced, the marks were awarded. This was fortunate for many candidates as some subsequent horrendous 'simplification' was perpetrated, including the correct $\ln(2y+3) + \frac{2y+8}{2y+3}$ becoming $\ln(2y+3) + \frac{8}{3}$.

(ii) This question assessed the specification item 'understand and use the relation $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \frac{\mathrm{d}x}{\mathrm{d}y}$'. It was not answered well in general and only 13% of candidates were able to earn all five marks. There were several major problems as far as candidates were concerned. Many candidates thought that the gradients could be found by substitution into the expression from part (i); many therefore claimed the gradient at *A* as 3.77 and rather fewer decided on 6 as the gradient at *B*.

Other candidates, with a little awareness of a difference between $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}x}{\mathrm{d}y}$, decided that all would be well if *x* and *y* were interchanged throughout and relevant *x*-values substituted into their adjusted expression from part (i). Another problem arose for those candidates trying to produce an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. There may already have been some 'simplification' as mentioned above but, for those still with a correct expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$, there was the problem of finding its reciprocal. A few did this correctly after expressing $\frac{\mathrm{d}x}{\mathrm{d}y}$ with a common denominator but, for far too many, $\frac{\mathrm{d}x}{\mathrm{d}y} = \ln(2y+3) + \frac{2y+8}{2y+3}$ became $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\ln(2y+3)} + \frac{2y+3}{2y+8}$.

A final problem concerned the values of *y* to be substituted. Most realised that $y = 0$ was appropriate for the point *A* but finding the *y*-value for *B* required more thought which, commendably, some candidates did display with a comment about $y = -4$ being impossible because it led to the logarithm of a negative number.

Successful candidates usually avoided all the algebraic problems by substituting each of the two *y*-values into the expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ and then finding the reciprocal of each numerical value. This uncomplicated approach was not seen very often.

8) (i) This apparently straightforward request was not answered well and 52% of candidates scored no marks. Whether the poor response was due to lack of knowledge of range or to the presence of the indeterminate constant *a* was not clear. Some candidates attempted to complete the square but this was not always done well and many did not know how to deduce the range from their version. The other popular approach involved differentiation; again candidates were not sure how to conclude and it was common for differentiation to lead to $x = -2a$ with a consequent statement that the range was $x > -2a$. An error that was seen many times in attempts to find the stationary point was $\mathrm{f}'(x) = 2x + 4a + 2a$.

(ii) In contrast to part (i), this part was answered well with 45% of candidates earning all six marks. Many more earned four or five marks, only faltering towards the end of their solutions. The vast majority formed the composition of the functions the right way round and generally coped well with the algebraic simplification. There were some slips and a few equated fg(3) to 0 rather than to 69 but most reached $a^2 = 25$ without too much trouble. There was no penalty at this stage for stating $a = \pm 5$. The neat use of f(*x*) in its completed square form to produce $\mathrm{fg}(x) = \mathrm{f}(4x - 2a) = (4x - 2a + 2a)^2 - 3a^2 = 16x^2 - 3a^2$ was noted a few times.

Somewhat strangely, after finding a, a significant number of candidates did no more on this question or found an expression for $g^{-1}(x)$ but then did not consider the equation $g^{-1}(x) = x$. Of course, it is not necessary to find the inverse of g; solving the equation $g(x) = x$ is an equivalent, and easier, process as a few alert candidates recognised.

9) **(i)** 24% of the candidates earned all four marks for this proof. Success needed thorough knowledge of the relevant trigonometry plus care with detail. Most candidates earned a mark for an appropriate identity involving $\sin 2\theta$ or $\cos 2\theta$ but, for many candidates, that was the only mark they earned. The main problem concerned the term $\cos^2(\theta + 45°)$; candidates realised that expansion was needed but, all too often, this was $\cos^2\theta\cos^2 45° - \sin^2\theta\sin^2 45°$. Other candidates did start with $(\cos\theta\cos 45° - \sin\theta\sin 45°)^2$, or the corresponding version involving the exact values of $\cos 45°$ and $\sin 45°$, but the expansion then often omitted the term involving $\sin\theta\cos\theta$. Once mistakes like these had been made, the identity could not be proved but many candidates did persevere with some involved, if doomed, attempts to try to reach $\sin^2\theta$.

A few candidates showed pleasing mathematical awareness by rewriting the first term of the left-hand side as $\frac{1}{2} + \frac{1}{2}\cos(2\theta + 90°)$ which simplifies to $\frac{1}{2} - \frac{1}{2}\sin 2\theta$; completion of the proof followed quickly.

(ii) Many candidates seemed unaware of the fact that the identity established in part (i) was related to the equation in this part and they tried manipulation of the given equation. Others did try to exploit the earlier part but, with a lack of attention to detail, proceeded to solve $6\sin^2\theta = 2$. Many candidates did proceed with the correct $6\sin^2\frac{1}{2}\theta = 2$ and 25% of all the candidates concluded with the two correct angles. Other candidates gave one correct answer $70.5°$ but omitted the other possibility of $-70.5°$.

(iii) This was a testing conclusion to the paper but 6% of candidates were equal to the challenge and answered accurately. There were many others who made significant progress, recognising that the value of k was less than $\frac{3}{2}$ but then concluding with $-\frac{3}{2} < k < \frac{3}{2}$, a result that overlooks the fact that $6\sin^2\frac{1}{3}\theta$ cannot be negative.

4724 Core Mathematics 4

General comments

As usual, there was a wide range of responses. Many candidates showed clarity of thought with concise presentation, though the number seemed fewer on this occasion - that is not to say that correct working was not present but there was a lack in presenting the work well; some other candidates appeared to have made little preparation for the examination though they were in a minority.

Misreading by candidates was less common than in previous examinations – though miscopying their own work is still an unwelcome feature and can lead to more complicated working than necessary.

Comments on individual questions

1) The vast majority recognised this question as one suitable for integration by parts, the main errors arising from the integrations of $\cos 3x$ and $\sin 3x$. Provided the method of integrating by parts was fully understood, some credit was given to candidates who used a wrong sign or 3 instead of $\frac{1}{3}$ in the integrals. Candidates were expected to simplify $\frac{1}{3}\left(\frac{1}{3}\cos 3x\right)$ and $- - \frac{1}{9}\cos 3x$ in their answers but, needless to say, they were not expected to multiply their result by 9 to make it look 'better'.

2) Most candidates were confident with the binomial expansion and were able to change $(9 - 16x)^{\frac{3}{2}}$ into a suitable form for expansion. Common errors in the expansion included careless simplification of the x^2 term (often because of cramped writing) and multiplication of this expansion by 9 instead of by 27.
A significant number of candidates completely ignored the request for the validity.

3) The first part was generally answered well and most obtained the correct expression for $\frac{dy}{dx}$ though a few equated $\frac{dy}{dx}$ to 0 at an earlier stage (so losing a simple mark). The derivation of $x^2 = 1$ or $y^4 = 4$ was well done but the final easy hurdle of obtaining the two (and only two) pairs of coordinates left much to be desired.

4) **(i)** Almost everyone knew what to do, but this question prompted a large number of scrappy solutions. Equations were labelled 1, 2 and 3; 2 was doubled and called 3; 1 and 2 were subtracted, m found and then substituted into... . Which equation was used? Which equation was used for confirming consistency? Candidates should work systematically and state clearly what they are doing.

(ii) The first question candidates had to ask themselves was "Which aspect of the line equations shall we use?" but only a minority stated the answer. Examiners had to fight their way through the myriad of evaluations to determine what was happening. Fortunately most produced the correct answer, though a not insignificant minority only gave the obtuse angle.

5) **(i)** This was generally done well though a few were unable to manipulate the equation $\frac{2}{3}\tan q = \frac{1}{2}$ into its simpler version $\tan q = \frac{3}{4}$. Apart from rounding errors, the actual coordinates were then relatively easy to find.

(ii) A large number of candidates assumed that the required cartesian equation had to be linear, or that inverse trigonometrical functions would be acceptable in the answer. A few said that $\cos\theta = \frac{y-1}{2}$ and then used it in the equation $\cos^2\theta + \sin^2\theta = 1$; although $\left(\frac{y-1}{2}\right)^2$ and $\left(\frac{1-y}{2}\right)^2$ were equivalent at that stage, an earlier mistake had been seen and was consequently penalised.

6) This question was relatively straightforward provided candidates were meticulous in the presentation of their work and in any algebraic manipulation that was necessary. Many did not achieve the transformation of the numerator $4x - 1$ into $2u - 3$. A not insignificant number cancelled the '2' in this numerator with the '2' produced by dx = $\frac{1}{2}$ du. The denominator of the transformed integral, $2u^5$, frequently became $(2u)^{-5}$ in the numerator. The '$\frac{1}{2}$' was frequently overlooked. Even though it was easy to separate the transformed integral into two relatively simple parts, many used the idea of integration by parts again.

It was interesting to note that some candidates managed to apply the correct limits to a correct function, only to get an incorrect answer. Were they using their calculators incorrectly, particularly with negative values?

7) (i) Three different methods were seen for this. The normal one, using the straightforward chain rule, proved successful for the majority. A few completed an initial double change into ln(sec x + tan x) and, again, were usually successful with the differentiation. An even smaller number changed the expression initially to just $\ln\left(\frac{1+\sin x}{\cos x}\right)$ but then had to cope with the differentiation of a quotient, which often proved a rather overwhelming prospect.

(ii) This was answered surprisingly well and the logarithm manipulation was not found to be a problem. The question said "Using this result" and candidates were expected to use either the given function or some aspect of their part (i) working. If they wished to use their part (i) working and the *List of Formulae* result $\int \sec x \, dx = \ln(\sec x + \tan x)$, they were expected to quote the converse in order to obtain full marks.

8) (i) This caused little problem for the majority but there were several who lost marks because they assumed that, seeing the same components (albeit in different positions) for their *AB* and *AD*, meant they had no more work to do, as the result was "obvious".

(ii) Those who found the mid-point correctly usually went on to find the equation correctly, and many who found an incorrect mid-point often knew what to do to obtain the direction vector.

(iii) This part was straightforward for those who had found the equation of the line correctly.

(iv) The correct answer was the least common of all the answers suggested.

9) (i) The majority of candidates used the method of 'separating the variables' and generally integrated each side correctly. However, many did not go further or failed to rearrange this equation correctly in order to express θ in terms of *t*, *k* and an arbitrary constant. Many candidates thought that '+ c' should appear only at the end. A very few inverted each side and were generally less successful because of the position of '*k*' immediately alongside the $\theta + 20$ in the denominator.

(ii) There were some very good and neat solutions to this part, particularly by those who read the question carefully before delving into its solution. Even though *k* was defined to be a positive constant, a large number of candidates obtained $k = -\frac{1}{20}$. A few interpreted the statement "…at this instant, the liquid is cooling …." to imply that there was a constant decrease of 3 degrees every minute.

However, many candidates attempted to use all the information correctly and found *k*, their own constant of integration and, finally, the time taken for the liquid to freeze.

(iii) Most candidates stated that *k* must be larger, but explanations were sketchy.

10) **(i)** This question commenced with "Use algebraic division….." and those candidates who did not follow this instruction were penalised. In general, the division was performed well and the positions of the quotient and remainder were rarely mixed up in the final expression.

(ii) The word "Hence" was used here and candidates were expected to use their expression from part (i) and evaluate the integral from that. The use of partial fractions was necessary but was applied by only a relatively small number of candidates.

4725 Further Pure Mathematics 1

General comments

Most candidates seemed well prepared for this paper. Completely correct solutions were seen to all questions and the majority of candidates gave correct solutions to at least half the questions, which resulted in a large number of candidates gaining high marks.

Candidates generally found the space in the answer booklet sufficient for their needs and only used additional sheets when a replacement solution was required.

Where an answer is given in the question, candidates should be aware that they must show sufficient working to justify their answer if full credit is to be gained. In particular, in question 10(i), often no working was seen to show why $u_4 = \frac{2}{7}$.

In general, candidates presented their work in a clear and logical way, but in question 8(ii) the layout when using the method of differences often meant that the cancelling of appropriate terms was not easily seen.

Comments on individual questions

1) **(i)** This was answered correctly by the majority of candidates, the most frequent errors being not doubling one element of **A** or using **I** rather than 3**I.** Some candidates used $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ for **I**.

(ii) The most frequent error was the omission of the determinant, while others made the leading diagonal negative instead of reversing the elements. Few candidates checked their answer by matrix multiplication of their answer with matrix **A**.

2) Completely correct solutions were given by a good number of candidates. Most attempted the difference of two terms, with $\sum 1$ being given as 1 rather than n the most common error. A significant minority expanded the correct expression to obtain a cubic, before attempting to factorise, rather than using a common factor of n or $\frac{1}{6}n$ as the first stage of factorisation.

3) **(i)** Most found $|z|$ correctly, but many candidates omitted a necessary negative sign in their value for arg z.

(ii) The conjugate was generally given as 2 + i, but a few candidates used $x + iy$ and $x - iy$ for z and z^*. Arithmetic slips in obtaining two linear equations or in solving their linear equations caused the most frequent loss of marks.

4) **(i)** This part was done correctly by the majority of candidates. Too many omitted " = 0" and so did not obtain an equation.

(ii) Most used the product of the roots of the derived equation, or used the sum and product of the roots of the given equation in an expanded expression. However a minority used the values from the derived equation in an expanded expression. A smaller minority found the roots of either the given or derived quadratic and attempted to use these to find the value of the given expression, usually without success.

5) The method of finding the determinant of a 3×3 matrix was well understood, with sign errors causing the majority of the loss of marks. Most knew that no unique solutions meant that they had to equate their determinant to zero, although some solved det $\mathbf{M} \neq 0$ or det $\mathbf{M} > 0$.

6) (i) The correct matrix was found by most candidates.

(ii) This part was found quite testing by many candidates. Some did not clearly indicate in their solution the identity of P and Q and the examiners had to assume that the first one given was P and the second one Q. The stretch was usually described correctly, but the shear was not. The expression "in the *y*-axis" would be part of the description of a reflection not a stretch. Candidates should be made aware that a shear has an invariant line, either the *x*- or *y*- axis, and that the image of one point must be stated correctly; "scale factor 2" is not acceptable when describing a shear. If transformation Q was given as the shear, its matrix was often incorrect, ie $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, and very few checked by matrix multiplication that their pair of matrices gave the answer to (i).

(iii) Most candidates showed that they understood matrix multiplication, but many obtained the same answer as (i), thus not realising that this pair of transformations is not commutative.

7) (i) Many candidates did not give sufficient detail in their sketches. The circle often had no clear indication of the radius, and the point (3, 1) was not shown on the line or half-line. Also, what may have been meant to be a half-line was often drawn with a portion to the right of (3, 1). Some candidates thought that both loci were circles and some that both were straight lines.

(ii) Most candidates realised that the inside of the circle was required, but the second inequality was often not interpreted as meaning that the portion above the line was to be shaded.

8) (i) The majority of candidates showed sufficient working to justify the given answer.

(ii) Candidates who displayed the terms in columns usually showed the correct cancelling process. Those who listed terms in rows often had difficulty in seeing the correct way to cancel.

(iii) Many candidates thought that the sum from 1 to ∞ was 1, rather than the correct value of 0. Some did not see that the sum from 2 to ∞ was required.

9) (i) Most derived the given answer correctly and showed sufficient working. Minor algebraic slips resulted in major loss of marks, but some showed little or no working to justify their answer.

(ii) The expression in (i) was generally used to obtain a fraction in which the values of the symmetric functions could be substituted. Sign errors caused the major loss of marks. Some used the substitution $x = \frac{1}{\sqrt{u}}$ and then rearranged correctly in order to square out and obtain a correct cubic equation. The correct value of the required expression can then be easily found. Some used a substitution $x = \frac{1}{u}$ and then a similar approach to (i) to find the value of the required expression. These last two methods usually resulted in more lengthy solutions.

10) (i) Most found the first two answers correctly, but too frequently no working was shown to justify the value for u_4.

(ii) A correct expression, in terms of *n*, was usually seen, but a minority gave an expression in terms of u_n .

(iii) Sufficient working to demonstrate that the result was true for $n = 1$ was generally seen, as was the working to derive the correct expression for u_{k+1} from the recurrence relation and their part (ii). However, candidates should be made aware that the work required in an induction question is mainly of the "given answer" type, so sufficient justification is essential. Many candidates did not give a satisfactory conclusion to the induction process.

4726 Further Pure Mathematics 2

General comments

This paper was rather more straightforward than some from previous years and there were some good marks with only a very few candidates recording very low marks.

Comments on three aspects of responses from candidates have been made before; again, marks were lost as a result of these shortcomings.

- Full working needs to be shown to achieve a given answer in a "show that..." question.
- Care needs to be taken over sketches. If they do not illustrate the essential characteristics needed, it is often because of a lack of care.
- There are occasions when candidates are not able to complete their answer to a part question within the space allotted. In such situations, candidates are required to complete their working on separate sheets. They should not use space that is allocated to answers for another part question.

These aspects remain as problems; further comments are offered in the body of the report.

Comments on individual questions

1) Most scored well on this first question. A few wrote as their partial fractions $A+\frac{B}{x-1}+\frac{Cx+D}{x^2+4}$. For those that obtained $A = 0$ all that was lost was a little time but it did add an extra opportunity for error.

2) (i) The vertical asymptote caused no difficulties. Most found the oblique asymptote by long division. It is, of course, incorrect to assert that as $x \to \infty$, $\frac{x^2-3}{x-1} \to \frac{x^2}{x} \to x$ which was seen in a few scripts.

(ii) "The coordinates" means ordered pairs. This was not always seen.

(iii) Some candidates were anticipating a different question and demonstrated that all values of y were possible. It was necessary to differentiate the function and to show that it could never be equal to zero. This is a "show that..." where working needs to be seen. Stating that $x^2-2x+3=0$ has no roots is not enough. It has to be demonstrated arithmetically that this is the case and the conclusion drawn from this fact.

(iv) The sketch provided often failed to meet the criteria required for full marks, usually because of careless drawing. Some failed to insert the points where the curve cut the axes and a significant number of candidates allowed their curve to drift away from the asymptotes for large values of x and y.

3) This was a standard question and most candidates scored well. Often the fact that one of the roots of the quadratic equation in e^x was invalid was missed.

4) (i) This standard reduction formula was usually well done, though a few candidates jumped to the result rather too soon, missing essential algebraic steps.

(ii) This part was also done well. A few found I_1 initially rather than the easier I_0. A very small number of candidates ignored part (i) completely and found I_3 by successive reductions.

5) (i) Most scored full marks on this part. Those that did not failed to differentiate a product. A few rewrote the function as the fraction $\frac{\sin x}{e^x}$ which was, of course, perfectly acceptable but gave extra opportunities for algebraic errors.

(ii) Most candidates confirmed that each side of the equality was $-2e^{-x}\cos x$. Having done so, a few failed to work out f″(0).

(iii) A significant number only found f‴(0). A few others made a guess as to what f‴(x) might look like. [–2f″(x) – 2f′(x) – 2f(x) was the most popular, closely followed by –3f″(x) – 3f′(x).] Most candidates failed to realise that all that was required was to differentiate f″(x) from part (ii).

(iv) Most obtained the method mark for using the Maclaurin series, albeit with incorrect values from the earlier parts. Just a few candidates ignored all the work in the first three parts and wrote down, and multiplied, the series expansions for $\sin x$ and e^x from the *List of Formulae*. This was, of course, acceptable but added time.

6) This was another standard question for which most scored full marks. A number of candidates failed to write down the standard form correctly. This is given in the *List of Formulae*. Candidates should be discouraged from relying on memory when it is not necessary!

7) (i) A number failed to note the precise demands of the question and drew curves for $q > \frac{1}{2}p$, or failed to state the equation of the line of symmetry or the coordinates of *P*. The sketch was also expected to show that candidates understood that $q = 0$ and $q = \frac{1}{2}p$ were tangents to the curve at the pole.

(ii) Most candidates scored well here, adopting the correct way to carry out the integration.

(iii) This part defeated many candidates. Because *P* was the point furthest from *O*, some candidates thought that $\frac{dr}{dq}$ had to be involved and even set equal to zero.

8) (i)(a) In order to confirm the root correct to 3 decimal places it was necessary to work to at least 4 decimal places. Some candidates failed to do so and lost marks needlessly.

(i)(b) This was another sketch requirement where some rather sloppy drawing meant that marks were lost. It was expected that initial values of *x* should be greater and less than α but this was not always clearly seen.

(ii)(a) A large number of candidates failed to score full marks here because of incorrect algebra. If an error is made which leads to an incorrect result, candidates should check carefully for their error and not just write down the result as given in the question. Some candidates also lost marks for the reasons outlined before: the final result was written down too soon, missing out essential algebraic steps.

(ii)(b) Most candidates scored well on this part.

4727 Further Pure Mathematics 3

General comments

As usual at the January series, only a small number of candidates sat this paper. Overall the paper was found to be slightly less demanding than the paper of a year ago. Almost all candidates were able to gain some marks on each question and there was less evidence of candidates who had completely omitted study on one section of the specification (something observable last year). Some candidates again lacked the level of confidence in Core 1 to Core 4 techniques that this paper presupposes. This was demonstrated by a lack of ability to differentiate products (Q6(i)), find the sum of a geometric series (Q7(i)(a)) and in a general lack of ease when manipulating algebra or working with radians.

Well-prepared candidates were able to tackle several of the questions which were standard (textbook) types. Some candidates were also able to demonstrate their problem solving skills and their ability to produce well-written mathematical arguments; questions 7(ii) and (iii) and 8(iii) targeted the former, while question 2(iii) and much of 7 and 8 assessed the latter. There did not appear to be any problem with the length of the paper with all candidates appearing to have sufficient time. Compared to last year, there were fewer scripts that were difficult to decipher because of poor handwriting.

Comments on individual questions

1) This first question should have been straightforward for any properly prepared candidate since both parts are listed as techniques required in the vectors section of the specification. So it was surprising how many candidates did not answer this with ease.

(i) Most candidates used the scalar product method to find an angle, although one or two, perfectly acceptably, chose to use the cross product. It was disappointing to find that some candidates thought that $|a||b|\sin q$ was the value of the scalar product. A few were unfamiliar with the description 'acute angle' or did not know the relationship between the angle between planes and the angle between their normals.

(ii) Most candidates knew that they needed to find the direction of the line by using the vector product of the normals to the planes; a few made errors in actually finding it. Finding a point on the line proved more of a challenge for weaker candidates with some fixing values for two of the variables before trying to find the third. Those candidates who attempted to find the line solely through simultaneous equations were far more likely to introduce errors.

2) **(i)** This question was generally answered well, particularly when it came to stating the identity. A common error with the order of the group was to say that it was 5 rather than 25.

(ii) A significant number of candidates, erroneously, gave the answer as –2 – 4i, failing to realise that this was not an element of the group. A smaller number, for some reason, thought that the inverse had to be the complex conjugate.

(iii) This part of the question allowed candidates to demonstrate the thoroughness of their mathematical arguments. Some, however tended not to realise that $5(a + bi) = e$, whilst a necessary condition, was not a sufficient one without giving further reasoning. The strongest candidates either used Lagrange's theorem to help them complete the demonstration, or were exhaustive in their exploration of the multiples of all possible $a + bi$, considering $a \neq 0$ or $b \neq 0$ (or both).

3) This was another standard question from which most candidates were able to gain high marks. Common errors which caused progress to be impossible were not recognising the need to initially divide throughout by x or neglecting the negative sign in the initial equation. Where candidates successfully heeded these two issues, they could usually work through to a general solution. Some of them, however, lost the final mark by neglecting to multiply their constant by the x^3 term when isolating y. Candidates could pick up the final method mark, even when they had been unsuccessful in finding the general solution, by demonstrating that they knew how to find the constant using the boundary condition.

4) **(i)** Many candidates knew a standard formula for this request and most were then able to use that formula successfully. Others found success by working from first principles; once they had compared the vector between two general points with the perpendicular to both lines, these candidates either used the dot product to simplify or solved the three equations in three unknowns. Marks were sometimes lost through sign errors, with some candidates being unaware of the sign for the **j** component of the vector product.

(ii) Finding the cartesian equation of the plane was generally answered well by candidates who had found the direction of the perpendicular in part (i). Some, however, left their answer in vector equation form.

5) **(i)** Most candidates were familiar with this standard problem and many gained full marks. A variety of notation can be described as 'polar form', so it was acceptable to write the answer in exponential form, CiS form or in polar coordinates with argument in the interval $[0, 2\pi)$ or $(-\pi, \pi]$. Candidates were, however, penalised for using degrees, omitting i (commonly noted in exponential form attempts) or for roots of the form $\cos\theta - \mathrm{i}\sin\theta$.

(ii) Although many candidates were able to tackle this question, few of them recognised that, with a quartic equation, there would only be four roots; consequently most candidates did not spot the fact that the value corresponding to $k = 0$ should be rejected.

6) Parts (ii) and (iii), which assessed the interpretation of a solution to a differential equation, were answered well only by the very strongest candidates.

(i) Although this question covered another standard technique, there was a significant minority of candidates who made little progress towards a solution. Some demonstrated a lack of knowledge of the standard form of solution resulting from complex roots to the auxiliary equation. Others took insufficient care when finding derivatives of the particular integral. As in previous sessions, there were some whose work resulted in "constants" which were functions of x and others who failed to see any problem with a final solution which contained complex coefficients.

(ii) Marks were gained by some candidates for a recognition that the $x\cos 2x$ term would dominate as x tended to infinity, but only a few were able to fully describe the behaviour of this function. Good answers often used a diagram to flesh out a verbal explanation.

(iii) This question was, in part, a way to assess candidates understanding of how to select the particular integral since this is the change that affects the shape of the graph as x tends to infinity. Candidates who could do part (ii) and who spotted this usually gained full marks.

7) **(i)** It was pleasing, in part (a), to see that most candidates spotted that they were dealing with a geometric progression and were able to gain the first two marks for this question. Converting from $\frac{a(1-r^n)}{1-r}$ (given in the *List of Formulae* MF1) to the final form was beyond some candidates. Part (b) was often attempted by using the formula in part (a) instead of realising the necessity to refer back to the original definition of S.

(ii) Those candidates who used the real part of (a) plus de Moivre's theorem could, with care concerning i in the denominator, produce the right-hand side from the left with relative ease. Some candidates, however, chose to use $\cos nq = \frac{z^n + z^{-n}}{2}$ for each term. If they were adept at algebraic manipulation, they could still work their way through to the solution by summing two geometric series. Some candidates, though, tended to make errors such as believing, mistakenly, that there was now one geometric series with $r = z - z^{-1}$.

(iii) Many candidates did not realise that it is insufficient to use (inexact) decimal calculator values for functions when trying to show the exact value of a root. Because they were not trying to solve the equation $\sin\frac{21q}{2} = \sin\frac{q}{2}$, they were then unable to make progress when it came to finding another root. Some candidates availed themselves of their knowledge of the identities $\sin x = \sin(p - x) = \sin(x + 2p)$ to produce neat, well argued solutions.

8) **(i)** The most elegant answers started from the given identity and worked from there to the new identity in clear steps with reference to the use of $a^4 = e$ or $w^2 = e$. Other well-written arguments worked from one side of the new identity to the other. Where candidates started with the identity to be shown and operated on both sides of the identity till both sides were shown to equate, they still gained the marks on this occasion. However, this practice should be discouraged.

(ii) There were a few candidates who both here - and sometimes in part (i) - falsely assumed commutativity. Most others recognised what was required and made a good attempt to simplify the squared terms they produced.

(iii) While many candidates recognised the need to use only elements of order 2 (plus *e*) in their subgroups, some of these had clearly not considered ensuring that their subset was closed. So it was common to see sets such as $\{e, aw, aw^2, aw^3\}$ alongside one correct cyclic subgroup. The strongest candidates often demonstrated a familiarity not only with general group theory, but also with the structure of both groups of order 4.

Overview – Mechanics

General comments

A high standard of work was apparent across all units this session, and in each the number of poor scripts was small. For the most part scripts contained accurate and well organised solutions to the questions posed. In each module however there were items which caused problems for many candidates, and which were – for the best – the only cause of loss of marks.

In Mechanics 1, it was apparent that that the force exerted by a string on a pulley, Q5 (iv), was an unfamiliar topic. Other problems arose from unusual situations, specifically Q3, parts (i) and (ii), and the final part of Q7.

Difficulties arising from context were apparent also in Mechanics 2, where problems were seen in many scripts with Q3 and Q8. In both questions situations normally tested on horizontal surfaces were instead set on slopes, and strategic thinking was required.

Motion in a circle also proved problematic in Mechanics 3, where it featured the less usual two particle configuration. Towards the end of that paper, the novel configuration of rods in Q6 and the use of SHM equations for a simple pendulum in Q7 proved taxing.

It is of course impossible for students to learn mechanics without reference to specific situations and contexts. It did however seem that even some good candidates were unable to use their knowledge effectively when faced by an unfamiliar situation. The switch from using specific situations to illustrate general principles, to using those principles to confront unusual questions, is perennial, and a difficulty not to be underestimated.

4728 Mechanics 1

General comments

Candidates were well prepared for the examination, a large majority displaying an excellent knowledge of the specification, and how to use that knowledge in the context of the paper. One feature of some wrong answers was that they seldom made a candidate look for an error, as noted in Q5 (ii) and Q7 (i).

There were no significant general areas of weakness. Particular questions which gave difficulty are described below. Misreading of the questions, and a somewhat cavalier attitude to sign, were a source of significant mark loss to some candidates. The choice of positive sense determines whether vector quantities will be evaluated as positive or negative. An unexplained change of sign in an answer may well lose marks, as the value given is a wrong solution to the candidate's equation.

Comments on individual questions

1) Most frequently candidates started the paper by gaining full marks, though 3 out of 5 was a common total among candidates who did not target the "bearing".
Some scripts contained a confusion between bearings and polar angles.

A small minority of candidates began by finding the resultant of the 5 N and 12 N forces, and then combining it with the 14 N force. This gives a diagram which should contain an obtuse angle and calculation which incorporates the ambiguous case of the sine rule. This was not done.

2) (i) This question was very well done, with constant acceleration formulae almost entirely absent.

(ii) Again this was done well, with nearly all candidates demonstrating the appropriate substitution.

3) (i) Though most scripts contained solutions based on resolving parallel to the plane, a significant minority resolved perpendicular to it, getting $T = 6.20$ as the answer. In this question, candidates having a clear diagram with forces were at a significant advantage. However, having 0.2 as the mass, or $T\cos30$ as the component of T, were common misreads.

(ii) Correct solutions were common. Some candidates who had used correct data in (i) used incorrect values in (ii). It was also quite usual for resolving perpendicular to the plane to be used again in (ii) after it had been (wrongly) used in (i).

(iii) Candidates who made errors in (i) or (ii) were able to gain full marks here, and most did. In this simple context candidates were able to avoid gross sign errors.

4) (i) Nearly all candidates gained both marks, getting their answers via factorisation.

(ii) Attempts to use constant acceleration were rare. Nearly all candidates, correctly, went beyond their value of the integration constant, explicitly finding v at time zero.

(iii) While most candidates started well, many did not make explicit the link between the change of sign for v and a change in direction of motion. Finding a value of t when v was zero was not regarded as showing a change of direction.

5 **(i)** A minority of scripts included the assertion that $u = 1.4$, but most contained a correct calculation of acceleration. Some candidates failed to calculate the tension in the string, while others erred in confusing the signs of their terms, and found $T = 4.41$ N.

(ii) Again this was answered well, though sign errors led to some wrong answers. Finding $m = 0.3$ seldom seemed to surprise a candidate

(iii) In only a few scripts did candidates demonstrate the upward velocity of *Q* at the instant when *P* struck the ground, or assume that the deceleration of *P* would continue to be 4.9. Fully correct work was often seen.

(iv)(a) This was the first major area of difficulty for many candidates, who used particle masses rather than the tension in the string.

(iv)(b) Though some candidates made this a complex situation, for many the answer was simple.

6 **(i)** There were few incorrect solutions.

(ii)(a) Though this was an uncommon request, most scripts showed correct solutions. Some indicated faulty logic: "As *P* has the greater momentum, it will be moving after the collision". A significant minority of candidates believed *P* to be in motion after the collision, with speed 0.667 m s^{-1}.

(ii)(b) This question took candidates away from formal specification topics (such as constant acceleration), and many were unsure how to proceed. There were however many correct answers gaining full marks.

(ii)(c) Speed-time diagrams were fairly common, as was a failure to label which line segment related to which particle. Many diagrams showed *P* and *Q* with zero initial velocity, and graph lines which sloped.

7 **(i)** This was well answered in the main, but candidates who found that *P* had an acceleration down the plane greater than *g* did not look for an error in their calculation. There were a significant number of scripts in which only one of *v* or *t* were found.

(ii)(a) There was a noticeable reluctance on the part of candidates to create a Newton's Second Law equation in which all the force terms were negative. This lead to the sly insertion of a minus sign into a calculation of the distance travelled by *P* before coming to rest.

(ii)(b) This proved too demanding for most candidates, hinging as it did on using the acceleration found in (i), and a distance calculated in (ii)(a). Some candidates who did this introductory work correctly then overlooked the vector nature of momentum. A significant minority used either the velocity calculated in (i) or the acceleration found in (ii)(a) in this part.

4729 Mechanics 2

General comments

A large number of very good candidates were entered for this module. There were many excellent scripts which showed thorough understanding and some of which scored full marks. Only a very small minority showed a lack of preparation for the rigours of the paper. Generally, marks were lost through candidates not answering fully the questions asked, particularly when more than one request was made. However, many candidates could improve their performance by taking greater care over presentation, notation and their diagrams. Q 3 and Q8(ii) proved to be the most challenging.

Comments on individual questions

1) **(i)** This question was well answered. Only a small minority of candidates failed to resolve the force.

(ii) Again well answered. Candidates either used their answer to (i) divided by time or used force component multiplied by speed with equal success.

2) **(i)** The majority of candidates knew how to convert the power to a driving force and then use it in a Newton's second law equation successfully.

(ii) This question was well done by the majority. Only a small minority attempted to use the acceleration given for part (i).

3) **(i)** Setting the motion on an inclined plane caused problems for a significant number of candidates. Solutions in which the speed of *A* before the collision was taken to be zero or found from using an acceleration of g were all too common. Many were able to use the coefficient of restitution correctly with only a few examples of wrong signs with velocities being used.

(ii) The usual approach seen was to use constant acceleration equations using their acceleration found in (i). It was surprising though that a significant number, having found acceleration as $g\sin 30$ in (i), used g in this part.

(iii) Most recognised the need to use conservation of momentum to solve this request and the signs of the individual terms were usually correct. A few candidates used the masses with the wrong velocities.

4) **(i)** The majority of candidates used a correct approach for this question. It was surprising the number who did not find the centre of mass correctly from using the wrong formula, using a radius of 6 cm, or using an angle of 90° instead of $^{\pi}/_{2}$. Other causes of lost marks were finding the area of a semicircle incorrectly and adding rather than finding the difference of the area of the square and semicircle to find the total area.

(ii) Most scored well in this part as candidates used their wrong value from part (i) correctly without penalty.

5) Candidates should be advised that the most profitable way of attempting this type of question is to take moments once and resolve twice. It is of course possible to have more than one moments equation, but candidates who do this are usually not as successful.

(i) Most candidates approached this by taking moments about *A*. Most problems arose from having the force at *P* in the wrong direction, or finding wrong distances.

(ii) Resolving in two directions was by far the more successful approach. Those who tried to take moments, although a valid method, were generally less successful. Examiners reported that the use of normal reaction as weight was seen as often as in previous series.

6) (i) This proved to be the most accessible question on the paper. Only a small minority were unfamiliar with the relationship between impulse and momentum.

(ii) Most candidates used the intended energy approach. This was the most successful method with only a minority having the change in kinetic energy with a wrong sign. It was possible to use constant acceleration to solve the problem if a general angle of the slope was used. Unfortunately many, wrongly, used acceleration as *g* and displacement as 0.3 m.

(iii) Two common methods were seen by examiners. The energy approach was usually well done, with common errors seen as using conservation of energy with wrong signs and also the potential energy of the particle used twice. The distance of 0.2/sin30 m in the work done by the frictional force was usually attempted. Candidates, who used constant acceleration combined with Newton's second law, commonly thought the displacement to be used was 0.2 m rather than 0.2/sin30 m.

7) (i) Many good solutions were seen. There was very little evidence of attempts to 'fudge' the given answer.

(ii) The connection between this and the previous part was not always appreciated and quite a few candidates started again. The most common error was to take $y = 2.1$ and some thought the trig identity to be used was $\sec^2\theta = 1 - \tan^2\theta$ but nevertheless still, wrongly, obtained the required result! Some candidates who could not show the given result were sensible enough to use it to find the angle.

(iii) Although the simple method using the expression for the *x* displacement was often seen, many chose to work vertically, often unsuccessfully since they failed to consider all stages of the motion. Some correctly used their expression for *y* displacement, a few simply used 22/14.

8) (i)(a) Some good solutions were seen. Some candidates either got a wrong angle or got a correct angle and resolved incorrectly, but amazingly still managed to get the correct result.

(i)(b) Finding a second appropriate equation proved difficult for quite a few. A remarkable number of combinations of values of *F* and *R* appear to produce $\mu = 0.336$! Some candidates lost marks here by eliminating both *R* and *F* from their equations to find just μ and so failing to find the values of *R* and *F* as required.

(ii) This proved to be the most difficult question on the paper with very few correct solutions seen by examiners. Only a minority realised the direction of *F* had to change in this situation and most simply used the same equations as before, or used one equation with the values of *R* and *F* found in (ii), or solved equations in which *F* (or *R*) was zero.

4730 Mechanics 3

General comments

Many of the candidates for this unit were very competent and very well prepared; these candidates found the paper well within their grasp. There were a number of others who struggled, either with the whole paper or with certain questions. There were many cases of candidates doing some questions completely correctly but making no attempt, or no attempt worth any credit at other questions – the questions they could and could not do varied. However, the questions found most difficult were Questions 4, 6 and 7 – especially the later parts. It may be that some candidates were having an initial attempt at the unit before having a more serious and better prepared attempt in June.

The presentation of the scripts was extremely good in many cases, but there were rather a lot of scripts where the writing was hard to read and the mathematical argument very hard to follow. An examiner can only give credit for work that can be read – and some of the work this series was very nearly totally illegible.

Comments on individual questions

1) Most candidates found this question straightforward, with about half using the cosine rule to find I and then the sine rule to find θ and about half finding expressions for $I\sin\theta$ and $I\cos\theta$ and solving these. A small number of candidates completed the diagram wrongly, and so ended up with wrong answers. Others made small errors, like omitting the mass from one or more term.

2) Almost all candidates did this question very well, working methodically through the three parts, with only an occasional sign error, or a confusion of sine and cosine. However, a small number of candidates thought that, after the collision, the motion of *B* was at right angles to the line of centres; the question clearly states that the direction is at right angles to *B*'s original direction of motion. These candidates were restricted to a method mark in each part.

3) Most candidates correctly used $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration in part (i), with a very small number making a sign error, missing the mass of 0.3 kg or using equations of motion for constant acceleration. Almost all of those who had the correct start went on to correctly establish the given expression. Most candidates established that the arbitrary constant was 0, though not doing this was not penalised in part (i). In part (ii) some candidates worked from the given answer in part (i) by expressing v as $\frac{\mathrm{d}x}{\mathrm{d}t}$, others started with $\frac{\mathrm{d}v}{\mathrm{d}t}=5x$ and, provided they managed to write $5x$ in terms of v they made progress; however, some candidates attempted to solve the differential equation written in terms of the 3 variables t, v and x. In this part candidates were required to either find the arbitrary constant, or show that it was 0 – depending on their method of solution. A number of candidates made mistakes involving the $\sqrt{5}$ factor, either losing it or putting it in the wrong place in an equation.

4) Many candidates did this question completely correctly. In part (i) a few made things more difficult than necessary for themselves by using a zero level for potential energy that was not the initial position, others made a sign error, or forgot about the kinetic energy of one of the particles. Quite a number of candidates made a sign error in the equation $0.4g\sin\theta - R = \frac{0.4v^2}{a}$ and so did not find a correct expression for R; a small number omitted the weight term. Almost all candidates knew that part (ii) depended on putting $R = 0$, and those who had part (i) correct invariably gained both marks in part (ii). A considerable number of candidates omitted this part.

5) Most candidates correctly established the extension of the string in part (i) and went on to find the speed of *Q* when it reached *P* and the common speed after that. A few candidates did not know how to use energy to find the speed of *Q*, and some tried to use energy to find the common speed. Part (ii) was done extremely well, with only a few candidates going wrong by forgetting about the initial elastic energy or by making errors in working out the kinetic or potential energy terms. A small number of candidates used a wrong '*X*', though some of those managed to correct for this later and arrive at the correct equation.

6) Although the working was often rather complicated in part (i), most candidates did manage to show that the force on *CD* at *D* is $\frac{1}{2}W$. While most then went on to find the force acting on *AB* at *B* a small number missed out this demand and others did very complicated and wrong work. Part (ii) was obvious to quite a number of candidates, but a large number of candidates did very complicated and usually wrong work to arrive at quite exotic wrong answers. Part (iii) was found quite challenging – only candidates with the correct answer to part (ii) were able to gain full marks, but other candidates were given method marks for correct working.

7) Part (i) was done well by many candidates. In part (ii), some candidates missed out the length of the pendulum, and a few others only quoted the SHM result; a few failed to work out the period of the motion. Part (iii) proved more challenging, though most candidates did tackle it by means of the equations for velocity and displacement in terms of cos ωt and sin ωt. Those who chose to use $\dot{\theta}^2 = \omega^2(\alpha^2 - \theta^2)$ were able to find the angular displacement but were not usually able to correctly find the time, since this required some manipulation involving the period of motion. For both methods, some candidates used a linear speed of -0.2 m s^{-1} when they should have used an angular speed of -0.25 rad s^{-1}.

Overview – Probability & Statistics

This was the last January sitting for these units. As usual the standards were high on S2 and S3, and in all three units many candidates were able to answer standard numerical questions well. Answers to hypothesis test questions are also improving.

There is some concern about candidates who use calculators with a wide range of statistical functions. Such calculators are of course permitted in examinations, but their use does not excuse candidates from the duty of showing full working. Candidates who write down an answer obtained from a calculator without showing the level of working that would be expected from a candidate without such a calculator (for instance, the appropriate standardisation formula in S2) risk losing several marks if their answer is wrong, even if it is nearly right.

The attention of Centres is particularly drawn to the desirability of explicit teaching of the use of the Formulae Book MF1 (see S1) and to the misunderstandings concerning the nature of the variables in a probability density function (see S2 and S3).

4732 Probability & Statistics 1

General comments

There were a few questions that were found more difficult than might have been expected, particularly questions 4 and 6. In addition to this, the question on Spearman's rank correlation coefficient (7) was not the usual "turn the handle" calculation but required some thought. As a result fewer candidates than usual scored high marks overall. The questions that required an answer given in words were fairly well attempted, except for question 7(ii) which required logical thought rather than interpretation.

A few candidates lost marks by premature rounding (eg in questions 3 and 5) or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Question 7(i) required some elementary algebraic manipulation which some candidates were unable to cope with.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formulae booklet, MF1, was useful in questions 3, 5 (for binomial tables) and 7. In question 3 a few candidates quoted their own (usually incorrect) formulae for b and/or r, rather than using the ones from MF1. Some thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Others tried to use the less convenient versions, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}}$ and $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1, despite the fact that the individual data values are not given in this question. These candidates completely misunderstood the formulae, interpreting them as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{\sqrt{\{(\Sigma x-\bar{x})^2\}\{(\Sigma y-\bar{y})^2\}}}$ and $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$.

In question 7(i), Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1-\frac{6\times\Sigma d^2}{n}$ or $1-\frac{6\times\Sigma d^2}{n^2(n-1)}$.

In question 5, some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formulae booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on individual questions

1) (i) Many candidates argued in a circle, using the given answer $P(X = 6) = \frac{3}{10}$ to find k and then using this value of k to derive the answer $\frac{3}{10}$. Others simply verified that if $P(X = 6) = \frac{3}{10}$, and the other probabilities are in proportion, then their sum is 1. Neither of these methods scored any marks. Many started with $\frac{1}{4}$ and continued with $\frac{1}{4}k = \frac{3}{10} \Rightarrow k = \frac{6}{5} \Rightarrow P(X = 6)$ $= \frac{6}{5} \times \frac{1}{4} = \frac{3}{10}$. A few found $\frac{1}{3}(1 - \frac{3}{10}) = \frac{7}{30}$ as the probability for each of the other three values.

(ii) Most candidates answered this standard question well. Incorrect probabilities from (i) were allowed, so long as their sum was 1. In the calculation of $\mathrm{Var}(X)$ a few candidates omitted to subtract $(E(X))^2$ and others subtracted $E(X)$. The usual error of dividing by 4 was occasionally seen. A few candidates assumed that the probabilities were each 0.25. Some used a formula for the mean and/or variance of the binomial distribution or the geometric distribution.

2) (i) Most candidates answered this part correctly. A few omitted the probability of success at either the first attempt or the third attempt. Others thought that the probability of success at the third attempt was $\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$ instead of $\frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$. Only a few chose the more elegant method using the complement.

(ii) Many good answers were seen. Some candidates appeared not to understand the difference between P(he passes on the 2nd attempt) and P(he passes on the 2nd attempt, given that he failed on the first), giving an answer of 0.58 – 0.4 = 0.18. A few formed an equation, but with the term "0.4*p*" instead of "0.6*p*". Some found the correct answer of 0.3 but then unnecessarily continued by using the formula $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6 \times 0.3}{0.6} = 0.3$. Others, having found the correct value of 0.3, continued with $0.6 \times 0.3 = 0.18$. If candidates did not make clear that this last line was only a check, rather than an answer, they were likely to lose the final mark. Many candidates gave incorrect attempts based on misunderstandings of conditional probability, such as $0.4 \div 0.58 = 0.690$.

3) (i) This part was answered very well on the whole. A few candidates made a sign error when substituting b (which is negative) in order to find a. Some simply lost the minus sign in b. Some found b and stopped. Others found r instead of what was asked. A few found r and then used this as their value of b.

(ii) Many candidates ignored the instruction to "use the regression line in the diagram" and used their equation from (i). A few candidates misinterpreted the situation, giving an answer such as 71 000.

(iii) Most candidates gave one correct answer, using the word "extrapolation" or some equivalent wording. However, many either gave no second reason or gave one that was, in effect, equivalent to their first reason (eg "The IMR will become negative"). Some candidates gave the valid second reason, namely that the diagram does not show good linear correlation.

(iv) This part was answered well by most candidates. A few used the original totals, just changing the value of n from 6 to 7. Others made the opposite error, finding new totals, but with $n = 6$. Sensibly, most wrote down their new totals such as Σxy, but some were incorrect and, without any indication of method, these lost a method mark.

(v) A few candidates thought that r would decrease because the values used in the formula would decrease, but most stated correctly that r would be unchanged.

4) (i)(a) Most candidates answered this part correctly.

(i)(b) Some candidates answered this very simple part correctly, although some used complicated (though correct) methods such as considering all possible cases: with 3 repetitions or 2 or none ($3 + {}^3C_2 \times 2 \times 3 + 3!$). Others used all manner of incorrect methods, such as ${}^3P_3$, ${}^9P_3$, $3! \times 3!$, $3! \times 3! \times 3!$, $3! \times 2+3 + 3 \times 2 + 3 \times 3$ and ${}^2P_2 \times 3 + {}^3P_3 + 3$.

(i)(c) A few candidates recognised that this was simply (i)(b) – 3 and gained both marks even if (i)(b) had been incorrect. Most candidates, however, did not use this method but started from scratch, giving either correct methods such as $3! + 6 \times 3$ or, more commonly, incorrect methods such as $3! \times 3! \times 2$, ${}^6C_3$, ${}^6P_3$, $6! \div 3!$, $5!$, $6 + 9$, $3! \times 3! \div 2!$, $3! \times 2$ and $2 \times 3! + 2 \times 3! + 2 \times 3! + 6$.

(ii) Very few candidates scored marks on this question. This is surprising in the light of the explicit phrase in the specification "solve problems about arrangements . . . . including those involving repetition". Only a few candidates correctly found the number of numbers with one pair of repeated digits ($\frac{4!}{2!} \times 3$). Even fewer found the number of numbers with two pairs of repeated digits ($\frac{4!}{2!2!} \times 3$). Some candidates attempted to list numbers in categories, e.g. those beginning with 1 or those containing 1 and 2, but the lists were frequently incomplete or included repetitions. A common incorrect method was $3 \times 3 \times 2 \times 2$ which fails to distinguish between cases in which the first two digits are the same and those in which they are different. Out of the vast number of incorrect methods seen, some examples are these: $3 \times 3 \times 2$, $3! \times 4 \times 2$, ${}^4P \times {}^3C_2$, $3^4 - 4 \times 3 \times 3$, $2 \times 4 + 2 \times 4 + 2 \times 4$, $4! \times 3! \div 12$ and $4! \times 3! \times 3! - 3! \times 3! \times 3! - 3! \times 23 - 3! \times 2^3$. Some incorrect methods resulted in answers in the millions.

5) (i)(a) This part was answered correctly by almost all candidates. A few unnecessarily attempted to use Σxp, sometimes successfully.

(i)(b) Most candidates answered this correctly, using either the tables or the formula. A few just read 0.8965 from the table without subtracting 0.6328.

(ii) Some correct solutions were seen. A small number of candidates gave an elegant method, using B(10, 0.25). However, most candidates gave partially correct solutions. Common such attempts were those which omitted $(P(X = 0))^2$ or omitted one case of P(sum = 0)×(P(sum = 1) or doubled $(P(X = 0))^2$ as well as doubling P(sum = 0)×(P(sum = 1). Some candidates found P(sum < 2) or P(sum = 2). Others just added $P(X = 0) + P(X = 1)$.

(iii) Some candidates used B(5, 0.25). Others understood that a different value of p was required, but chose the answer to (ii) instead of (i) for that value. Others misread the question to read "10 such sums are chosen . . ." and found P(exactly three values of $(X_1 + X_2)$ are 2).

6) (i) This question gave rise to many different approaches, only some of which were valid. Many candidates appreciated the need to find the total area, but many did not appear to understand what units they were using. Some used cm^2 or small squares but others took the scale from the x–axis and assumed a scale for the y–axis. In many cases it appeared that candidates were not aware that they were actually choosing a scale in their calculation. Marking was generous and many candidates scored two marks for attempting to find the total area (in any units) and relating this correctly to the total frequency of 800. However, because of the muddle over scales, few knew how to take the final step and find a. Some candidates considered only one block without considering the total area of the histogram. Others only considered the heights of the blocks rather than their areas. Many candidates used the range (80) and found $800 \div 80$, which led to an incorrect method in almost every case.

(ii) Few fully correct answers were seen. Most candidates found half the total area or frequency. Many identified the correct class (50–56) but some of these just gave the midpoint of this class as the median. Others tried to find exactly where in the class the median was situated, but only some of these could handle the necessary proportion calculation. A few candidates found the mid–point of the range, giving an answer of 60.

7) (i) Most candidates correctly attempted to form an equation by equating the formula for r_s to $\frac{63}{65}$. Many did not know what to do with the Σd^2. Some substituted $\Sigma d^2 = 6$ (or 8) by looking at the table. Others recognised $d = 1$ but thought that $\Sigma d^2 = 1$. Some understood that $d = 1$ for n pairs and while a few put $\Sigma d^2 = 1^n = 1$, many found the correct value of $\Sigma d^2 = n$. From then on the problem was one of algebra. Some cancelled the n immediately and went on to solve the equation correctly. Others did not cancel and found themselves faced with a cubic equation. Some cancelled at that stage although some did so incorrectly. Many algebraic errors crept in.

A few candidates simply tried a few values of n and generally found the correct value in the end.

(ii)(a) Many candidates answered this part correctly, often referring to a straight line or to perfect correlation. A few thought that the statement is false because r and r_s are independent of each other.

(ii)(b) Candidates struggled to give a convincing explanation here. Some unwisely ignored the instruction to "use a diagram" thus compounding their difficulties. A few clearly understood the point but drew a diagram in which one point was out of position, thus invalidating their argument. Others drew a diagram with $r » –1$ and $r_s = –1$. Some drew incorrect diagrams, showing apparently randomly scattered points, which they claimed showed cases in which $r_s = 1$ but $r \neq 1$. Some gave separate diagrams for r and r_s. Some candidates thought that the statement is true for a reason such as "r_s is r for ranks".

8) (i)(a) Most candidates answered this correctly, although a few gave $0.9^5 \times 0.1$.

(i)(b) Geometric distribution questions involving "before" or "after" often cause problems. Candidates are confused as to whether a "1 –" is needed. Others think that since it is a geometric situation, "$\times\ p$" must be included. Also sometimes there is confusion over the power. In fact most candidates answered this question correctly, with a few giving 0.9^4 or $1 – 0.9^5$ or $0.9^5 \times 0.1$. Some used the long method (ie the complement method), but (as usual) a few of these omitted a term or added an extra term.

(i)(c) Only a few candidates used the simplest method which involves SS, FSS, SFS. Few candidates answered this question totally correctly although many gave partially correct answers. Some gave only $0.1^2 \times 0.9$. Many gave $3 \times 0.1^2 \times 0.9$ but omitted $+ 0.1^3$. Many included terms such as 0.1×0.9^2. Some used the complement method, but most of these only gave $1 – 0.9^3$, omitting to subtract $3 \times 0.9^2 \times 0.1$ also.

(ii)(a) This question was well answered by most candidates. A few misread and thought Jill went first. Others included success for the wrong girl or for both girls.

(ii)(b) Many candidates were confused as to how many failures were necessary for each girl. Others included success for the wrong girl or for both girls.

4733 Probability & Statistics 2

General comments

This was a challenging paper in several ways, but there were many candidates who were able to display a very good ability to handle techniques and who could understand most of the finer points involved. The proportion of correct continuity corrections was gratifyingly high. Answers to hypothesis tests were of a high standard, with most candidates stating hypotheses and conclusions carefully and correctly.

Candidates who rely on calculators, rather than tables, for probabilities need to ensure that they show full details of working. As this paper does not assume the use of calculators with an inbuilt normal probability function, it is expected that all candidates show the standardisation of normal variables. Candidates who merely say, for instance, P(> 29.98) = 0.3274 risk losing *all* subsequent marks (even as many as 5) if their answer is wrong – even if it is close. The same is true for calculations involving the Poisson formula.

As mentioned in previous reports, modelling assumptions for the Poisson distribution continue to be poorly understood, and the same is plainly true for the concept of the probability density function.

Comments on individual questions

1) Most knew what to do, though many lost marks by failure to spell out the answers for the critical region and the significance level. Some wrongly attempted a right-hand tail.

2) (i) Almost everyone scored full marks here.

(ii) Many got this right, but almost as many used a standard deviation of $\sigma/\sqrt{10}$, which is wrong.

3) (i) Many scored full marks here, but some failed to give sufficient detail. DVDs should be numbered “sequentially”, or “from 1 to 9000”, or similar; it should be stated that numbers falling outside the range 1 to 9000 are rejected; and “select numbers randomly” is not an acceptable alternative to “select using random numbers”.

3) (ii) This was invariably answered well, with a very large number of candidates choosing the correct continuity correction.

4) (i) This simple request revealed many misunderstandings. It is clear that the letter x had no meaning in this context for many candidates. The correct answer is that x represents a value, or values, taken by the random variable X.

(ii) Almost always correct.

(iii) Generally done very well, but some failed to square the mean (even when they had written down a formula involving μ^2), and some omitted it altogether. Examiners find that candidates who try to use a single formula, involving the subtraction of the square of an integral, are more likely to get the wrong answer; this formula seems to be too complicated for the majority of candidates.

A number of candidates failed to realise that the final answer had to be given in terms of a only, and not k.

5) (i) Generally very well done. The continuity correction was very often correct; with these numbers it makes little difference, but it should still be included.

(ii) Full marks here were common, either by using the intended Poisson approximation (as used by a large majority) or the exact binomial.

6) (i) This was generally very well handled. Few confused $\bar{x}$ and m, or p and z. The most common mistake was to take 12 as the standard deviation rather than the variance.

It was also pleasing that so many candidates realised that the necessary assumption was that the standard deviation remained the same. Those who said "all other factors must remain the same" also gained full credit.

A continuity correction (of –1/60) is correct in this context, but those who did not use it (the vast majority) were not penalised. Almost nobody attempted to use the total rather than the average score.

Candidates whose final conclusion was that "there was significant evidence that Gordon's mean score had not improved" lost the final mark.

(ii) There remains confusion between whether use of the CLT is *necessary* (because you are not told that the parent distribution is normal) and whether it is *possible* (as n is large). It is baffling that so many candidates answer that the parent distribution was known to be normal, when there had been no mention of the normal distribution.

Some candidates continue to have a wrong idea of what the CLT says, often seeming to think that the CLT refers to the n divisor in the standard error.

7) (i) Most candidates could get under way with this, correctly using z-values. Those whose calculators gave them answers to more than 4 SF made life very difficult for themselves; it was expected that the tabular values $z = 2$ and -1 would be used, although of course a final answer such as s = 5.0003Ön would gain full marks.

Examiners continue to feel that candidates would benefit from being taught how best to set out, and solve, two equations of this sort, especially as this is an absolutely standard type of question. Far too many make very heavy weather of substitution, when elimination is far preferable.

(ii) This question caused many candidates to think hard. Although there are three unknowns, only two probabilities are needed. Some could see this at once; many attempted to use Aidan's view and abandoned it only when they found that it didn't work.

8) (i) Very few candidates knew what the word "random" means. It simply means "not following a predictable pattern", and does not imply either "independent" or "constant average rate". (The term "random *sample*" does have extra implications of this sort, but that is not the term used here.)

(ii) As usual this question was poorly answered. Most candidates rightly said that traffic light failures had to be independent, and could get a further mark either by saying that it would be true because traffic lights were not on the same circuit, or that it wouldn't because they were – or similar arguments. Examiners were not testing candidates' knowledge of traffic lights but of their understanding of the assumptions.

The second condition is that failures must occur at constant average rate. Many omitted the word "average", and inadvertently showed the necessity for it by saying things like "it will not hold as you will not have exactly the same number of failures in each hour". Many candidates wrote comments that addressed independence rather than constant average rate, such as "in a storm there would be more failures". Further, the issue is whether the average rate is constant within one day, and not from day to day, so it was wrong to say "it wouldn't happen because failures would be more common in winter".

As has been repeatedly stated in these Reports, "singly" should not be used as a condition (despite its appearance in textbooks). This context shows clearly that if it is not part of the "independence" condition it is meaningless, as the probability that two lights fail at *exactly* the same instant is vanishingly small. As this is inherent in the scenario, it is wrong to state it as an assumption.

(iii) Pleasingly, this question was found very straightforward by many candidates, and full marks were common. Some made life exceptionally difficult for themselves by not cancelling the factor of $\mathrm{e}^{-/}$, often taking logs or attempting to apply the laws of indices, and almost always incorrectly.

9) (i) Almost always right.

(ii) Often right, though some used 1 – P(³ 12) rather than 1 – P(£ 13), and others failed to subtract from 1.

(iii) This was a challenging final question. Only one or two candidates saw the short method: the only way that $p = 0.5$ for the second test is if there is a Type II error on the first test. Most attempted to divide the scenario up into separate cases, and a pleasingly large number of correct answers were seen, although too many considered only two cases instead of three or four, and many could not deal with the factors of 0.2 and 0.8.

4734 Probability & Statistics 3

General comments

There were 43 candidates, roughly the same as other January sessions. 5 candidates scored 70 or more out of 72. 27 candidates scored 50 or more.

Most candidates have learned to give their conclusion to significance tests in context, and not to be over-assertive.
Most candidates used the normal distribution correctly in questions 1(ii) and 7(iii).

Comments on individual questions

1) (i) Most candidates answered the first part correctly. In the second part, a few made the common error of saying Var(S) = 25Var(X) + 4Var(Y) + 362 = 408, but most answered correctly.

(ii) Several candidates failed to recognise that X is a continuous distribution and made the error $P(X \geq 2) = 1 - P(X \leq 1)$.

2) Usually well-answered, but a few lost marks for premature rounding. A few candidates did not use a paired t-test and were able to score a maximum of 2 marks.

3) (i) Many candidates scored full marks. Of those who did not, many failed to obtain a pooled estimate of variance and/or used an incorrect value of t, or even a z-value.

(ii) Most knew that the population variances should be equal, but some did not use the word population, or mention 'scores' or 'schemes' in their answers and thus did not gain this mark.

4) (i) Most candidates gained most of the marks in this part. The most common error was obtaining an incorrect range for y. In recent series candidates have learned to answer this type of question well, understanding the difference between Y and y.

(ii) Those who tried to evaluate $\int_0^1 \frac{3}{2\sqrt{x}}dx$ usually scored better than those using $\int y^2 g(y)\,dy$, many having the wrong limits obtained in (i).

5) (i) As usual, most candidates answered the question on confidence limits for a proportion well.

(ii) Most gave an acceptable answer to this part, realising that consideration of a large number of intervals was necessary.

(iii) Most candidates obtained the correct answer to this part. A few used an incorrect z value.

6) Well answered by most candidates. About one-quarter of the candidates did not recognise that a χ^2 test was necessary and tried to carry out various tests based on proportions. Most of these scored no marks.

7) (i) Most answered this part correctly. Of those who did not, some used an incorrect z value, others used $\frac{1}{16}$ or 1 instead of $\frac{1}{4}$.

(ii) More candidates obtained the correct variance than the correct mean. Common errors were $\mu - \mu$ and $\mu_x - \mu_y$.

(iii) This was the most difficult question on the paper. Many considered $\bar{x} + 1.163 < \bar{y} + 1.234$ and the equivalent statement with a – sign, or only one inequality.

8) (i) Most scored well on this part. Some did not use Yates' correction, others did not take the modulus of the difference between *O* and *E* before subtracting 0.5.

(ii) Many candidates did not pool the samples. A special ruling allowing 5/7 was in place for these candidates. Otherwise a good discriminator, better candidates scoring at least 6/7 weaker ones scoring from 0 to 2.

4736 Decision Mathematics 1

General comments

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the Printed Answer Book. Those candidates who used extension sheets usually labelled their answers to show which part of which question the work referred to. The quality of written answers was rather better than in previous sessions.

Comments on individual questions

1) **(i)** Most candidates carried out the shuttle sort correctly and recorded the comparisons and swaps accurately. A few candidates used bubble sort or sorted into increasing order and one or two shuttled from the right hand end of the list. The comparisons and swaps should be recorded in figures, not as tally marks. Inevitably there were a few misreads and some candidates who miscopied their own values or lost values in the course of carrying out the algorithm.

(ii) Generally answered correctly, with the sacks appearing in the correct boxes in the correct order.

(iii) Several candidates were able to answer this appropriately, often by first trying $W = 128/4 = 32$ and then realising why it had to be 33. Those who started with an arbitrary higher value often suggested $W = 35$, not realising that the 4^{th} and 5^{th} weights (in decreasing order) could be packed together.

2) **(i)** Most candidates were able to draw suitable graphs, often these preserved the shape of the original tetromino, although any topologically equivalent graphs (in the correct answer spaces) were acceptable.

(ii) Many candidates thought that (1) was not possible, this seemed to have arisen because they had assumed that the vertices corresponded to the centres of the faces in the relative positions as drawn. Graph (1) could, however, have represented a line of squares, as in picture (A) in the diagram for part (i), for example. It was generally appreciated that (2) and (4) did represent tetrominoes while (3), (5) and (6) did not. The explanations of why these did not represent tetrominoes often referred back to the mistaken idea that the vertices showed the relative positions of the centres of the faces.

(iii) This question asked candidates to identify a tetromino from the ones shown. They needed to choose a letter A to D, from the first set of diagrams, or possibly draw a picture of the tetromino that they had chosen.
In part (a), C had a graph that was not a tree. In part (b), A and D had graphs that were the same tree structure.

(iv) There were several candidates who found two or three of the appropriate graphs and some who found all four. Some candidates repeated graphs they had already drawn but with a different orientation, for example the graph that looks like W and the graph that looks like L are the same. Some candidates carefully drew the four pentominoes with distinct graphs but did not actually draw the graphs.

3) **(i)** Nearly all the candidates were able to apply Dijkstra's algorithm accurately to find the required path and its weight.

(ii) There were several correct answers. Some candidates forgot to subtract the weight of the arc that had been removed from the given total. A few candidates started from scratch and wrote down a route and then added up its weight, these usually made a slip at some point.

(iii) Again, there were several correct answers. Some candidates used 230 again as the total weight, and some assumed that the least weight route from *A* to *G* was the same as in part (ii). The point here was that the route of weight 41 was no longer available and an alternative route had to be found, such a route must either start *AD* or *AF* and must then use the least weight route to get from there to *G*, this essentially meant checking just the possibilities: *ADEG*, *ADFG* and *AFG*, of these *AFG* = 47 is the best.

4) (i) The question had asked for a cycle (a closed path). Many candidates were able to apply the nearest neighbour method correctly to get a path from Pam through all the other employees, but they did not always close the cycle by returning to Pam at the end. Some candidates only showed their working on the table and did not actually write down their cycle (or their incomplete cycle), where the order of selection was apparent candidates were given credit for the method, but to achieve full marks the cycle needed to be written down: PBAGCFEDG.

(ii) There were several requests in this part, and some candidates did not do all of them, they were asked to apply Prim's algorithm to the table (starting by crossing out the row for P), to list the arcs in the order they were chosen (note, this asked for arcs not vertices), to draw the spanning tree and to write down its weight.

(iii) The lower bound was for Pam and the seven employees and candidates were asked to start by finding a mst with Gita removed. Some candidates removed Pam, or sometimes Alan, instead of Gita. Other candidates constructed the mst on the reduced network but then did not reinsert Gita by using the two least weight arcs from G.

(iv) The candidates who had found the correct minimum spanning tree usually realised who was in which team. Sometimes they just redrew the tree without any indication of how this related to the teams. The minimum elapsed time to contact all employees should have been 40 minutes, this is achieved by Pam contacting Caz, then while Caz rings each of her team, Pam can contact Bob and Bob can contact each of his team. There was no requirement in this part for anyone to ring back to Pam at the end.

5) (i) The profit was £1 on each box of cupcakes, so the expression to be maximised was $x + y + z$. Several candidates just picked out the values 24, 20 and 12 from the stem. This would have led to an appropriate expression if the profit had been £1 on each cupcake (although this seems a little excessive).

(ii) Most candidates were able to convincingly achieve the given expression, a few played around with the given values in a less than convincing way. The best answers were those that included both numerical values and some written explanation.

(iii) Many candidates were able to get the second constraint as $48x + 60y + 48z \leq 3000$ and then reduce this to $4x + 5y + 4z \leq 250$. Some candidates achieved the constraint with 3000 on the rhs but then only divided by 4.

(iv) The tableau should have 3 rows and 7 columns. Some candidates omitted the *P* column, the Simplex tableau is a shorthand for a set of matrix operations really and so the *P* column does need to be included.

(v) The question asked candidates to choose a pivot from the *x*-column, despite this some chose to use the *y*-column. Candidates should show the values of the ratios, 69.44 and 62.5, rather than giving a general description of the method. Having chosen the entry on which to pivot, the row operation was required for each row, including the objective row and the pivot row. The operations could be given in abbreviated forms, for example R2-18pr, provided that (for the rows other than the pivot) they were of the form current row $\pm$ appropriate multiple of (original or new) pivot row. The numerical work of most candidates was accurate.

(vi) Candidates were asked to write down the values that their first iteration gave for x, y and z. This was a simple case of reading off from a tableau, there was no need for simultaneous equations. Some candidates did not know how to interpret the tableau and gave the coefficients from the objective row. A small, but not insignificant, number of candidates claimed that they still had x=0, y= 0, z=0.
Candidates should have achieved x=62.5, y=0, z=0; as only complete boxes could be sold, the maximum profit was £62, this meant that as well as excess topping there were also decorations left over. Some candidates were able to calculate these excess amounts, usually by working out how much topping and how many decorations had been used in making 62 boxes each containing 24 miniature cupcakes. The candidates who used the slack values generally did not scale the constraints back up to 5000 and 3000, and rarely allowed for the half box that could not be sold.

(vii) Some candidates were able to achieve full marks on this part, although fully correct solutions were rare. By rearranging $P = x+y+z$, the variable x can be eliminated from the second constraint to give an inequality involving just P and y. For P to be as large as possible we try setting $P = 62$, the values of x, y and z then follow.

4737 Decision Mathematics 2

General comments

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the Printed Answer Book. The quality of written answers was substantially better than in previous series.

Comments on individual questions

1) (i) The bipartite graph was usually correct, a few candidates missed out a single arc, often the arc joining *J* to *B*.

(ii) Many candidates found the alternating path *D-F-A-I* and drew the corresponding incomplete matching. Some candidates found a longer alternating path, such as *D-F-A-G-B-J*, this only got 1 mark because the question had asked for the shortest alternating path.

(iii) Most candidates were able to construct the complete matching, even if they had not found the alternating path in part (ii).

(iv) Candidates were usually able to explain why there is just one complete matching, generally they started at either *D* or *J* and followed the logic around the diagram from there.

2) (i) Candidates were able to complete the immediate predecessors.

(ii) Most candidates were able to complete the passes, including dealing with the dummy activities.

(iii) Candidates generally realised that *B* and *E* had 2 minutes of float and could be delayed without affecting the project completion time.

(iv) Here the delay was on a critical activity so it caused a 2 minute delay in the project completion time, as would a similar delay on any other critical activity.

(v) Some candidates assumed that a delay in *C* would have no effect on the project completion time as *C* is not a critical activity, however most candidates realised that there was only 1 minute of float on *C* and the other minute would be a delay in the completion time.

3) (i) There were very few arithmetic errors in subtracting the given values from 10, a few candidates subtracted 10 from each value and did not seem concerned that they had achieved negative values.

(ii) This was done well. Some candidates did not explain how the various tables were formed and some went into far too much detail, all that was needed was phrases like ‘reduce rows’, ‘augment by 2’, etc.

(iii) Some candidates assumed that the values found in part (i) must be the guesses, this is not what the question says however. Starting from the original table and deleting the row for L and the column for R enables an allocation with 23 guesses to be found, this cannot be bettered since any allocation with 24 would require K and M to both be the nurse.

4) (i) The supersource was usually correctly added to feed the two sources with appropriate arc weights.

(ii) Many candidates just wrote down values with no evidence of working. Several candidates had clearly not dealt with the arcs in which the flow across the cut was from *T* to *S*.

(iii) As in part (ii), there was often little evidence of the method used. The arcs that join vertices from set *X* to vertices from set *Y* were S_1B, S_2C, *AD*, *EB* and *EC*. Some candidates using the graphical method tried to include *AE*, but this arc was cut twice, if it was cut at all, so it cancelled itself out.

(iv) Most candidates were able to show an appropriate flow.

(v) Some candidates gave the largest individual arc capacity on the route and others gave the lowest flow in any of the arcs on the route. Most candidates were able to state that the flow of 13 could be augmented by another 6+7=13.

(vi) The maximum flow was 30, several candidates claimed 26 but this was usually because they had assumed that the flow in arc *DE* was from *D* to *E*. Undirected arcs may be dealt with by treating them as a pair of directed arcs. An example of a feasible flow of 30 litres per second is obtained by sending 9 along S_2CET, 2 along S_1BCET, 8 along S_1BET, 1 along S_1AET, 4 along S_1BEDT and 6 along S_1ADT.

5) (i) Most candidates answered this correctly, giving both the value 4 and the choice of the diamond card.

(ii) Candidates usually understood that the square card dominated over the diamond card for Rose, in the sense that she won more points by playing square than by playing diamond for each of Colin's choices. Most candidates were able to say that Colin should not play the triangle card and write out the reduced table. The question had made three requests and candidates needed to explicitly answer all three of these to get the marks in this part.

(iii) Most candidates understood what they needed to do here, although there were some arithmetic errors. Colin should have ended up with two play-safe strategies and candidates needed to write down both of these. Candidates should show the numerical values when demonstrating that the game is unstable.

(iv) Many candidates were able to explain that 6 had been added throughout the original matrix to make the entries non-negative, then the square column had been used to give the expression in the question.

(v) The value of *m* was usually correct and most candidates subtracted 6 to get the optimal value for *M*. In this particular case, all three expressions gave $m \leq 149/24$, it would be more usual to have at least two different values and then the smallest value would be the appropriate choice.

6) (i) Candidates often understood that this referred to there being no houses stored overnight on Thursday night, and could usually explain why there must be at least one house stored. A few candidates thought that it referred to Friday night.

(ii) There were several completely correct answers to this part, although other candidates were clearly very confused about what was happening.

(iii) The dynamic programming should have started at stage 4, some candidates tried to show (5; 0), but nothing has happened until there is travel from (5; 0) to (4; 1) or (4; 2).

(iv) Many candidates were able to find the appropriate production plan.

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