OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS

4726 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Express $\operatorname{sech} 2x$ in terms of exponentials and hence, by using the substitution $u = e^{2x}$, find $\int \operatorname{sech} 2x \, dx$. **[5]**

2 A curve has polar equation $r = \cos\theta \sin 2\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. Find

(i) the equations of the tangents at the pole, **[2]**

(ii) the maximum value of r, **[4]**

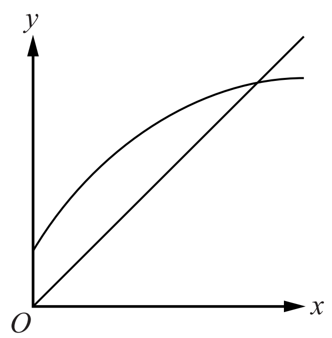
(iii) a cartesian equation of the curve, in a form not involving fractions. **[3]**

3 **(i)** By quoting results given in the List of Formulae (MF1), prove that $\tanh 2x \equiv \dfrac{2\tanh x}{1 + \tanh^2 x}$. **[2]**

(ii) Solve the equation $5\tanh 2x = 1 + 6\tanh x$, giving your answers in logarithmic form. **[6]**

4 It is given that the equation $x^4 - 2x - 1 = 0$ has only one positive root, α, and $1.3 < \alpha < 1.5$.

(i)



The diagram shows a sketch of $y = x$ and $y = \sqrt[4]{2x + 1}$ for $x \geqslant 0$. Use the iteration $x_{n+1} = \sqrt[4]{2x_n + 1}$ with $x_1 = 1.35$ to find x_2 and x_3, correct to 4 decimal places. On the copy of the diagram show how the iteration converges to α. **[3]**

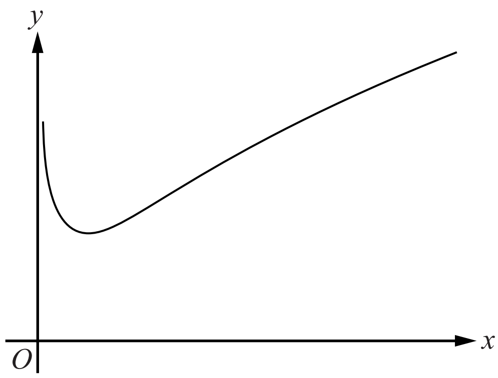
(ii) For the same equation, the iteration $x_{n+1} = \frac{1}{2}\left(x_n^4 - 1\right)$ with $x_1 = 1.35$ gives $x_2 = 1.1608$ and $x_3 = 0.4077$, correct to 4 decimal places. Draw a sketch of $y = x$ and $y = \frac{1}{2}\left(x^4 - 1\right)$ for $x \geqslant 0$, and show how this iteration does not converge to α. **[2]**

(iii) Find the positive root of the equation $x^4 - 2x - 1 = 0$ by using the Newton-Raphson method with $x_1 = 1.35$, giving the root correct to 4 decimal places. **[4]**

5 A function is defined by $f(x) = \sinh^{-1} x + \sinh^{-1}\left(\frac{1}{x}\right)$, for $x \neq 0$.

(i) When $x > 0$, show that the value of $f(x)$ for which $f'(x) = 0$ is $2\ln\left(1 + \sqrt{2}\right)$. **[5]**

(ii)



The diagram shows the graph of $y = f(x)$ for $x > 0$. Sketch the graph of $y = f(x)$ for $x < 0$ and state the range of values that $f(x)$ can take for $x \neq 0$. **[3]**

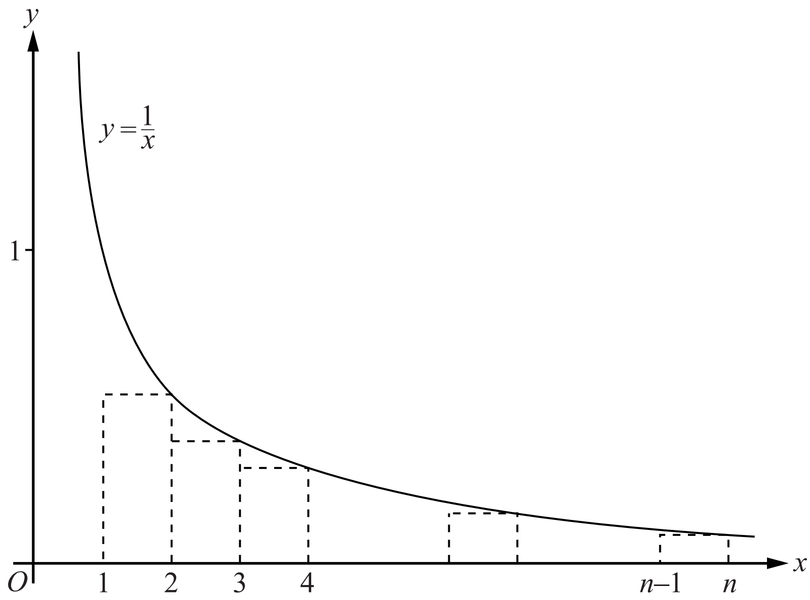
6 It is given that, for non-negative integers n,

$$I_n = \int_0^{\pi} x^n \sin x \, dx.$$

(i) Prove that, for $n \geqslant 2$, $I_n = \pi^n - n(n-1)I_{n-2}$. **[5]**

(ii) Find I_5 in terms of π. **[4]**

7



The diagram shows the curve $y = \frac{1}{x}$ for $x > 0$ and a set of $(n - 1)$ rectangles of unit width below the curve. These rectangles can be used to obtain an inequality of the form

$$\frac{1}{a} + \frac{1}{a+1} + \frac{1}{a+2} + \ldots + \frac{1}{b} < \int_1^n \frac{1}{x}\,dx.$$

Another set of rectangles can be used similarly to obtain

$$\int_1^n \frac{1}{x}\,dx < \frac{1}{c} + \frac{1}{c+1} + \frac{1}{c+2} + \ldots + \frac{1}{d}.$$

(i) Write down the values of the constants a and c, and express b and d in terms of n. **[3]**

The function f is defined by $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} - \ln n$, for positive integers n.

(ii) Use your answers to part **(i)** to obtain upper and lower bounds for $f(n)$. **[4]**

(iii) By using the first 2 terms of the Maclaurin series for $\ln(1 + x)$ show that, for large n,

$$f(n+1) - f(n) \approx -\frac{n-1}{2n^2(n+1)}.$$ **[5]**

8 The curve C_1 has equation $y = \frac{\mathrm{p}(x)}{\mathrm{q}(x)}$, where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are polynomials of degree 2 and 1 respectively. The asymptotes of the curve are $x = -2$ and $y = \frac{1}{2}x + 1$, and the curve passes through the point $\left(-1, \frac{17}{2}\right)$.

(i) Express the equation of C_1 in the form $y = \frac{\mathrm{p}(x)}{\mathrm{q}(x)}$. **[4]**

(ii) For the curve C_1, find the range of values that y can take. **[4]**

Another curve, C_2, has equation $y^2 = \frac{\mathrm{p}(x)}{\mathrm{q}(x)}$, where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are the polynomials found in part **(i)**.

(iii) It is given that C_2 intersects the line $y = \frac{1}{2}x + 1$ exactly once. Find the coordinates of the point of intersection. **[4]**

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THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.